Micro Comp **(First attempt)** **May 30, 2017**

1 Part A: Short answer or explanation. 4 questions.

1. (Jones) A consumer of 2 goods has indirect utility function $v(p_1, p_2, y) = \frac{p_1+p_2}{p_1 p_2} y$. What is her Marshallian demand, Hicksian demand and expenditure function?

2. (Jones) Show that the price elasticities of a conditional factor demand sum up to zero.

3. (Pape) Suppose the following game is infinitely repeated. Suppose players discount with some

	L	R
U	$5, 4$	$0, 3$
D	$7, 0$	$7, 1$

   common, known discount rate $\delta \in (0, 1)$. Is there an SPNE in which (U, L) is played every period? If so, find one such SPNE and the set of δ which support this SPNE. If not, argue why not.

4. (Pape) Farmers use wells to draw water from the ground to use to grow crops. There is a cost associated with drawing water from a well. When water is drawn from the ground, it can pull in water from wells that belong to neighbors (the water flows underground). Would you expect the socially optimal amount of water to be drawn from wells, or will more be drawn than is optimal, or will less be drawn than is optimal? Explain.

2 Part B: Mathematical or advanced questions. 4 questions.

5. (Jones) A consumer consumes n goods, denoted by $x_1, \ldots, x_n$, and leisure, denoted by S. Her utility function $u(x_1, \ldots, x_n, S)$ is strictly quasiconcave and strictly increasing. Her income can only be derived from her supply of labor, denoted by L. Because of her time endowment T being fixed, $S + L = T$ must hold. Let w denote the wage she earns per unit of labor, and $(p_1, \ldots, p_n)$ the price vector for the n consumption goods. Let $L(w)$ denote the optimal amount of labor she supplies given the wage rate w, holding all other parameters constant. If leisure is an inferior good, what will the slope of her labor supply function $L(w)$?

6. (Jones) A consumer's preferences on consumption bundles $\mathbf{x} \in R^n_+$ admits a strictly quasiconcave utility representation $u(x)$. Let $p \in R^n_{++}$ denote the price vector and $y \in R_+$ denote the income. Show that if $u(x)$ is homogeneous of degree 1, then the indirect utility function $v(p, y)$ and the Marshallian demand function $\mathbf{x}(p, y)$ take the following forms:

$$v(p, y) = yv(p, 1),$$
$$\mathbf{x}(p, y) = y\mathbf{x}(p, 1).$$

7. (Pape) Consider an economy with $I = 10$ agents, indexed $i = 1, \ldots, 10$. There are $L = 11$ goods, indexed $l = 0, 1, \ldots, 10$. Good $l = 0$ is a standard consumption good that are equally valuable to all agents, and goods $l > 0$ are specialized goods that are of differing value to different agents. In particular, the utility function of agent i is:

$$u_i(x_0, x_1, \ldots, x_10) = x_0 + \sum_{l=1}^{10} \left[(3 - |l - i|)\, x_l\right]$$

Each agent starts with the same endowment vector; one hundred units of the first good and one unit each of each subsequent good. I.e. $\omega = (100, 1, \ldots, 1)^T$. Find a Walrasian equilibrium of this economy.

8. (Pape) Consider a betting card game between players A and B. There are three cards, labeled 1, 2, and 3. Initially, each player is given a card, which is private information. Cards are given to players with equal probability (but without replacement: that is, there is only one of each card.)

   After cards are distributed, A chooses Bet or Pass. Then B chooses Bet or Pass. Then, cards are revealed. The player with the higher card Wins, and the player with the lower card Loses. Table 1 gives the payoff of Winning and Losing as a function of whether one bet.

   Find a Nash Equilibrium of this game. Explain your reasoning.

	Bet	Pass
Win	2	0
Lose	-1	0

Table 1: Betting Payoffs.

3 Part C: Longer questions. 4 questions.

9. (Jones) A consumer plans to buy goods 1 and 2 at market prices p_1 and p_2, but her wealth w is uncertain. Her utility from the goods is given by $u(x_1, x_2) = x_1^a x_2^b$, where $0 < a + b < 1$, $0 < a$, and $0 < b$.

   (a) Assume that, once w is known, she will make optimal purchases for goods 1 and 2 at the (certain) prices p_1 and p_2. Call her in this case the *unconstrained consumer*.

      i. What are her Marshallian demands $x_1(p_1, p_2, w)$ and $x_2(p_1, p_2, w)$? Graphically represent them in Figure 1 for given prices p_1 and p_2, and wealth w^0.

      ii. Because the only uncertainty concerns her wealth, we can write the maximized utility of the unconstrained consumer as a function of w (with prices as parameters), to be denoted $\beta(w)$. What is this function?

      iii. For the function $\beta(w)$, compute: its first-order derivative $\beta'(w)$; its second-order derivative $\beta''(w)$; its coefficient of absolute risk aversion at w, denoted $r^A(w)$.

   (b) Consider a given wealth level w^0, and let $x_1^0 \equiv x_1(p_1, p_2, w^0)$. Assume now that as her wealth changes, she is constrained to consume the exact amount $x_1^0 \equiv x_1(p_1, p_2, w^0)$ of good 1 (perhaps because of previous commitments, or by institutional restrictions), spending what is left of her wealth on good 2. Refer to her in this case as the *constrained consumer*.

      i. Graphically represent her constrained choice in Figure 1.

      ii. Now her maximized utility is a different function of wealth, to be denoted $\beta_c(w)$. What is this function?

      iii. For the function $\beta_c(w)$, compute its first-order derivative $\beta_c'(w)$. How do $\beta_c'(w^0)$ and $\beta'(w^0)$ compare? Explain. Graphically illustrate the comparison between $\beta_c'(w^0)$ and $\beta'(w^0)$ in the (wealth, utility) plane in Figure 2.

      iv. Compute the second-order derivative $\beta_c''(w)$ and evaluate it at w^0.

      v. Compute the coefficient of absolute risk aversion $r_c^A(w^0)$ of β_c evaluated at w^0.

   (c) Compare $r^A(w^0)$ and $r_c^A(w^0)$. Who is more risk averse (at w^0), the constrained consumer, or the unconstrained consumer? What is the intuition for this result?

10. (Jones) Consider the following variant of a first-price auction. Sealed bids are collected. The highest bidder pays his bid but receives the object only if the outcome of the toss of a fair coin is heads. If the outcome is tails, the seller keeps the object and the high bidder's bid. Assume bidders symmetry.

   (a) Find the unique symmetric equilibrium bidding function. Interpret.

   (b) Do bidders bid higher or lower than in a first-price auction?

   (c) Find an expression for the seller's expected revenue.

   (d) Show that the seller's expected revenue is exactly half of that of a standard first-price auction.

11. (Pape) Consider a game with three players: Player 1 and Player 2 are identical firms who produce a product q at a constant marginal cost $c = 1$. Player 3 is a government regulator (more below). Player 1 and Player 2 produce in a market with a downward-sloping demand curve $p_d = 4 - Q$, where $Q = q_1 + q_2$, where q_i is the quantity produced by player i.

    The first round of play is the 'bribery' round. The government regulator, player 3, will consider accepting a bribe. He chooses a value x, and then he contacts Player 1, and says if 1 pays him at least x, then he will allow Player 1 to select her quantity first. If Player 1 refuses, he makes the same offer to Player 2 with a possibly different value y. If Player 2 also refuses, then they will choose simultaneously.

    After the bribery round concludes, then quantities are selected, and demand and then profits are calculated, and final payoffs are rewarded.

    Find a SPNE of this game.

12. (Pape) Consider this setting, which is similar to the 2x2 production model. There are two goods, X and Y, sold on the world market for exogenous prices p_x and p_y. Suppose there are three representative consumers, one endowed with $\bar{L}$ units of labor, and one endowed with $\bar{K}$ units of capital, and one endowed with the ownership of both firms (see below). Their utility functions are identical: $u(x, y) = xy$.

    Suppose there are two representative firms, one which produces X and one which produces Y. Their production functions are:

    $$f_x(K_x, L_x) = \alpha \ln K_x + (1 - \alpha) \ln L_x$$
    $$f_y(K_y, L_y) = \beta \ln K_y + (1 - \beta) \ln L_y$$

    with $0 < \alpha < \beta < 1$. For simplicity, assume that both firms are producing where the production functions are positive.

    Like in the 2x2 production model, capital and labor must be supplied locally but unlimited x and y are available from world markets at their exogenous prices; so x and y do not need to be produced locally in order to be purchased and consumed by the consumers.

    Suppose p_x increases. What happens to the local production of X? The local production of Y? Explain and compare to the standard 2x2 production model setting. Is the first consumer made better or worse off, or is it ambiguous? The second consumer? The third consumer? Explain and compare to the standard 2x2 production model setting.