Micro Comp (First attempt) May 29, 2018

This exam is organized into four sections, one for each quarter. Each section includes a short answer question and a longer, in-depth question.

1 Part A: Consumer and Producer Theory. (Pape)

1. Consider the following utility function over two goods:

$$u(x,y) \quad = \ln(x) + (10-y)^2$$

   (a) Are the preferences which correspond to this utility function transitive? Continuous? Monotonic? Explain each in turn.

   (b) Evaluate the following claim: y is a normal good. Is that claim always, never, or sometimes true? Explain *carefully.*

2. Suppose there are two people, A and B, who live in the same household. There are two goods: leisure l and premium cable TV packages x. A and B are each endowed with the number of hours $H > 1$ which they can allocate toward working or liesure; each hour that person i allocates toward working earns them a wage of $w_i > 0$, where $w_A > w_B$. Those earnings can be put toward purchase of x, which has a normalized price of 1.

   Since A and B live in the same household, premium cable TV packages x purchased by *either one* is consumed by both. However, they each only consume their own leisure. Suppose they have the same utility function: $u_i(x, l_i) = \alpha \ln(x) + (1-\alpha)\ln(l_i)$, $0 \leq \alpha \leq 1$, where $x = x_A + x_B$.

   (a) Suppose that they jointly decide xs and ls to maximize total utility $u_A + u_B$. Carefully set up the joint utility maximization problem. Should A work more, B work more, should they work the same amount, or is the answer indeterminate? Explain and justify your answer.

   (b) Now suppose that they choose their own xs and ls separately (although they still both consume total x.) Will A work more or less than she did in the previous part? Will B work more or less than she did in the previous part? Explain and justify your answer.

   (c) Are A and B better of under the regime in part a or in part b, or is the answer indeterminate? Explain.

2 Part B: Game Theory. (Pape)

3. Consider an $N > 2$ player game. The game is infinitely repeated, and all players discount future payoffs with the same $\delta \in (0, 1)$. Each round, players simultaneously choose one of two actions: A or B. The payoff for choosing each action is, unsuprisingly, a function of how many players choose each action, as follows:

$$\begin{aligned} PO\,(A|n \text{ opponents choose } B) &= 2n \\ PO\,(B|N-1 \text{ opponents choose } B) &= N \\ PO\,(B|n < N-1 \text{ opponents choose } B) &= 0 \end{aligned}$$

   (Note that since there are N players, each player has $N-1$ opponents.)

   (a) Find an SPNE in which all players play A on the equilibrium path, or argue why none exists. Does this SPNE only exist for some values of δ or N? Which values?

   (b) Find an SPNE in which all players play B on the equilibrium path, or argue why none exists. Does this SPNE only exist for some values of δ or N? Which values?

4. A seller offers a potential buyer an item at a price $p \geq 0$. The seller's payoff is p if the buyer accepts, and 0 otherwise. A type θ buyer ($0 \leq \theta \leq 1$) gets payoff $v + \theta - \frac{\theta^2}{2} - p$ if she accepts the seller's offer and $-\frac{\theta^2}{2}$ if she rejects the offer. Both agents know v. The buyer knows her type before the seller makes the offer, but the seller does not. The seller's initial belief about the buyer's type θ is represented by a uniform probability distribution on $[0, 1]$. Both agents are risk neutral.

   (a) Draw a game tree that can represent the interaction described above.

   (b) Explain what a pure strategy is for the buyer and give an example of such a strategy.

   (c) Find a subgame perfect equilibrium (SPE) of the game. Is it possible that there is a SPE in which the seller offers a price $p < v$?

   (d) Is there a Nash equilibrium of the game that is not a SPE? If not, show why not. If so, find an NE and compare the players' payoffs to what they get in the SPE of part c.

   Suppose for the remainder of the problem that the agents' interaction is the same as above except that before the seller announces an offered price, the buyer can choose her type θ from the interval $[0, 1]$. (Assume the seller knows the buyer can choose her type.) As before, the seller does not know the buyer's type when it announces its price offer.

   (e) Explain what a pure strategy is for the buyer in this new game and give an example of such a strategy.

   (f) Is there a pure strategy SPE for this game? Explain.

3 Part C: Information economics. (Tonguc)

5. For the following list of pairs of lotteries from (A)-(C), explain whether one lottery in the pair will be unambiguously preferred to the other lottery by the following economic agents:

   (i) Any expected utility maximizer

   (ii) Any risk neutral expected utility maximizer

   (iii) Any risk averse expected utility maximizer

   Justify your answer.

   (A) **Lottery A1** yields the random payoff \$$x$, where \$$x$ varies according to a distribution $F(x)$ with support [10, 20].

   Lottery A2 yields the random payoff \$$y$, where \$$y$ varies according to a distribution $G(y)$ with support [25, 26].

   (B) **Lottery B1** yields the random payoff \$$x$, where \$$x$ varies according to the normal distribution with mean 10 and variance 2.

   Lottery B2 yields the random payoff \$$y$, where \$$y$ varies according to the normal distribution with mean 10 and variance 3.

   (C) **Lottery C1** yields the random payoff \$$x$, where \$$x$ varies according to the distribution $F(x)$ with mean 10 and support over [5, 15].

   Lottery C2 is the degenerate lottery yielding the certain payoff \$$y = 5.1$.

6. A farmer farms land for an absentee owner. The farmer can choose effort levels $e \in \{3, 4\}$. She has expected utility over money and effort given by $u(w, e) = \sqrt{w} - e$. Farm profits take three possible random values, $\pi_1 < \pi_2 < \pi_3$. If the farmer chooses effort level $e = 3$, the distribution over profits is given by $p = (1/3, 1/2, 1/6)$ while if she selects effort level $e = 4$, the distribution over profits is $P = (1/6, 1/2, 1/3)$, where $p_i = Prob[\pi = \pi_i | e = 3]$ and $P_i = Prob[\pi = \pi_i | e = 4]$. The owner wishes to maximize expected net profits, $E[\pi - w]$. The farmer will work as long as she receives non-negative payoff.

   (a) Show that the two distributions generated by $e = 3$ and $e = 4$ satisfy a First Order Stochastic Dominance property and a monotone likelihood property.

   (b) Find the cost minimizing wage scheme assuming that effort can be observed and contracted on. Find a condition on π_i, $i = 1, 2, 3$ which implies that when effort can be contracted on, the owner would choose to induce $e = 4$.

   (c) Suppose that effort cannot be contracted on. Determine the cost-minimizing wage schedule for each effort level assuming that effort cannot be contracted on.

   (d) What is the risk premium of the 'gamble' that the farmer is being offered?

   (e) Find values of π_1, π_2, π_3 such that the owner would choose to induce $e = 4$ if effort could be contracted on, but would choose to induce $e = 3$ if effort could not be contracted on.

4 Part D: General Equilibrium. (Tonguc)

7. Consider an exchange economy with two identical consumers, $i = A, B$. Their common utility function is $u_i(x_1, x_2) = x_1^{\alpha} x_2^{(1-\alpha)}$ for $0 < \alpha < 1$. There are 10 units of x_1 and 10 units of x_2 in the economy. Find endowments ω_A and ω_B, where $\omega_A \neq \omega_B$ and Walrasian equilibrium prices that will support the allocation $((5,5),(5,5))$.

8. Imagine a three consumer economy (i=A,B,C) in which the first commodity is gardening services (x), the consumption of which makes one's yard more beautiful, and the second good is food (f). Suppose that two of the consumers (A and B) in this economy live in adjacent houses, while the third consumer lives on the other side of a large mountain. Consumption of gardening services by the third consumer (C) generates no externality for the other consumers, but consumption of gardening services by A and B generates a positive externality for their neighbor. To be precise, A and B have utility functions

$$U_i(x, f) = 0.5 \ln x_A + 0.5 \ln x_B + f_i, \ i = A, B$$

while consumer C has utility function

$$U_i(x, f) = 0.5 \ln x_C + f_C$$

Suppose that each consumer is initially allocated one unit of gardening services and two units of food.

   (a) Find the Walrasian Equilibrium.

   (b) Find the set of Pareto optimal allocations. Is the equilibrium allocation in part (a) Pareto optimal?