Policy-Seeking Parties in a Parliamentary Democracy with Proportional Representation: A Valence-Uncertainty Model

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Abstract

We develop a unidimensional spatial model of multiparty parliamentary elections under Proportional Representation, in which policy-seeking parties project that the median parliamentary party will implement its policy position. The parties are assumed to be uncertain about the electoral impact of valence issues. The underlying assumptions of the model – which highlights the importance of the median party in parliament – are supported by theoretical work by Cho and Duggan and are consistent with empirical work by McDonald and Budge. Under these assumptions, we prove the existence of a Nash equilibrium under quite general concavity conditions and we derive a centripetal effects of valence result, that parties will slightly moderate their positions when they grow weaker along measured valence dimensions of evaluation. We report computations of party equilibria, and we contrast our model and its implications for policy-seeking parties with results on vote-seeking parties recently reported by Schofield and Sened.
1. Introduction

In the past decade scholars who analyze politicians’ policy strategies have become increasingly aware of the strategic importance of so-called valence dimensions of voters’ candidate and party evaluations. Valence dimensions, a term first coined by Stokes (1963, 1992), refer to dimensions “on which parties or leaders are differentiated not by what they advocate, but by the degree to which they are linked in the public’s mind with conditions, goals, or symbols of which almost everyone approves or disapproves” (1992, page 143). Valence dimensions include such factors as parties’ and party leaders’ images with respect to honesty, competence, charisma, and unity. These dimensions contrast with position dimensions such as tax policy, foreign policy, and debates over immigration controls and abortion policy, on which “parties or leaders are differentiated by their advocacy of alternative positions” (Stokes, 1992, p. 143).

Political parties that are widely viewed as competent, trustworthy, and united may enjoy election advantages that are not directly tied to the positions they stake out on positional dimensions, while parties with poor reputations along valence dimensions suffer electoral disadvantages.¹ There is extensive empirical research that confirms the crucial importance of valence dimensions in shaping election outcomes (Pierce, 2000; Johnston, 2000; Crewe and King, 1994; Clark, 2005).

Several recent spatial modeling studies have explored the implications of valence dimensions for parties’ and candidates’ strategies along positional dimensions, some in the context of two-party elections (Londregan and Romer, 1993; Macdonald and Rabinowitz, 1998; Ansolabehere and Snyder, 2000; Berger, Munger, and Potthoff, 1999; Groseclose, 2001; Adams, Merrill, and Grofman, 2005, chapters 11-12), and others in the context of multiparty elections (Schofield, 2003; Schofield and Sened, 2005a b, forthcoming). To date, however, we are unaware of any such multiparty studies that consider the positional strategies of policy-seeking parties, i.e., parties that seek office in order to implement their desired policies rather than proposing policies in order to win office. That is what we present here. Specifically, we develop a spatial model of multiparty competition under Proportional Representation in a parliamentary democracy, in which the political parties vary in terms of their valence-related attributes, and

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¹ In the literature on two-candidate elections – particularly those with links to U.S. Congressional elections – valence advantages are typically ascribed to incumbent candidates, who have been found empirically to be stronger than challengers in terms of name recognition, fund-raising ability, and established records for constituent service (see Fiorina, 1977).
where furthermore the parties are uncertain, at the time they select their policy positions, about what their valence images will ultimately be on Election Day.

A crucial feature of our model is that the parties believe that the median parliamentary party – i.e. the party that controls the median legislator along the unidimensional positional continuum – will dominate the post-election policy-making process. Our assumption of the primacy of the median legislator, which traces back to Duncan Black (1958), contrasts with alternative models of policy-making that emphasize the primacy of the parties formally invested in the government (Austen-Smith and Banks, 1988) or the policy primacy of the formateur, i.e. the party charged with forming the government (Baron, 1998; Diermeier and Feddersen, 1998). In Section 2 we discuss empirical findings by McDonald and Budge (2005) along with theoretical results by Cho and Duggan (2004), which suggest that our assumption of the policy primacy of the median parliamentary party may be reasonable for real world democracies.

We explore several questions about policy-seeking parties’ positional strategies in Proportional Representation elections, when parties project a dominant policy-making role for the median parliamentary party: Is this political context likely to support a stable configuration of party positional strategies, and if so what are the characteristics of such positional configurations? How do parties adjust their positions in response to ebbs and flows in their valence images?, and, How do our conclusions about policy-seeking parties’ strategies compare with results on parties’ vote-seeking strategies? Our study produces three central conclusions.

First, we show that a Nash equilibrium configuration of positional strategies exists given quite general assumptions about parties and voters.

Second, with respect to parties’ positional dynamics, we show – using a combination of theoretical and simulation methods – that when a policy-seeking party’s valence images deteriorates – which may occur due to scandals, weak leadership, or intra-party divisions – then such a party has strategic incentives to moderate its positions, an effect we label the centripetal valence effects result. Conversely, we show that parties with enhanced valence images are motivated to shift to more extreme positions.

Third, we report computations of equilibrium configurations, and we show that, for realistic model parameters, these equilibria resemble the actual configurations that we observe in real world party systems in Israel, Germany, Spain, and the Scandinavian countries. We also contrast our model and its implications for policy-seeking parties with results on vote-seeking parties recently reported by Schofield and his co-authors (Schofield, 2003, 2005; Schofield and
Sened, 2005a b, forthcoming). This discussion suggests that, counter-intuitively, policy-seeking and vote-seeking motivations can motivate similar sets of strategies in multiparty elections. However we outline an empirical test that potentially allows one to distinguish between vote- and policy-seeking motivations.

Our findings have interesting implications for spatial modeling, for party strategies, and for the connections between formal theory and behavioral voting research. With respect to spatial modeling, we show that the two-party models with valence dimensions and policy-seeking candidates developed by Londregan and Romer (1993) and Groseclose (2001) can, with appropriate adjustments, be extended to multiparty elections in parliamentary democracies. With respect to party strategies, numerical calculations suggest that under a range of assumptions, equilibrium positions are similar to the parties' preferred positions, although contracted toward the center. But we prove that parties with depressed valence attributes will slightly moderate their positions – a prediction that is contrary to predictions derived for vote-seeking parties contesting multiparty elections. Finally, and related to the previous point, our model highlights the fact that empirical voting research on valence issues is crucial for understanding party strategies. As we discuss below, behavioral researchers are divided about the electoral impact that valence dimensions exert in real world elections. Our results suggest that this debate has important implications for party strategies.

2. Policy-Seeking Parties in Multiparty Elections under Proportional Representation: A Valence-Uncertainty Model

Assumptions on seat allocations and policy outputs. We specify a model in which parties and voters locate along a one-dimensional positional continuum – which we label the Left-Right policy continuum – and each voter supports the party that she prefers based upon her evaluations of the parties’ policy proximities’ and their valence attributes, using a decision rule that we specify below. Unlike plurality voting systems where candidates are selected from single-member districts, Proportional Representation (PR) voting systems allocate seats in parliament in rough proportion to the parties’ vote shares. For simplicity we will assume that parliamentary seat share is exactly proportional to vote share, i.e., that the PR system is perfectly proportional.\(^2\) We also assume that policy outputs are determined entirely by the parliament, an as-

\(^2\) The degree to which real world PR systems approach perfect proportionality depends on several factors, notably district magnitude (i.e., the number of seats awarded per district) and the existence (or absence) of electoral
sumption that is best approximated in parliamentary democracies with unicameral legislatures. Among the parliamentary democracies that most closely approximate our model of unicameral policy dominance with a PR voting system are those of Israel, Spain, India, Italy, Austria, Portugal, and the Scandinavian countries. Finally, we specify that the number of seats in the parliament is odd, so that there exists a single location for the median legislator.

In order to specify policy-seeking utilities for the parties, we must specify how the parties’ policy positions and their parliamentary seat shares – which by assumption are equivalent to their vote shares – influence policy outputs. Here we assume that there are $K$ policy-seeking parties with preferred positions $R_1, \ldots, R_K$ and policy positions (strategies) $s_1, \ldots, s_K$. We define the median parliamentary party (MPP) as that party that, together with all the parties with policy positions to its left, can form a majority and that could also form a majority if, alternatively, it were combined with all the parties with policy positions to its right. We further assume that the parties project that following the election the MPP dominates the policy-making process, so that it will succeed in implementing its pre-election policy position (we justify this assumption below). Thus a party $k$’s utility $U_k$ for an election outcome is equivalent to its utility for the policy position of the MPP. Defining $f(s_j, R_k)$ as party $k$’s utility for party $j$’s policy position $s_j$, where $f(s_j, R_k)$ is assumed to be concave and to peak at $R_k$, it follows that $k$’s utility for an election outcome is:

Thresholds defined in terms of the minimum percentage of the national vote a party must win in order to guarantee parliamentary representation (see Taagepera and Shugert, 1989). Among the most perfectly proportional systems are those of Israel, the Netherlands, and the Scandinavian countries (see Lijphart, 1999, Appendix A).

3 Parliamentary democracies that feature unicameral legislatures include Denmark, Sweden, New Zealand, Greece, Israel, Finland, Luxembourg, and Portugal. In addition, many parliamentary democracies feature asymmetric bicameralism, i.e., they have two chambers but one of these is dominant. Examples include Norway, Austria, Britain, Canada, France, India, and Ireland (see Lijphart, 1999, Chapter 11).

4 Thus, if the $s_k$’s are ordered so that $s_1 \leq s_2 \leq \ldots \leq s_K$, then the MPP is that party $k_M$ such that parties $1, \ldots, k_M$ and parties $k_M, k_M + 1, \ldots, K$ each include a majority of the seats in parliament.

5 We say that a function $U$ is concave and peaks at $x_0$ if it is continuous, and if for all $x$ in the domain of $U$ for which $x \neq x_0$, $\frac{\partial^2 U}{\partial x^2}(x) \leq 0$ and $U(x_0) > U(x)$. Note that if $U$ is concave and peaks at $x_0$, then $U$ is strictly increasing on the left of $x_0$ and strictly decreasing on the right, i.e., if $x_1 < x_2 \leq x_0$, then $U(x_1) < U(x_2)$ and if $x_0 \leq x_1 < x_2$, then $U(x_1) > U(x_2)$.
Party $k$’s utility $= f(s_j, R_k)$ if party $j$ is the MPP.

**Assumptions on voters.** We assume that voters’ party evaluations depend on their evaluations of the parties’ policy positions, plus a valence component. Specifically, for each voter $i$ with policy preference $x_i$, the policy distance component of $i$’s evaluation of party $j$ is given as $ag(s_j, x_i)$, where $g(s_j, x_i)$ represents $i$’s utility for party $j$’s position $s_j$, and $a$ is a non-negative parameter denoting the salience of the policy dimension relative to the valence dimension. We assume that for each voter $i$, $g(s_j, x_i)$ is concave and peaks at $x_i$. We assume no abstention.

The valence component of voter $i$’s evaluation of party $j$ is assumed to be the same for all voters and has two components: the party’s measured valence characteristics, $V_j$, which the parties know at the time they select their policy strategies, and which we label the party’s valence image; and unmeasured characteristics $\varepsilon_j$, which the parties do not know at the time they choose their policy positions. Thus:

$$V_i \text{’s utility for party } j = ag(s_j, V_i) + V_j + \varepsilon_j.$$ (1)

Our distinction between the measured and unmeasured components of valence plausibly captures the information environment party elites confront as they devise their strategies. At the time that parties commit to their policy strategies – which is typically well in advance of the election – political elites are likely to have formed general impressions about the parties’ comparative valence images, based upon their contacts with constituents, public opinion polls, media coverage, and conversations with fellow elites. Such information forms the basis for the measured valence component $V_j$. At the same time, this wealth of information – which may point in conflicting directions – plausibly leaves elites uncertain of the parties’ precise valence-

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6 Parties in parliamentary democracies publish detailed policy programmes several weeks in advance of the election. These policy programmes, furthermore, usually hew closely to the policy positions that the party has staked out at its most recent annual party conference, which can be up to a year in advance of the election. These considerations suggest that parties’ policy strategies are largely determined a minimum of several weeks – and perhaps as long as several months – prior to the election.
related reputations. Moreover, elites are well aware that parties’ valence images can be significantly affected by the election campaign that follows the selection of party strategies, as well as by late-breaking political scandals or crises.\(^7\) The unmeasured valence component \(\varepsilon_j\) captures this uncertainty.

Note that our model specifies that voters prefer the party that offers the most attractive combination of policies and valence characteristics, so that a voter may prefer a party that is less attractive on pure policy grounds (relative to its competitors) if this party has strong valence-related characteristics along such dimensions as competence, integrity, and unity. We assume that all voters vote sincerely. The following remarks develop two important implications of our model, and also support our assumption of sincere voting:

**Remark 1.** When all individuals vote sincerely, then the party that is supported by the median voter will be the MPP.\(^8\)

**Remark 2.** The situation where all individuals vote sincerely is an equilibrium in voters’ utility-maximizing strategies, i.e., no voter can increase her utility by voting insincerely while all other voters vote sincerely.\(^9\)

\(^7\) Two striking recent examples of such phenomena occurred during the course of the German parliamentary election campaign in September, 2002, and the Spanish election campaign in March, 2004. In Germany the disciplined, forceful campaign waged by the SDP and its leader, Gerhard Schroeder, enhanced the party’s valence image and helped it achieve an unexpectedly strong election result. In Spain, the governing parties’ response to the Madrid train bombing – which occurred just days before the election – was widely believed to have harmed these parties’ reputations for competence and honesty, thereby contributing to their unexpectedly poor showings.

\(^8\) Denote by \(j_M\) the party supported by the median voter. Because, for each party, the voter utility \(g(s_j, x_i)\) declines as the voter position \(x_i\) recedes from the party position while the valence component \(V_j + \varepsilon_j\) is identical across voters, it follows that all voters located to the left of the median voter prefer party \(j_M\) to all parties whose policy positions lie to the right of \(j_M\), and vice versa. Hence party \(j_M\) is the MPP under sincere voting.

\(^9\) This equilibrium among voters is not to be confused with the equilibrium among parties that is the primary topic of this paper. Denote by \(j_M\) the MPP under sincere voting. Assume that all voters other than a focal voter are sincere. A focal voter who prefers \(j_M\) – which includes the median voter – cannot improve her utility by switching her vote. Second, a focal voter who prefers a party located to the left of \(j_M\) can only alter the identity of the MPP by switching her vote to a party located to the right of \(j_M\). However, given that 1) all voters who prefer a party located to the left of \(j_M\) must themselves be located to the left of the median voter’s position \(m\), and, 2) all
Finally, we assume that for each party $j$, the unknown component $\varepsilon_j$ of voters’ valence evaluations is selected independently over parties from a type 1 extreme value distribution. This assumption, which has been employed extensively both in empirical studies of voting behavior (Whitten and Palmer, 1996; Adams and Merrill, 1999; Schofield and Sened, 2005; Quinn and Martin, 2002) and in spatial models of multiparty competition (Merrill and Adams, 2001; Schofield and Sened, 2005a b), implies that voters’ choice probabilities can be represented via a logit function. Specifically, the probability $P_k$ that the median voter votes for party $k$ – which by Remark 1 is $k$’s probability of being the MPP – is given by the logit probability function\(^{10}\).

\[
P_k = \frac{\exp\left(\text{ag}(s_k, m) + V_k\right)}{\sum_{j=1}^{K} \exp\left(\text{ag}(s_j, m) + V_j\right)},
\]

and party $k$’s expected policy utility $U_k$ is

\[
U_k = \sum_{j=1}^{K} P_j f(s_j, R_k).\]

We note that our model of voting and policy outputs in multiparty parliamentary democracies represents a direct extension of Londregan and Romer’s (1993) two-candidate model. As with our model, the Londregan-Romer model posits that the candidate who is supported by the median voter implements his pre-election policy proposal, and that, from the candidates’ perspectives, the uncertainty over the election outcome revolves entirely around voters’ valence considerations – that is, the candidates know the voter distribution with certainty but are unsure about voters’ comparative evaluations of the candidates’ valence-related attrib-

\[\text{voters located to the left of the median voter prefer } j_M \text{ to all parties located right of } j_M \text{ (see footnote 9), it follows that no focal voter located to the left of } m \text{ can have an incentive to strategically switch her vote. A similar argument applies to a focal voter located to the right of } m.\]

\[\text{\^{10}}\text{ See Train (1986, Chapter 3) for a proof that the logit model implies choice probabilities of the functional form given by equation 2.}\]

\[\text{\^{11}}\text{ For example, under quadratic-loss utility for parties and voters, } U_k = -\sum_{j=1}^{K} P_j (s_j - R_k)^2, \text{ where }\]

\[
P_k = \frac{\exp\left(-a(s_k - m)^2 + V_k\right)}{\sum_{j=1}^{K} \exp\left(-a(s_j - m)^2 + V_j\right)}.
\]
utes. Also in common with our model, the Londregan-Romer model allows the candidates to differ in terms of their measured valence attributes (the \( V_j \) term in equation 1).

**Policy outputs and the median parliamentary party.** Our assumption that the MPP controls policy outputs contrasts with alternative models of policy-making in parliaments – both theoretical and empirical – which emphasize the policy primacy of the parties in the governing coalition (Powell, 2000; Huber and Powell, 1994; Austen-Smith and Banks, 1988); the central importance of the formateur, i.e., the party charged with forming the government (Baron, 1998; Diermeier and Feddersen, 1998); or the dominance of the party with jurisdiction over the relevant government ministry (Laver and Shepsle, 1996). Choosing between these competing models is difficult, because the empirical literature on the relationship between parties’ policy positions and government policy outputs is under-developed.\(^{13}\) McDonald and Budge (2005), however, report empirical results from a study of twenty-one postwar democracies – which is to our knowledge the only extensive, cross-national, study that analyzes the links between parties’ policy positions and government policy outputs.\(^{14}\) These authors analyze government policy outputs in three areas – central government spending, social spending, and international policy – and find more instances of statistically significant associations with the MPP assumption than with the position of the government or with that of the relevant government minis-

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\(^{12}\) In the Londregan-Romer model this uncertainty relates to voters’ evaluations of the candidates’ abilities to perform constituent service, although we do not restrict the meaning of valence in this way. By contrast, Groseclose (2001) analyzes a model in which candidates’ valence-related characteristics are known with certainty at the time they select their policy positions, and the candidates’ uncertainty is over the location of the median voter.

\(^{13}\) By contrast there is a voluminous literature that analyzes the composition and survival of governing coalitions (Gamson, 1961; Riker, 1962; Axelrod, 1970; Browne and Franklin, 1973; Warwick, 1998; Laver and Shepsle, 1996). Warwick (2001, pages 1213-1214) explains this disparity by noting that “the greater attention these matters [explaining the composition and survival of governing coalitions] has received largely relates to the fact that they are readily measurable. The tasks of determining which parties entered a government, how portfolios were distributed among them, and how long the government survived in office are miniscule compared with those of measuring its policy output.”

\(^{14}\) A related literature analyzes the relationship between parties’ policy programmes and the published policy declarations of the government (see Budge and Laver, 1992; McDonald and Budge, 2005, Chapter 8; Warwick, 2001). Note, however, that these studies take as their dependent variable the government’s policy declarations rather than the actual government policy outputs that were observed. Policy-seeking parties are presumably concerned with actual policy outputs as opposed to policy promises.
tries, and that overall support for an MPP assumption is comparable to that for these alternative assumptions (see McDonald and Budge, 2005, Tables 12.3-12.5).\footnote{We note that the authors did not assess the policy influence of the formateur, and thus they do not compare the relative policy influence of this variable versus the MPP.}

In addition to the McDonald-Budge results summarized above, we note that Cho and Duggan (2004) have recently presented important theoretical results that, as legislators become arbitrarily patient, a large class of bargaining models of distributive politics collapse to the position of the median legislator. This result, which runs counter to the folk theorem for repeated games that any possible division of resources can be supported as a subgame perfect equilibrium outcome, also supports our assumption of the policy primacy of the MPP. Thus we have both theoretical and empirical reasons to believe that our model is relevant to policy-making in real world democracies.

3. Policy-Seeking Equilibrium in Parliamentary Elections: Theoretical Results

We now consider the existence and characteristics of equilibrium in policy-seeking parties’ strategies in parliamentary elections under PR, for the model of voting behavior and government policy outputs developed in Section 2. We also explore comparative statics on how parties react to changes in their measured valence characteristics. The theoretical results on existence and characteristics of equilibria presented in this section are proved in Appendix A; the comparative statics results are proved in Appendix B.

Existence and characteristics of policy-seeking equilibrium strategies

The following theorem provides sufficient conditions for existence of a Nash equilibrium, i.e., a configuration of party strategies $s_1, \ldots, s_K$ such that no party can increase its expected utility by unilaterally changing its policy position. Let $I$ denote a closed bounded interval containing the voter ideal points and the party preferences $R_k$ for $k = 1, \ldots, K$. For $k = 1, \ldots, K$, define the interval $I_k = [R_k, m]$ if $R_k \leq m$ and $I_k = [m, R_k]$ if $R_k \geq m$, where $m$ is the location of the median voter. Note that $I_k \subseteq I$.

**Theorem 1 (Existence of Nash equilibrium).** If for each party $k$, $k = 1, \ldots, K$, $f(s_k, R_k)$ is concave and peaks at $R_k$, and $g(s_k, m)$ is concave and peaks at $m$, then there exists a set of
party strategies \( \mathbf{s}^* = (s_1^*, ..., s_K^*) \in I_1 \times I_2 \times ... \times I_K \) such that \( U_k(\mathbf{s}^*) \) is the maximum over \( I \) for each \( k \), i.e., \( \mathbf{s}^* \) is a Nash equilibrium.

In words, the theorem states that a Nash equilibrium is guaranteed provided that both voters and parties have concave policy utility functions that peak at their respective ideal points, as we assume in our model. These conditions are weak in the sense that they are satisfied by most commonly-used policy distance functions, including the linear and quadratic loss specifications.

For the proof, which is given in Appendix A, the theorem is broken into three assertions, or lemmas, each of which is of interest in its own right and is stated here. Lemma 1 shows that if for all \( k, k = 1, ..., K \), \( f(s_k, R_k) \) and \( g(s_k, m) \) are concave and peak at their ideal points, then \( U_k \) is single-peaked\(^{16} \) on \( I_k \). Lemma 2 shows that if \( f(s_k, R_k) \) and \( g(s_k, m) \) are concave and peak at their ideal points (and hence are single-peaked), then \( U_k \) peaks at a point in \( I_k \). Finally Lemma 3 shows that if \( U_k \) is both single-peaked on \( I_k \) and peaks at a point in \( I_k \) for each party \( k \), then there exists a set of party strategies \( \mathbf{s}^* = (s_1^*, ..., s_K^*) \) that is a Nash equilibrium.

Lemma 1 provides conditions that guarantee that each party’s expected utility function will be single-peaked (i.e., strictly quasi-concave), conditional on fixed locations of the remaining parties. Such properties are typical of conditions used in proving the existence of Nash equilibria (Wittman 1990: 67; Roemer, 2001: 57).

Lemma 2 states that each party’s policy-seeking equilibrium strategy is located between its preferred position and the median voter’s position – i.e. that each party’s policy-seeking optimum is similar to, but at least as moderate as, its sincere policy preference. This result, which mirrors the Londregan-Romer (1993) result on two-candidate elections with valence-related uncertainty, makes intuitive sense: policy-seeking parties must balance their desire to present the policies that reflect their sincere preferences against the need to propose

\(^{16}\) If \( U \) is continuous on a closed bounded interval \( I \), then \( U \) is single-peaked (or, equivalently, strictly quasi-concave) if \( U \) has a unique local maximum on \( I \) (see Roemer, 2001: 18). In particular, if \( U \) is single-peaked, there exists \( x_0 \in I \) such that \( U(x_0) > U(x) \) for all \( x \in I, x \neq x_0 \). Note that if a continuous function is concave and peaks at \( x_0 \), then it is single-peaked.
moderate policies that increase their chances of capturing the median voter’s support (which is necessary for them to be the median parliamentary party).

Lemma 3 is a general result that applies to any single-peaked (i.e., strictly quasi-concave) utility function $U_k$ known to peak on closed intervals that may vary over $k$. The lemma states that a Nash equilibrium exists in such a situation.

**Comparative statics: The centripetal effects of valence result**

Theorem 1 establishes existence conditions for a Nash equilibrium. Under such an equilibrium, no party has an incentive to move its position while other parties remain fixed. If, however, a party’s valence image deteriorates, its electoral prospects are also diminished. A strategic move toward the median voter might be expected to help balance this loss, augmenting the party’s probability of being the median parliamentary party, although such a move would carry the party further from its preferred policy location. That a small movement in this direction that represents this strategic trade-off is in general beneficial to a party whose valence image has deteriorated is established by the following theorem.

**Theorem 2 (Centripetal Valence Effects).** Assume that the parties’ and voters’ policy loss utility functions $f(s_k, R_k)$ and $g(s_k, m)$ are single-peaked around their ideal points. Then for any party $k$ whose optimal position $s_k^*$ lies strictly between $R_k$ and $m$, if the measured component $V_k$ of the party’s valence score decreases (increases), party $k$ improves its expected utility by shifting unilaterally toward (away from) the median voter’s position $m$, with all rival parties fixed at their optimal positions and their measured valence components held constant.

In words, the Centripetal Valence Effect result is that when parties enhance their valence images they then have policy-seeking incentives to shift slightly, unilaterally to more extreme positions (relative to the median voter), while parties whose valence images deteriorate are motivated to slightly moderate their policies. We note that this result is similar to Londregan and Romer’s (1993) result on two-candidate elections (see also Wittman, 1990, Theorem 4). However, when one party alters its position, the other parties can be expected to adjust theirs, with all parties moving to a new equilibrium. Explicit formulas for such changes in equilibrium have proved mathematically intractable, but numerical calculations (reported below) suggest strongly that when a focal party shifts due to a change in valence, the other parties move as well and move in the same direction as the focal party (some parties may remain
Thus, for example, if a leftist party loses valence and responds by moving toward the median, i.e., to the right, then the other parties also shift to the right, if they move at all. Thus, relative to one another, optimal party positions change very little when one of them gains or loses valence.

We emphasize that movements in response to plausible changes in valence are small perturbations on an array of dispersed optimal positions primarily influenced by the preferred locations of the parties. Typically, the optimal positions are similar to the preferred positions, but somewhat less dispersed, especially for the parties who favor the more extreme positions.

4. Implications of the Theory and Simulation Results

_Equilibrium configurations and comparative statics: Illustrative examples_

What do our theoretical results imply about party strategies for plausible election scenarios, and how do strategies differ for policy-seeking as opposed to vote-seeking motivations? Consider a situation in which there are two high-valence parties and one or more low-valence parties – a situation that obtains in many real world party systems, including those of Germany, Israel, Spain, Britain, Norway, and Sweden.

Assuming vote maximization, as do Schofield, 2003, 2005 and Schofield and Sened, 2005a b, the two major parties – following Downsian arguments – will move toward the center. Vote-seeking, low-valence (small) parties will then avoid the location of the major players, where they would compete for vote-share on valence alone and lose. Instead they will seek a niche on the policy periphery, where their policy advantage among nearby voters can offset their valence disadvantage. Thus, Schofield shows that when parties maximize votes, low-

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17 In every scenario we have tested using quadratic voter utility, all parties have moved in the same direction as the focal party; using linear utility, some parties remain fixed while others move in the same direction as that of the focal party. See examples below.

18 By contrast, under deterministic voting, policy-seeking parties are motivated to move in to the location of the median voter except for the party with the highest valence, which moves just far enough away from the median voter, in the direction of its preferred position, that its valence advantage over the other parties trumps their spatial advantage. This conclusion does not carry over to the valence-uncertainty model because uncertainty about valence effects permits each party to trade off its policy preference against the probability that it will be the MPP.

19 We note that Schofield’s work on multiparty elections also considers parties’ policy-seeking objectives as well as their expectations about post-election coalition negotiations. However here we consider only Schofield’s conclusions about vote-maximizing parties.
valence parties have incentives to locate sharply away from the center of the voter distribution, while high-valence parties typically have incentives to present moderate positions. Schofield’s results thereby suggest that a weak valence image exerts a centrifugal force on vote-seeking parties, one that pulls them away from the center of the voter distribution.

Schofield and Sened report empirical applications to Israel demonstrating that an equilibrium in vote-maximizing strategies exists in which the two high-valence parties – Labour and Likud – present moderate policies, while the low-valence parties are located farther from the center of the voter distribution (Schofield and Sened, 2005a).

When vote-maximizing motivations are replaced with the policy-seeking motivations that we analyze in this paper, the parties face a trade-off. Each party attempts to balance policy and the probability of attracting the median voter – an effort that leads most parties to seek either center-left or center-right positions, i.e., positions that are neither in the ideological center nor at the extremes. The divergent properties of policy-seeking motivations were first studied by Wittman (1977, 1983) and have been extended to valence models by Londregan and Romer (1993) and others. The multi-party, valence-uncertainty model developed in this paper is a generalization of these models.

At first glance Schofield’s results, compared to our own, seem to imply that valence considerations create diametrically opposite strategic incentives for vote-seeking parties compared to policy-seeking parties. However this is only partly true. To grasp the connection between Schofield’s results and our Centripetal Valence Effects (CVE) theorem, we must consider both what the CVE theorem implies about party strategies and also what the theorem does not imply. The CVE theorem is a comparative statics result that states that, all other factors being equal, a policy-seeking party’s optimal strategy is to unilaterally moderate its position when its valence image deteriorates, and to shift to a more radical position when its valence image improves. However the CVE theorem makes no predictions about the relative positioning of different parties, nor does it imply that low-valence parties will inevitably present moderate positions. The reason the CVE theorem does not imply these outcomes is because, in comparing optimal policy positions across parties, there is a crucial factor that is not equal: the

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20 Schofield shows that the degree of policy moderation by high-valence parties depends on the specifics of the election context. However if there is more than one high-valence party – as is typically the case in competitive multiparty systems – then the competing high-valence parties will typically not converge all the way to the center of the voter distribution.

21 An exception is a party whose sincere preference is in the center; see examples in Table 2 below.
parties’ sincere policy preferences. As the computations we present below make clear, it can be rational, under the valence-uncertainty model, for a low-valence, policy-seeking, party to present sharply noncentrist policies provided that it has noncentrist policy preferences; similarly, it can be rational for a high-valence party to present a moderate policy, provided that this high-valence party has moderate policy preferences (indeed in this latter case policy moderation is invariably an optimal strategy).

**Numerical examples**

To investigate the optimal behavior of all parties as their valence images change, we use numerical calculation because analytic analysis becomes intractable. We note that the parameter space for our model is huge, and thus we make no claim that the examples presented here provide a comprehensive computational sweep of how the model works as we step through key parameter settings (but see footnote 23 below for a discussion of this issue). For these calculations we consider four parties – labeled A, B, C, and D – and we specify the conventional 1-7 scale, quadratic-loss utility for voters and for parties, and that the median voter’s position is \( m=4 \) with the policy-salience parameter set to \( a = 0.25 \). Table 1 reports equilibrium strategies for several valence configurations of parties whose preferred policy positions are \( R_A = 1, \ R_B = 3, \ R_C = 5, \ R_D = 7 \). To clarify the results reported in Table 1, scenario 1 (presented in the top row) is a “generic” scenario where the parties’ valence images are set to the equal values \( V_A = V_B = V_C = V_D = 0 \) (see column 2), and the table reports that for this scenario

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22 The key variables include the number of parties; the policy salience coefficient \( a \); the specification of the voters’ policy loss function \( g(s_j, x_j) \) (i.e. linear losses, quadratic losses, or other); the parties’ sincere policy preferences \( R_1, \ldots, R_K \); and parties’ measured valence characteristics \( V_1, \ldots, V_K \).

23 The parameter \( a=0.25 \) is suggested by empirical studies on voting (Adams and Merrill, 2003; Lacy and Burden, 1999). Substantively, this value implies that if the median voter M is located three units closer to Party A than to Party B along the 1-7 Left-Right scale, and these parties have equal valence images (i.e. \( V_A = V_B \)), then the probability that M will prefer A to B on Election Day is approximately 90%. We note that realistic variations in the specified value of \( a \) did not substantially affect the parties’ equilibrium positions (decreasing \( a \) resulted in somewhat more dispersed positions and increasing \( a \) somewhat depressed party dispersion). With respect to variations in the other model parameters used for our examples, we found that: 1) Results for linear loss utility for parties were similar to those for quadratic losses, but somewhat more dispersed; 2) Results for larger party systems (i.e. more than four parties) we somewhat more dispersed. Results for alternative sets of assumptions about the parties’ valence images and their preferred policies are reported below.
the equilibrium configuration is \( \{s^*_A = 2.85, s^*_B = 3.31, s^*_C = 4.69, s^*_D = 5.15\} \) (see columns 3-6), and that the parties’ equilibrium probabilities of being the MPP are

\[ P^*_A = .224, \quad P^*_B = .276, \quad P^*_C = .276, \quad P^*_D = .224 \]

(see the far right column). Table 1 reports results for ten additional scenarios (Scenarios 2A-2E and 3A-3E) – to be discussed in detail below – in which we vary the parties’ valence images.

The computations reported in Table 1 reveal three striking patterns. First, for all parameters that were investigated, the parties’ optimal strategies at equilibrium are highly dispersed. Each party attempts to balance its policy preference with its likelihood of being the MPP, resulting in two groupings: two parties (A and B) who present moderate to sharply leftist positions, and two rightist parties (C and D) who present moderate to sharply rightist positions. Second, the parties’ optimal positions vary only modestly as a function of their valence images. Thus in the 11 scenarios that are presented in Table 1, Party A’s equilibrium position varies only between 2.42 and 2.91, despite the fact that A’s valence image varies sharply across these scenarios, as is evident from the fact that its equilibrium probability of being the MPP varies between .01 and .27; similarly, Party B’s equilibrium position varies only between 3.10 and 3.33, despite the fact that B’s equilibrium probability of being the MPP varies between .28 and .85. The same patterns obtain for parties C and D. Third, note that in every scenario the parties with the most extreme policy preferences, A and D, present significantly more radical policies strategies than do the parties with moderate policy preferences (B and C) – regardless of the parties’ relative valence images. The latter two patterns underline an important feature of policy competition under the valence-uncertainty model: namely, that for realistic model parameters, policy-seeking parties’ optimal strategies vary only modestly as a function of their valence images \( V \) – even though valence images have a massive effect on the parties’ probabilities of being the MPP – while these policy optima vary substantially as a function of their sincere policy preferences \( R \).

*Illustrative example: Spatial competition with strong center parties*

In scenario 2A in Table 1, the parties’ sincere policy preferences are the same as in Scenario 1, but their valence scores have been changed to \( V_A = 0, \quad V_B = 2, \quad V_C = 2, \quad V_D = 0 \) (see column 2) – i.e. the two parties with moderate policy preferences (B and C) are assumed to have much stronger valence images than are the two parties with extreme preferences (A and
Scenario 2A thereby plausibly captures the strategic situation in Israel described above—in which the moderate, high valence Labor and Likud parties compete in a party system that also features several small, radical, parties—and is also relevant to the Spanish party system which features two large parties—the Socialists and the Conservatives—who are viewed as holding moderate policy preferences, along with smaller, more radical parties such as the Communists and the Popular Coalition. In this scenario, the parties’ policy-seeking strategies are nearly identical to those for scenario 1: Once again the high-valence parties B and C present moderate positions ($s_B^* = 3.28$, $s_C^* = 4.72$) that reflect their sincere policy preferences, and the low-valence parties A and D present more radical positions ($s_A^* = 2.83$, $s_D^* = 5.17$), although these positions are more moderate than their sincere preferences.

The values in the rightmost column of Table 1 show each party’s probability of being the median parliamentary party (MPP) when all parties locate at their equilibrium positions; this shows that for scenario 2A the two radical, low-valence parties are extremely unlikely to be the MPP, so that the moderate, high-valence parties are overwhelmingly likely to control government policy outputs following the election. This equilibrium configuration conforms well to the policy configurations that we actually observe in the Israeli and Spanish party systems—in which the major parties present moderate policies—and it illustrates the fact that, under the valence-uncertainty model, policy-seeking parties with poor valence images may present sharply non-centrist positions—the same result that Schofield obtains for vote-seeking parties.

Why, in this illustrative example, do the high-valence parties present moderate positions while the low-valence parties present sharply non-centrist policies? The moderate positioning by the high-valence parties B and C is easily explained by the fact that these parties’ moderate policy-seeking optima are quite similar to their sincere policy preferences; it would clearly be irrational for these parties to present positions that are more extreme than their sincere preferences, since this would simultaneously depress their chances of being the MPP and would oblige them to implement less desirable policies in the event they are the MPP. At the same time these parties do not moderate all the way to the center, which is at some distance from their sincere preferences. The modestly more extreme positioning by the low-valence parties reflects a compromise between these parties’ sincere beliefs and their desire to be the MPP. Note

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24 Substantively, the settings ($V_A = 0, V_B = 2, V_C = 2, V_D = 0$) imply that if the median voter M is indifferent between a high-valence party (B or C) and a low-valence party (A or D) on policy grounds, the probability that M will prefer the high-valence party on Election Day is roughly 88%.
first that at equilibrium the low-valence parties do in fact moderate their positions, relative to their sincere preferences. However because these low-valence parties hold extreme preferences, their strategic policy compromises still leave them presenting sharply noncentrist positions that are more radical than those of the moderate, high-valence parties.

Results when the parties’ valence images are varied. With the parties’ preferred positions fixed, we assess the Centripetal Valence Effect by varying the parties’ valence images, first of the far left party A (scenarios 2B-2C), and then that of the center-left party B (scenarios 2D-2E). We see from Table 1 that as the valence image of either left-of-center party is increased, the optimum strategies at equilibrium not only for the focal party but also for all parties shift slightly to the left.25 (Similarly, if the valence of a right-of-center party is increased, the optimum strategies of all parties shift right.) Intuitively, the party with enhanced valence has the leeway to move in its preferred direction; i.e., it trades away some of its increased likelihood of winning due to increased valence for a more desirable policy position. When, say, a leftist party gains valence, parties to its right also move left to make up for their loss of valence relative to the focal party.26 This generalizes the finding in Adams, Merrill, and Grofman (2005; Ch. 11; see also Smirnov and Fowler, forthcoming) for two-party contests: the valence-advantaged party has the leeway to move to a more extreme location (toward its preferred policy position) while a valence-disadvantaged party becomes more moderate. Note, however, that in the examples presented in scenarios 2A-2E the parties’ equilibrium positions change

25 We note that the CVE is a comparative statics result about unilateral policy shifts by the focal party in response to changes in its valence image \( V \), rather than a result on changes in the global equilibrium configuration in response to changes in \( V \), which is what we report in Table 1. If we fix parties B, C, and D at their equilibrium positions for scenario 2A, and then compute Party A’s optimal position for a unilateral policy shift for the valence scenarios 2B and 2C, then for scenario 2B Party A’s optimal position is \( s_e=2.79 \), and for scenario 2C Party A’s optimal position is \( s_e=2.70 \) – positions that are virtually identical to the equilibrium positions reported in Table 1, and that support the CVE result. Computations on Party B’s optimal unilateral policy shifts for scenarios 2D-2E, with the rival parties fixed at their optimal positions for scenario 2A, support identical substantive conclusions.

26 The following may explain why, when Party B’s valence image improves, party A also shifts to the left. A shift to the left by Party A "transfers" proportionately more of its probability \( P_A \) of being MPP to party B as Party B’s valence (and hence vote-share) increases than it transfers to parties on the right. This is because of the IIA property of the probability function under the valence-uncertainty model. Since B’s center-left policy strategy is more agreeable to Party A than are the right-wing policy strategies of parties C-D, it follows that the “expected policy cost” of the decline in \( P_A \) that occurs when A shifts left decreases, and so Party A benefits by shifting to a more radical position.
only slightly even as their probabilities of being the MPP vary wildly – perhaps the most interesting pattern in these examples, and one that carries over to alternative sets of model parameters (see note 23).

Illustrative example: Spatial competition with both strong and weak moderate parties

With the parties’ preferred positions still fixed at locations \( R_A = 1, R_B = 3, R_C = 5, R_D = 7 \), as in the previous examples, we explore an alternative situation in scenario 3A: namely, one in which parties B and D (the center-left and far right parties) have high valence scores \( V_B = V_D = 2 \), whereas parties A and C (the extreme left and center-right parties) have low valence scores \( V_A = V_C = 0 \). This scenario plausibly captures the strategic situation in Germany, which features four major parties – the Greens, the Social Democratic Party (SDP), the Free Democrats (FDP), and the CDU/CSU – of which the two smallest are the Greens who espouse radical left-wing policies, and the FDP which currently espouses center-right positions.\(^{27}\) The parties’ equilibrium strategies for this scenario, \( \{s_A^* = 2.91, s_B^* = 3.33, s_C^* = 4.72, s_D^* = 5.21\} \), are similar to those reported for scenarios 1 and 2, even though the parties’ valence images, and their equilibrium probabilities of being the MPP, differ dramatically in scenario 3A compared to scenarios 1-2. As before there is greater distance between the optimal positions of the two leftist parties and the two rightist parties than there is within either of these pairs.

With the parties’ preferred positions fixed, we again assess the centripetal valence effect by varying the parties’ valence images, first of the far left party A (scenarios 3B-3C), and then that of the center-left party B (scenarios 3D-3E). We again see from Table 1 that as the valence of either left-of-center party is increased, the optimum strategies at equilibrium for all parties shift to the left.

[TABLE 1 ABOUT HERE]

In toto, the computations presented in Table 1 suggest that, for a wide range of scenarios, parties that hold extreme policy preferences have policy-seeking incentives to present more radical

\(^{27}\) Scenario 2A is also relevant to policy competition in postwar Norway, which features two proto-coalitions: A leftist proto-coalition consisting of the large, moderately leftist Labor Party, and the small, sharply left-wing Socialists (who typically support Labor from outside the government); and a right-wing proto-coalition that is an-
policies than do parties with moderate policy preferences. These results appear consistent with the party strategies that we actually observe in PR-based parliamentary democracies such as Israel, Spain, Germany, and Norway. Our computations also support our Centripetal Valence Effects (CVE) result, that parties are motivated to moderate their policy strategies when their valence images deteriorate, and shift to more radical positions when their valence images improve. The computations also demonstrate that the CVE result is compatible with radical policy positioning by valence-disadvantaged parties.

Finally, in Table 2, we explore the effects on equilibrium strategies of varying the party-preference configuration. Because the effects of varying the valence images were found to be similar to those for our initial example with dispersed party preferences reported in Table 1, we present results in Table 2 only for one valence configuration, that with equal valence images: \( V_A = V_B = V_C = V_D = 0 \). Scenario 1 in Table 2 is the base configuration of dispersed party preferences (1, 3, 5, 7), given here for comparison. Scenarios 2 and 3 depict polarized preferences; the corresponding equilibrium strategies are likewise polarized, although less extreme than the preferences. Scenarios 4 and 5 represent asymmetrical party preferences; again, equilibrium strategies are similar, but more moderate, than the preferred positions. These results suggest that over a wide range of party-preference and valence configurations, equilibrium strategies for the MPP model resemble the parties' sincere preferences but are more moderate.

\[ \text{[TABLE 2 ABOUT HERE]} \]

5. Conclusion

We have developed a spatial model of policy-seeking parties contesting multiparty parliamentary elections under Proportional Representation (PR), in which the parties differ in terms of their measured valence attributes and where moreover they are uncertain about the electoral impact of valence issues. The key assumption in our model – which represents a direct extension of Londregan and Romer’s (1993) two-candidate model to the multiparty context – is that party elites believe that the median parliamentary party will control government policy outputs. In both the multi-party and two-party models, the utility of a party is a weighted mean of its utilities for the declared positions of the parties, where the weights are the probabilities

chored by the large, sharply right-wing Conservative Party, along with small parties that are moderate on Left-Right issues, the Center Party, the Christian Peoples’ Party, and (in earlier periods) the Liberals.
that each party wins the vote of the median voter. In the two-party case, the party who wins the median voter receives a majority of the vote and hence controls the government. In the multi-party case, the party who attracts the median voter is the median party in parliament, and by our assumption controls governmental output.\footnote{The conditional logit implementations of the two models differ in one respect: the error term in the multi-party model is the same for all voters whereas there are independent error terms for voters in the two-party model.}

We have shown that the multi-party, valence-uncertainty model motivates sincere voting, and that it supports a Nash equilibrium for party strategies under fairly general conditions. Numerical calculations suggest strongly that the configuration of equilibrium strategies under policy-seeking motivations resembles the configuration of the parties' sincere preferences, but these strategies are more moderate than the preferred positions, especially for those parties with extreme preferences. These calculations also suggest that, for realistic model parameters, the computed equilibria resemble the actual configurations that we observe in real world party systems in Israel, Germany, Spain, and Norway. This effect tends to encourage a number of parties to locate in either center-left or center-right positions. Furthermore, the model generates a prediction that we label the \textit{Centripetal Valence Effects} hypothesis: namely, that parties whose valence images deteriorate have policy-seeking motivations to moderate their policies, compared with valence-advantaged parties.

Our model could be extended to encompass additional complications, such as restrictions on party positioning (Kollman, Miller, and Page, 1992; Laver, 2005), uncertainty over the location of the median voter (Wittman, 1983; Groseclose, 2001; Smirnov and Fowler, forthcoming), or politicians with mixtures of expressive and instrumental policy motivations (see, e.g., Brennan and Lomansky, 1993).\footnote{Numerical calculations for a valence-uncertainty model in which parties have mixed vote-maximizing and policy-seeking motivations yielded equilibrium strategies intermediate between dispersed strategies under policy-seeking and centrist strategies under vote-maximization.} In addition, it would be useful to compare how parties' strategic incentives in our model differ from their incentives in models which employ alternative assumptions about policy outputs, such as the policy primacy of the formateur (Baron and Ferejohn, 1989; Merrill and Adams, 2005) or the primacy of the governing parties (Austen-Smith and Banks, 1998). I would also be interesting to conduct a comprehensive computational sweep to determine how party equilibrium changes as we step through the numerous parameter settings. Finally, it would be extremely interesting to empirically evaluate our centripetal valence effects hypothesis, although such empirical tests must await the development of
cross-national, overtime measures of fluctuations in real world parties’ valence images – measures that are not currently available.\textsuperscript{30}

In this paper, we have analyzed the strategies of policy-seeking parties in parliamentary democracies with PR voting systems and a policy-dominant median parliamentary party. We find that such a model is likely to support a stable configuration of party policy strategies, and that moreover it illuminates a non-obvious empirical pattern, in which governing parties tend to moderate their policies compared with opposition parties.

\textsuperscript{30} In an earlier version of this paper we reported empirical analyses suggesting that governing parties in PR-based parliamentary democracies tend to moderate their policy positions over time – a finding that appears consistent with the CVE hypothesis, given that there is extensive research documenting that governing parties lose votes in subsequent elections for reasons that are plausibly related to deteriorations in their valence images (Paldam, 1991; Clark, 2005; McDonald and Budge, 2005). However, while these results are consistent with the CVE hypothesis they are hardly conclusive given that there are many alternative reasons why governing parties may moderate their positions over time.
Appendix A. Proof of the Existence of a Policy-Seeking Equilibrium

In this appendix we prove the existence of a Nash equilibrium for policy-seeking parties, given concavity assumptions on the voters’ and parties’ utility functions. Assume that there are \( K \) policy-seeking parties with preferred positions \( R_1, ..., R_K \) and voter ideal points on a closed bounded interval \( I \), as well as measured valence characteristics \( V_1, ..., V_K \). Define the interval \( I_k = [R_k, m] \) if \( R_k \leq m \) and \( I_k = [m, R_k] \) if \( R_k \geq m \), where \( m \) is the location of the median voter. Note that \( I_k \subseteq I \) for each \( k \). Let \( P_1, ..., P_K \) denote the parties’ respective probabilities of being the median parliamentary party. Denote by \( a \) the common policy-salience parameter for all parties. Let \( f = f(s_j, R_k) \) denote party \( j \)'s policy utility function for strategy \( s_j \) for \( j = 1, ..., K \) and \( ag(s_j, m) + V_j \) denote the median voter’s utility function for party \( j \).

Thus, as given in formulas 2 and 3 in section 2, party \( k \)'s expected policy utility is

\[
U_k = \sum_{j=1}^{K} P_j f(s_j, R_k),
\]

where

\[
P_k = \frac{\exp(ag(s_k, m) + V_k)}{\sum_{j=1}^{K} \exp(ag(s_j, m) + V_j)}.
\]

**Theorem 1 (Existence of Nash equilibrium).** If \( f(s_k, R_k) \) is concave and peaks at \( R_k \), and \( g(s_k, m) \) is concave and peaks at \( m \), for each \( k, k = 1, ..., K \), then there exists a set of party strategies \( s^* = (s_1^*, ..., s_K^*) \in I_1 \times I_2 \times ... \times I_K \) such that \( U_k(s^*) \) is the maximum over \( I \) for each \( k \), i.e., \( s^* \) is a Nash equilibrium.

**Proof.** Lemma 1 below shows that if for all \( k, k = 1, ..., K \), \( f(s_k, R_k) \) and \( g(s_k, m) \) are concave and peak at their ideal points, then \( U_k \) is single-peaked on \( I_k \). Lemma 2 shows that if \( f(s_k, R_k) \) and \( g(s_k, m) \) are concave and peak at their ideal points, and hence are single-peaked, then \( U_k \) peaks at a point in \( I_k \). Finally Lemma 3 shows that if \( U_k \) is single-peaked on
\( I_k \) and peaks at a point in \( I_k \) for each \( k \), then there exists a set of party strategies
\[ s^* = (s_1^*, \ldots, s_k^*) \]
that is a Nash equilibrium. These three lemmas establish the theorem.

**Lemma 1.** If \( f(s_k, R_k) \) is concave and peaks at \( R_k \), and if \( g(s_k, m) \) is concave and peaks at \( m \), then \( U_k \) is single-peaked on \( I_k \), i.e., on \([R_k, m]\) or \([m, R_k]\).

**Proof.** Without loss of generality we assume that \( R_k \leq m \). Note that on the interval \([R_k, m]\),
\[
\frac{\partial U_k}{\partial s_k} = a \frac{\partial g}{\partial s_k} P_k (1 - P_k) f(s_k, R_k) + P_k \frac{\partial f(s_k, R_k)}{\partial s_k} - a \frac{\partial g}{\partial s_k} P_k \sum_{j \neq k} P_j f(s_j, R_k)
\]
\[
= P_k \left( a \frac{\partial g}{\partial s_k} (1 - P_k) f(s_k, R_k) - \frac{(1 - P_k) \sum_{j \neq k} P_j f(s_j, R_k)}{(1 - P_k)} \right) + \frac{\partial f(s_k, R_k)}{\partial s_k},
\]
where \( c = \frac{\sum_{j \neq k} P_j f(s_j, R_k)}{(1 - P_k)} \) is a constant because of the independence-of-irrelevant-alternatives property of the conditional logit function.\(^{31}\) Because \( P_k > 0 \) (by equation A2), it follows from

\[^{31}\text{Because of the definition of } P_j \text{ under condition logit (see equation A2), ratios between } P_j \text{'s for } j \neq k \text{ remain the same when } P_k \text{ changes. Coupled with the fact that } \sum_{j \neq k} P_j = (1 - P_k), \text{ this implies that } P_j = (1 - P_k) \overline{P}_j, \]

where the \( \overline{P}_j \) are constants that do not depend on \( P_k \). Thus,
\[
c = \frac{\sum_{j \neq k} P_j f(s_j, R_k)}{(1 - P_k)} = \frac{(1 - P_k)}{(1 - P_k)} \sum_{j \neq k} \overline{P}_j f(s_j, R_k) = \sum_{j \neq k} \overline{P}_j f(s_j, R_k), \]
which is a constant, i.e., it does not depend on \( s_k \).
equation A3 that the sign of $\frac{\partial U_k}{\partial s_k}$ is the same as the sign of

$$a \frac{\partial g}{\partial s_k} (1 - P_k) [f(s_k, R_k) - c] + \frac{\partial f(s_k, R_k)}{\partial s_k}.$$ 

Because $f$ is strictly decreasing on $[R_k, m]$, if $f(s_k, R_k) - c$ is ever zero for $x \in (R_k, m)$, then there exists a unique value of $s_k \in (R_k, m)$, say, $\bar{s}_k$ such that $f(s_k, R_k) - c \geq 0$ for $x \in [R_k, \bar{s}_k]$ and $f(s_k, R_k) - c \leq 0$ for $x \in [\bar{s}_k, m]$. If, instead, $f(s_k, R_k) - c \geq 0$ for all $x \in [R_k, m]$, we define $\bar{s}_k = m$, and if $f(s_k, R_k) - c \leq 0$ for all $x \in [R_k, m]$, we define $\bar{s}_k = R_k$. First, we show that $\frac{1}{P_k} \frac{\partial U_k}{\partial s_k}$ is strictly decreasing over the interval, $[R_k, \bar{s}_k]$ hence $\frac{\partial U_k}{\partial s_k}(x) = 0$ can occur for at most one value of $x \in [R_k, \bar{s}_k]$ (this statement is vacuous if $\bar{s}_k = R_k$). In turn, $U_k$ has a single maximum in $[R_k, \bar{s}_k]$ and thus is single-peaked in that interval.

To see that $\frac{1}{P_k} \frac{\partial U_k}{\partial s_k}$ is strictly decreasing in the interval $[R_k, \bar{s}_k]$, note that $a \frac{\partial g(s_k, m)}{\partial s_k}$, $(1 - P_k)$, and $[f(s_k, R_k) - c]$ are all non-negative, strictly decreasing functions of $s_k$ on $[R_k, \bar{s}_k]$. These statements follow, respectively, because $g$ is concave and peaks at $m$, $P_k$ is strictly increasing by equation (A2), and $f(s_k, R_k)$ is strictly decreasing on $[R_k, \bar{s}_k]$ (because $f$ is concave and peaks at $R_k$; see footnote 13). It follows that the product of these functions,

$$a \frac{\partial g}{\partial s_k} (1 - P_k) [f(s_k, R_k) - c],$$

is strictly decreasing. Furthermore, $\frac{\partial f(s_k, R_k)}{\partial s_k}$ is also a non-increasing function because $f$ is concave. It follows that

$$a \frac{\partial g}{\partial s_k} (1 - P_k) [f(s_k, R_k) - c] + \frac{\partial f(s_k, R_k)}{\partial s_k}$$

is strictly decreasing on $[R_k, \bar{s}_k]$. Hence

$$a \frac{\partial g}{\partial s_k} (1 - P_k) [f(s_k, R_k) - c] + \frac{\partial f(s_k, R_k)}{\partial s_k} = 0$$

for at most a single location $s_k$ in the interval $[R_k, \bar{s}_k]$, i.e., $U_k$ is single-peaked in the interval $[R_k, \bar{s}_k]$.

Next we show that $U_k$ is a decreasing function in the interval $[\bar{s}_k, m]$ by showing that

$$\frac{\partial U_k}{\partial s_k} < 0$$

in the interval $(\bar{s}_k, m]$, i.e., that
\[
\frac{\partial U_k}{\partial s_k} = P_k \left( a \frac{\partial g}{\partial s_k} (1 - P_k) [f(s_k, R_k) - c]) + \frac{\partial f(s_k, R_k)}{\partial s_k} \right) < 0
\]  
(A4)

(this statement is vacuous if \( \bar{s}_k = m \)). Because \( g \) is increasing on \([R_k, m]\), \( \frac{\partial g}{\partial s_k} \) and \((1 - P_k)\) are non-negative on this interval and \([f(s_k, R_k) - c] \) is non-positive on \([\bar{s}_k, m]\). Hence

\[
a \frac{\partial g}{\partial s_k} (1 - P_k) [f(s_k, R_k) - c] \leq 0 \text{ on } [\bar{s}_k, m].
\]

Because \( f \) is strictly decreasing, \( \frac{\partial f(s_k, R_k)}{\partial s_k} < 0 \) on \((\bar{s}_k, m]\), so that \( \frac{\partial U_k}{\partial s_k} < 0 \) on \((\bar{s}_k, m]\).

Finally, we argue that, because \( U_k \) is single-peaked on \([R_k, \bar{s}_k]\) and decreasing on \([\bar{s}_k, m]\), that \( U_k \) is single-peaked on the entire interval \([R_k, m]\). Suppose the single peak of \( U_k \) on \([R_k, \bar{s}_k]\) occurs at the point \( \bar{x} \). Then \( U_k \) is decreasing on \([\bar{x}, \bar{s}_k]\), and hence decreasing on \([\bar{x}, m]\), so \( U_k \) has a single peak at \( \bar{x} \) in \([R_k, m]\) and is single-peaked. This concludes the proof of Lemma 1.

**Lemma 2.** For each \( k \), if the \( k \)th party’s utility functions \( f(x, R_k) \) and the median voter’s policy utility function \( g(x, m) \) are continuous and single-peaked around their ideal points, and if \( U_k \) peaks at \( s_k \) given fixed \( x_j \in \textbf{I}, j \neq k \), then \( s_k \) lies in the closed interval between its preferred position \( R_k \) and that of the median voter \( m \), i.e., \( s_k \in I_k \).

**Proof.** Without loss of generality, assume \( R_k \leq m \), in which case the Lemma states that \( R_k \leq s_k \leq m \). We make use of the fact that

\[
U_k = \sum_{j=1}^{K} p_j f(s_j, R_k) = p_k f(s_k, R_k) + \sum_{j \neq k} p_j f(s_j, R_k).
\]  
(A5)

Without loss of generality, because we can add a constant to a utility function if necessary, we may assume that \( f(R_k, R_k) = 0 \). To see that \( R_k \leq s_k \), note that shifting \( s_k \) from location \( R_k \) to a location \(< R_k\) decreases \( f(s_k, R_k) \) from 0 to a negative value, but has no effect on \( f(s_j, R_k) \).
This shift also results in a decrease in $P_k$ and a corresponding increase in each $P_j$ for $j \neq k$ (the latter occurs because of the form of equation A2 that defines the $P_k$ and the fact that the $g(s_j, m)$ do not change). Thus, under the shift of $s_k$ from location $R_k$ to a location $< R_k$, both terms of $U_k$ decrease. It follows that $R_k \leq s_k$.

To see that $s_k \leq m$, note first that if $R_k = m$, then the preceding argument can be used to show that $s_k \leq m$. Because $g$ is continuous and single-peaked with its peak at $m$, we may choose $m_1$ and $m_2$ in $I$ so that $R_k < m_1 < m < m_2$ and $g(m_1, m) = g(m_2, m) < g(m, m)$. If $R_k < m$, note that shifting $s_k$ from the location $m_2$ to the location $m_1$ increases $f(s_k, R_k)$, since $m_1$ is closer to $R_k$ than is $m_2$ and by assumption $f(s_k, R_k)$ is single-peaked and peaks at $R_k$. Note, however, that the shift from $m_2$ to $m_1$ has no effect on $P_k$, because $g(m_1, m) = g(m_2, m)$. It follows that $P_k f(m_1, R_k) > P_k f(m_2, R_k)$. Furthermore, given that $P_k$ is identical for the strategies $m_1$ to $m_2$, it follows that each $P_j$ for $j \neq k$ is also identical for $m_1$ to $m_2$ (the latter occurs because of the form of equation A2 that defines the $P_k$ and the fact that the $g(s_j, m)$ do not change). This implies in turn that $\sum_{j \neq k} P_j f(s_j, R_k)$ has identical values for $m_1$ to $m_2$. Thus, under the shift of $s_k$ from location $m_2$ to $m_1$, the first term of $U_k$ in equation A5 increases while the term $\sum_{j \neq k} P_j f(s_j, R_k)$ remains constant. It follows that the strategy $s_k = m_2$ cannot be optimal. This establishes that $s_k \leq m$ and completes the proof of Lemma 2.

**Lemma 3 (Single-peakedness implies existence of a Nash equilibrium).** Suppose that $U: I^K \rightarrow \mathbb{R}$ is continuous and defines for each party $k = 1, \ldots, K$, a conditional utility function $U_k : I \rightarrow \mathbb{R}$ that is single-peaked, i.e., for fixed $s_j \in I, j \neq k$, $U_k$ is a single-peaked function of $s_k$. Suppose, further, that the optimum for each utility function $U_k$ lies in $I_k$. Then there exist a set of party strategies $s^* = (s_1^*, \ldots, s_K^*) \in I_1 \times I_2 \times \ldots \times I_K$ such that $U_k(s^*)$ is the maximum over $I$ for each $k$, i.e., $s^*$ is a Nash equilibrium.

**Proof.** Define $\phi_k(s_2, \ldots, s_k) = \tilde{s}_1$, where $\tilde{s}_1$ maximizes $U_1$ for fixed $s_2, \ldots, s_K$, and in general define...
\[ \phi_k(s_1, \ldots, s_{k-1}, s_{k+1}, \ldots, s_K) = \bar{s}_k, \quad (A6) \]

where \( \bar{s}_k \) maximizes \( U_k \) for fixed \( s_1, \ldots, s_{k-1}, s_{k+1}, \ldots, s_K \). Note that for each \( k \), \( \phi_k \) maps \( I_k \) into \( I_k \).

We first show that because \( U_k \) is single-peaked, each \( \phi_k \) is continuous. Without loss of generality, we let \( k = 1 \). If \( \phi_1 \) were not continuous, then for each \( \varepsilon > 0 \), there exists a sequence \( s_n \rightarrow s_0 \) where \( s_n = (s_{2n}, \ldots, s_{Kn}) \in I_2 \times \cdots \times I_K \) and \( s_0 = (s_{20}, \ldots, s_{K0}) \in I_2 \times \cdots \times I_K \) such that \( |\phi_1(s_n) - \phi_1(s_0)| \geq \varepsilon \). Define \( a_n = \phi_1(s_n) \). Because the subset of \( I_1 \times I_2 \times \cdots \times I_K \) consisting of points whose first coordinate is at least \( \varepsilon \) distant from \( \phi_1(s_0) \) is compact, there is a point \( a_0 \in I_1 \) with \( |a_0 - \phi_1(s_0)| \geq \varepsilon \) and a subsequence of \( \{s_n\} \), call it also \( \{s_n\} \), such that \( a_n \rightarrow a_0 \).

Thus \( (a_n, s_{2n}, \ldots, s_{Kn}) \rightarrow (a_0, s_{2n}, \ldots, s_{Kn}) \). For each \( n \), since \( U \) is single-peaked, conditional on fixed \( s_n \),

\[ U(a_n, s_{2n}, \ldots, s_{Kn}) > U(\phi_1(s_0), s_{2n}, \ldots, s_{Kn}) \quad (A7) \]

By continuity of \( U \), the two terms in inequality (A7) converge to \( U(a_0, s_{2n}, \ldots, s_{Kn}) \) and \( U(\phi_1(s_0), s_{2n}, \ldots, s_{Kn}) \), respectively. It follows that

\[ U(a_0, s_{2n}, \ldots, s_{Kn}) \geq U(\phi_1(s_0), s_{2n}, \ldots, s_{Kn}) \quad (A8) \]

But since \( U \) is single-peaked for fixed \( s_0 \),

\[ U(\phi_1(s_0), s_{2n}, \ldots, s_{Kn}) > U(a_0, s_{2n}, \ldots, s_{Kn}), \]

which contradicts inequality (A8). This proves that \( \phi_1 \) is continuous.

Next define:

\[ \phi(s_1, \ldots, s_K) = (\phi_1(s_2, \ldots, s_K), \ldots, \phi_k(s_1, \ldots, s_{k-1}, s_{k+1}, \ldots, s_K), \ldots, \phi_K(s_1, \ldots, s_{K-1})). \]
Thus the vector-valued function $\phi$ is continuous and maps the convex, compact set $I_1 \times I_2 \times \ldots \times I_K$ into itself. Hence, by the Brouwer fixed-point theorem, $\phi$ has a fixed point $(s_1^*, \ldots, s_K^*)$ in $I_1 \times I_2 \times \ldots \times I_K$, i.e.,

$$\phi(s_1^*, \ldots, s_K^*) = (s_1^*, \ldots, s_K^*).$$

Thus, $s_k^*$ maximizes $U_k$ over $I$ because $s_k^* = \phi_k(s_1^*, \ldots, s_{k-1}^*, s_{k+1}^*, \ldots, s_K^*)$, so that $s_k^*$ is by definition the value of $s_k$ that maximizes $U_k$ when the $s_j^*, j \neq k$ are fixed.

---

32 The Brouwer fixed-point theorem states that every continuous function of a convex, compact set into itself has a fixed point (Arrow and Hahn, *General Competitive Analysis* (1971): p. 28).
Appendix B. A Result on Comparative Statics

**Theorem 2 (Centripetal Valence Effects).** Assume that the parties’ and voters’ policy loss utility functions are single-peaked around their ideal points. Then for any party \( k \) whose optimal position \( s_k^* \) lies strictly between \( R_k \) and \( m \), if the measured component \( V_k \) of the party’s valence score decreases, party \( k \) improves its expected utility by shifting unilaterally toward the median voter’s position \( m \), with all rival parties at their optimal positions and their measured valence components held constant.\(^{33}\)

**Proof.** We assume, without loss of generality, that \( R_k < m \). For \( s_k = s_k^* \), then \( \frac{\partial U_k}{\partial s_k} = 0 \), i.e.,

\[
\frac{\partial U_k}{\partial s_k} = a \frac{\partial g}{\partial s_k}(s_k, m)P_k(1 - P_k)f(s_k, R_k) + \frac{\partial f}{\partial s_k}(s_k, R_k)P_k - a \frac{\partial g}{\partial s_k}(s_k, m)P_k \sum_{j \neq k} P_j f(s_j, R_k) = 0
\]

Thus, solving for \( \sum_{j \neq k} P_j f(s_j, R_k) \) and simplifying, we obtain

\[
\sum_{j \neq k} P_j f(s_j, R_k) = \frac{a \frac{\partial g}{\partial s_k}(s_k, m)(1 - P_k)f(s_k, R_k) + \frac{\partial f}{\partial s_k}(s_k, R_k)}{a \frac{\partial g}{\partial s_k}(s_k, m)} = (1 - P_k)f(s_k, R_k) + \frac{\partial f}{\partial s_k}(s_k, R_k)
\]

for \( s_k = s_k^* \).

Now, suppose that \( V_k \), the measured component of party \( k \)’s valence score, increases from \( V_k \) to \( V_k' = V_k + \varepsilon \) (where \( \varepsilon > 0 \)). This in turn increases \( P_k \), the probability that \( k \) is the median party from \( P_k \) to \( P_k' = P_k + p \), where \( p > 0 \) because of the form of equation A2. We show that

\(^{33}\) If \( s_k = m \) or \( s_k = R_k \), then an increase in \( V_k \) will not change the location of Party \( k \)’s optimal position.
when party $k$ is located at its optimal position $s_k = s_k^*$ for $V_k$, it must be the case that $\frac{\partial U_k}{\partial s_k} < 0$ for $s_k = s_k^*$ and $V_k' = V_k + \varepsilon$, i.e., party $k$ can increase its expected utility $U_k$ by shifting unilaterally to the left of $s_k^*$, away from the median voter’s position.

Because the independence-of-irrelevant-alternatives property applies to the probabilities $P_j'$ (where $P_j'$ is the probability that a party $j \neq k$ will be the median party following the exogenous increase in $V_k$), there is a $\delta$, $0 < \delta < 1$, such that $P_j' = \delta P_j$ for all $j \neq k$. It follows that

$$1 - P_k - p = \sum_{j \neq k} P_j' = \delta \sum_{j \neq k} P_j = \delta(1 - P_k).$$

Solving for $\delta$ and substituting in $P_j' = \delta P_j$ yields

$$P_j' = \left(1 - \frac{p}{1 - P_k}\right)P_j.$$

Following the exogenous increase in $V_k$, the derivative $\frac{\partial U_k}{\partial s_k}$, evaluated at $s_k = s_k^*$, is given by

$$\frac{\partial U_k}{\partial s_k} = a \frac{\partial g}{\partial s_k} (s_k, m)P_k'(1 - P_k') f(s_k, R_k) + \frac{\partial f}{\partial s_k}(s_k, R_k)P_k' - a \frac{\partial g}{\partial s_k} (s_k, m)P_k' \sum_{j \neq k} P_j' f(s_j, R_k)$$

$$= a \frac{\partial g}{\partial s_k} (s_k, m)(P_k + p)(1 - P_k - p) f(s_k, R_k) + \frac{\partial f}{\partial s_k}(s_k, R_k)(P_k + p) - a \frac{\partial g}{\partial s_k} (s_k, m)(P_k + p) \sum_{j \neq k} P_j f(s_j, R_k)[1 - p/(1 - P_k)].$$

(A10).

By equation A9, when $s_k = s_k^*$, $\sum_{j \neq k} P_j f(s_j, R_k) = (1 - P_k) f(s_k, R_k) + \frac{\partial f}{\partial s_k}(s_k, R_k)$. Substituting this equality into equation A10 and rearranging terms yields

$$\frac{\partial U_k}{\partial s_k} = a \frac{\partial g}{\partial s_k} (s_k, m)(P_k + p)(1 - P_k - p) f(s_k, R_k) + \frac{\partial f}{\partial s_k}(s_k, R_k)(P_k + p).$$

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\[-a \frac{\partial g}{\partial s_k}(s_k, m)(P_k + p)[1 - p/(1 - P_k)] \left[ (1 - P_k)f(s_k, R_k) + \frac{\partial f}{\partial s_k}(s_k, R_k) \frac{\partial g}{\partial s_k}(s_k, m) \right] \]

\[= \frac{\partial f}{\partial s_k}(s_k, R_k)(P_k + p) \frac{p}{1 - P_k} \]

for \( \frac{\partial U_k}{\partial s_k} \) evaluated at \( s_k = s_k^* \). Since \( \frac{\partial f}{\partial s_k}(s_k^*, R_k) \) is negative for \( R_k < s_k^* < m \), \( \frac{\partial U_k}{\partial s_k} \), evaluated at \( s_k = s_k^* \), is negative as well. Therefore party \( k \) can increase its expected utility \( U_k \) by shifting unilaterally to the left of \( s_k^* \), away from the median voter’s position. This completes the proof of Theorem 2.
References


Axelrod, Robert. 1970. Conflict of Interest. Chicago: Markham


Table 1: Equilibrium Positions for Dispersed Party Preferences and Selected Valence Images

**Party preferences:** \(( R_A = 1, \ R_B = 3, \ R_C = 5, \ R_D = 7 )\)

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Valence Images ((V_A, V_B, V_C, V_D))</th>
<th>Equilibrium Positions (s_A^<em>, s_B^</em>, s_C^<em>, s_D^</em>)</th>
<th>Equilibrium MPP Probabilities ((P_A^<em>, P_B^</em>, P_C^<em>, P_D^</em>))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>((0,0,0,0))</td>
<td>2.85, 3.31, 4.69, 5.15</td>
<td>((.224, .276, .276, .224))</td>
</tr>
<tr>
<td>(2A)</td>
<td>((0,2,2,0))</td>
<td>2.83, 3.28, 4.72, 5.17</td>
<td>((.049, .451, .451, .049))</td>
</tr>
<tr>
<td>(2B)</td>
<td>((1,2,2,0))</td>
<td>2.78, 3.26, 4.69, 5.12</td>
<td>((.120, .413, .420, .046))</td>
</tr>
<tr>
<td>(2C)</td>
<td>((2,2,2,0))</td>
<td>2.67, 3.22, 4.63, 5.01</td>
<td>((.256, .342, .361, .042))</td>
</tr>
<tr>
<td>(2D)</td>
<td>((0,3,2,0))</td>
<td>2.62, 3.17, 4.63, 5.00</td>
<td>((.025, .676, .268, .031))</td>
</tr>
<tr>
<td>(2E)</td>
<td>((0,4,2,0))</td>
<td>2.42, 3.09, 4.57, 4.89</td>
<td>((.010, .845, .130, .016))</td>
</tr>
<tr>
<td>(3A)</td>
<td>((0,2,0,2))</td>
<td>2.91, 3.33, 4.72, 5.21</td>
<td>((.056, .495, .066, .384))</td>
</tr>
<tr>
<td>(3B)</td>
<td>((1,2,0,2))</td>
<td>2.85, 3.31, 4.69, 5.15</td>
<td>((.133, .446, .060, .361))</td>
</tr>
<tr>
<td>(3C)</td>
<td>((2,2,0,2))</td>
<td>2.73, 3.26, 4.63, 5.03</td>
<td>((.275, .359, .050, .316))</td>
</tr>
<tr>
<td>(3D)</td>
<td>((0,3,0,2))</td>
<td>2.67, 3.21, 4.64, 5.02</td>
<td>((.011, .715, .038, .237))</td>
</tr>
<tr>
<td>(3E)</td>
<td>((0,4,0,2))</td>
<td>2.44, 3.10, 4.58, 4.89</td>
<td>((.010, .856, .018, .116))</td>
</tr>
</tbody>
</table>

**Notes.** For these computations parties and voters were assumed to have quadratic policy losses, the median voter’s position was \(m=4\), and the policy salience parameter was \(a=0.25\).
Table 2: Equilibrium Positions for Selected Party-Preference Configurations and Fixed Valence Images.

**Valence Images:** $V_A = V_B = V_C = V_D = 0$

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Party Preferences $(R_A, R_B, R_C, R_D)$</th>
<th>Equilibrium Positions $s_A^*$</th>
<th>$s_B^*$</th>
<th>$s_C^*$</th>
<th>$s_D^*$</th>
<th>Equilibrium MPP Probabilities $(P_A^<em>, P_B^</em>, P_C^<em>, P_D^</em>)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>$(1,3,5,7)$</td>
<td>2.85</td>
<td>3.31</td>
<td>4.69</td>
<td>5.15</td>
<td>(.224, .276, .276, .224)</td>
</tr>
<tr>
<td>(2)</td>
<td>$(3,3,5,5)$</td>
<td>3.27</td>
<td>3.27</td>
<td>4.73</td>
<td>4.73</td>
<td>(.250, .250, .250, .250)</td>
</tr>
<tr>
<td>(3)</td>
<td>$(1,1,7,7)$</td>
<td>2.88</td>
<td>2.88</td>
<td>5.12</td>
<td>5.12</td>
<td>(.250, .250, .250, .250)</td>
</tr>
<tr>
<td>(4)</td>
<td>$(2,3,4,6)$</td>
<td>2.86</td>
<td>3.22</td>
<td>4.00</td>
<td>4.95</td>
<td>(.214, .254, .296, .236)</td>
</tr>
<tr>
<td>(5)</td>
<td>$(2,2,3,6)$</td>
<td>2.73</td>
<td>2.73</td>
<td>3.19</td>
<td>4.81</td>
<td>(.220, .220, .279, .280)</td>
</tr>
</tbody>
</table>

**Notes.** For these computations parties and voters were assumed to have quadratic policy losses, the median voter’s position was $m=4$, and the policy salience parameter was $\alpha=0.25$. 