Prof. Kuhn

Question 1 (60 points)

A farmer can produce output in one of two ways, either with or without a cow. If the cow is alive, the proudction function is given by $y = \bar{A}l$ where l is labor input. If the cow is dead the farmer produces with a production function of $y = \underline{A}l$ with $\bar{A} > \underline{A}$.

The output produced in any period can be split up between consumption c and feed f for the cow. The farmer derives utility from consumption, and disutility from labor; specifically, the farmer's preferences are described by $u\left(c - v\frac{l^2}{2}\right)$ where $u\left(\cdot\right)$ is a strictly increasing and strictly concave function and v is a scalar parameter.

The benefit of using the output for feed is that it increases the likelihood that the cow stays alive: The probability that the cow survives into the next period is given by $\delta\left(f\right)$ where $\delta\left(\cdot\right)$ is an increasing and concave function with $\delta\left(0\right) = 0$.

Denote with V^a the farmer's value function when the cow is alive, and with V^d the value function when the cow is dead.

1. Write down the dynamic programming problem for this farmer. What are the state variables, what are the control variables? (15)

2. Write down the first-order conditions for the choice of l for both a live and a dead cow and explain their economic meaning. Show that l is larger when the cow is alive. Write down the first-order condition for the choice of f, and explain its economic meaning. (15)

3. Show that $V^a > V^d$. (10)

4. As $\underline{A}$ increases, what happens to l and f when the cow is alive? (10)

5. If there is a minimum consumption constraint requiring that $c \geq \underline{c}$ for some level of minumum consumption $\underline{c}$, is it possible that the farmer works more when the cow is dead than when it is alive? (10)

Question 2 (60 points)

Consider a three-period economy with uncertainty. There is one representative consumer who derives utility from consumption and leisure at all dates and states of the world. The consumer's utility function is

$$u\left(c_1, l_1\right) + \beta E\left[u\left(c_2, l_2\right)\right] + \beta^2 E\left[u\left(c_3, l_3\right)\right]$$

where c_1 and l_1 are (nonrandom) consumption and leisure at time 1 and c_t and l_t for $t = 2, 3$ are the random variables for consumption and leisure in periods 2 and 3.

Production takes place in a representative firm which rents labor and sells the output. These output goods are produced according to a production function

of $y = zn$ where n is the amount of labor supplied by the household and z is a productivity shock. The household's labor supply equals the total time endowment, which is 1, minus the chosen leisure time.

The productivity shock takes on one of two possible values in each period so that we can write $z \in \{z_H, z_L\}$. The respective probabilities are π^H and $\pi^L = 1 - \pi^H$. The process is iid over time (which implies that z_2 and z_3 are independent random variables), and productivity in the first period is $z_1 = z_H$ with certainty.

Suppose that the economy has an Arrow-Debreu market structure, in which households and firms trade at time zero.

1. Carefully define a competitive equilibrium for this economy. How many markets are there, and how many prices? Is the equilibrium allocation in this economy Pareto optimal? (20)

2. What is the risk-free interest rate at time 2 (that is, the interest rate between dates 2 and 3), conditional on the realization of z_2? What is the price of a bond bought at time 2 in the state z_H that pays one unit of consumption at time 3 in state z_L?
   (Hint: Proceed in two steps. First, take the Arrow-Debreu prices as given. Express the risk-free rate and the bond price as a function of these prices. Second, use the household's optimality conditions to substitute out the prices, thereby relating interest rate and bond price to parameters and equilibrium quantities.) (20)

3. Simplify the setup in two ways. First, assume that there are only two periods in the model (that is, ignore the third period). Second, suppose that $u(c, l) = \log c + \log l$. Solve for all the equilibrium prices and quantities in this economy. (20)