Prof. Kuhn

Question 1 (60 points)

Consider the problem of an infinitely lived consumer with habit persistence. Specifically, a household's current-period utility is $u(c_t, c_{t-1})$, where the utility funcion depends not only on current consumption c_t, but also on past consumption c_{t-1}. Because the consumer "gets used to" previous consumption levels, the partial derivative with respect to past consumption is negative, $\partial u(c_t, c_{t-1}) / \partial c_{t-1} < 0$. The consumer receives a constant endowment of e goods each period. There is a bond market, on which 1-period bonds are traded; specifically, in period t the household can buy or sell bonds b_{t+1} that promise 1 unit of the good at period $t+1$, at a price of q_t.

1. Carefully state the household's sequential problem. Write down the first-order conditions and find the Euler equation. (20)

2. Set up the associated Bellman equation. What are the state and control variables? Use the value function as an alternative way to derive the Euler equation. (20)

Now assume that there is a large number of households that all face the same problem as outlined above. Additionally, take as given the utility function $u(c_t, c_{t-1}) = \frac{\left(\frac{c_t}{c_{t-1}^{\gamma}}\right)^{1-\sigma}}{1-\sigma}$ where $\sigma > 1$ and $\gamma \in [0,1]$ measures the degree to which past consumption enters current utility.

3. Solve for consumption and the bond price in a steady-state equilibrium in which those values are constant over time. How does the steady-state bond price depend on the habit parameter γ? (10)

4. Now consider the case that, as a one-time event, last period's endowment was higher than usual; in other words $e_{t-1} > e$, and $e_{t+j} = e \, \forall j \geq 0$. What happens to the bond price in period t? (10)

Question 2 (60 points)

In an economy, a large mass of identical households work and consume. However, because working long hours is considered prestigious, a household considers not only her own hours worked l, but also the number of hours worked by the average worker $\bar{l}$ in her preferences: An individual household's utility function is given by

$$U\left(c, l, \bar{l}\right) = u(c) - v(l) + k\left[l - \bar{l}\right]$$

where the function $u(\cdot)$ is strictly increasing and strictly concave, and the function $v(\cdot)$ is strictly increasing and strictly convex (both functions are twice differentiable). The constant parameter k is positive, $k \geq 0$.

The perfectly competitive, representative firm hires workers at the wage w. It has a linear production function of $y = Al$.

1. Carefully define a competitive equilibrium for this economy. Characterize it using the agents' first-order conditions. Is the economy Pareto-optimal? (15)

2. State the Social Planner's problem, and derive the first-order conditions. (15)

3. Do households work more, fewer, or just the same number of hours in the competitive equilibrium compared to the Planner's solution? (15)

4. Now assume that $u(c) = \log c$, $v(l) = 2l^2$, and that $k = \frac{1}{4}$ and $A = 2$. The government considers introducing a tax τ_L on the households' labor income (which can be a subsidy if $\tau_L < 0$). The government redistributes the tax revenue through a lump-sum transfer on households T (again possibly negative). Can the government implement the allocation from the Planner's solution? If yes, what level of τ_L should be chosen? (15)