May 2008
Macro comprehensive exam RETAKE
Hanes' questions

1) Consider the Ramsey (Ramsey-Cass-Koopmans) model, with a rate of growth g in the labor-efficiency parameter A, and a rate of population growth n.

Assume that the economy is initially in a long-run steady state. Define c and k in the usual way.

a) Suppose there is an unexpected increase in the rate of population growth n.

i) Recall the phase diagram with k on the horizontal axis and c on the vertical axis. Using such a graph, show what happens to the economy in response to the event. Use the following symbols to label points:

(1) to label the point that is the initial LRSS before the event
(2) to label the point that is the combination of c and k immediately after the event
(3) to label a combination of c and k some time after the event, but before the new LRSS
(4) to label the point that is the new LRSS after the event.

ii) Consider what happens over time to consumption per person C/L. Draw a graph with the log of (C/L) on the vertical axis and time on the horizontal axis. Use t_0 to denote the point in time that the event occurs. Show what happens.

b) Suppose there is an unexpected increase in the rate of labor-efficiency growth g. Answer as for part a). That is:

i) Using a phase diagram, show what happens to the economy in response to the event, with:
(1) to label the point that is the initial LRSS before the event
(2) to label the point that is the combination of c and k immediately after the event
(3) to label a combination of c and k some time after the event, but before the new LRSS
(4) to label the point that is the new LRSS after the event.

ii) Draw a graph with the log of (C/L) on the vertical axis and time on the horizontal axis. Use t_0 to denote the point in time that the event occurs. Show what happens.

2) Consider a closed-economy IS-LM model with a fixed price level and a fixed money supply M, described by the following expressions:

$$\frac{M}{P} = L(i,Y) \quad where \quad L_i<0,\ L_Y>0$$

$$Y = E(Y,r,G,T) \quad where \quad 0 < E_Y < 1\ ,\ E_r < 0\ ,\ E_G>0,\ E_T<0$$

As usual, r denotes the real interest rate.

Derive $\partial Y/\partial P$ assuming that people believe the price level will return to a long-run steady-state value $\bar{P}$, so that $\pi^e = (\bar{P} - P)/P$.

3) Assume that $y_t = \rho y_{t-1} - \beta_t r_t$ where y is the output gap, r denotes the *gap* between the real interest rate and the natural rate of interest

and $\pi_t = {}_{t-1}\pi_t^e + \alpha y_t$

The central bank sets the real interest rate r_t to minimize a loss function of the ordinary type, with a desired inflation rate equal to *zero*:

$$L = E\left[\frac{1}{2}y_t^2 + \frac{1}{2}\pi^2\right]$$

At the beginning of period t, the central bank observes the realized value of y_{t-1}, then sets the real interest rate r_t. The central bank knows the true value of ρ, but it does *not* know the true value of β_t. The central bank only knows that β_t will be equal to a long-run mean value $\bar{\beta}$ *plus* a short-run disturbance e_t which is an i.i.d. random variable with mean zero and variance σ_e^2.

Finally, assume that ${}_{t-1}\pi_t^e$ is *always equal to zero, every period.*

a) Assuming that ${}_{t-1}\pi_t^e = 0$, write an expression that gives loss L in terms of r_t, y_{t-1}, σ_ε^2 and σ_e^2.

b) Using your answer from a), find the value of r_t that the central bank will set to minimize this loss.

c) Using your answer from b), what will be the resulting values of y_t and π_t, given the realized value of e_t?

d) If you observed many periods of time-series data from this economy, would you conclude that the central bank was following the "Taylor principle"? Why or why not?

e) Now consider the information available to the public at the time they form their expectation ${}_{t-1}\pi_t^e$. Assume that, at the time they form this expectation, they do *not* know the realized value of y_{t-1}, but they *do* know y_{t-2}. Given this information, is ${}_{t-1}\pi_t^e = 0$ a *rational expectation*? Explain why or why not.

4) Consider the effect of an increase in the foreign real interest rate r^* in an open economy with a floating exchange rate and *static* exchange-rate expectations (that is $\dot{\varepsilon}^e = 0$), in each of the specific cases listed below.

a) A "small" open economy with perfect capital mobility. The central bank follows an interest-rate rule $r(Y,\pi)$ *where* $r_Y > 0$, $r_\pi > 0$.

b) A "small" open economy with perfect capital mobility. The central bank holds the money supply M^S fixed. The price level P remains fixed and expected inflation π^e is always zero.

c) A "large" open economy with imperfect capital mobility. The central bank follows an interest-rate rule $r(Y,\pi)$ *where* $r_Y > 0$, $r_\pi > 0$. Net exports depends only on the exchange rate, that is net exports can be described by $NX(\varepsilon)$.

d) A "large" open economy with imperfect capital mobility. The central bank holds the money supply M^S fixed. The price level P remains fixed and expected inflation π^e is always zero. Net exports depends only on the exchange rate, that is net exports can be described by $NX(\varepsilon)$.

5) In the Diamond OLG model, it is possible for an economy to have "too much" capital in its LRSS. Explain.

6) In the "new Keynesian" model that Clarida, Gali and Gertler use to analyse monetary policy,

$$x_t = -\varphi r_t + {}_{t}\pi^{e}_{t+1} + g_t$$

$$\pi_t = {}_{t}\pi^{e}_{t+1} + \lambda x_t + u_t$$

where x is the output gap, g is the "spending shock," u is the "cost-push" shock, and the central bank can observe the values of both g and u when setting the real interest rate.
Suppose the economy is subject *only* to cost-push shocks: g is always zero.

a) Describe what the central bank will do, and the results for the economy, if the central bank has "discretion" (it cannot "pre-commit" in its policy actions).
b) How is policy different if the central bank can "pre-commit" in its policy actions?

7) For each of the following Phillips curves, tell me:

i) The model or models that generate that Phillips curve.

ii) In words, describe the microeconomic assumptions of that model or models: what are the relevant agents; what is the structure of product markets; what are the key constraints on the agents' information or behavior.

iii) What are the parameters of the model that determine the Phillips curve's output-gap coefficient? I am not looking for Greek letters here; I want you to tell me in words what the parameter or parameters mean.

a) $\pi_t = \beta y_t + \lambda \sum_{j=0}^{\infty} (1-\lambda)^j E_{t-1-j}(\pi_t + \alpha \Delta y_t)$

b) $\pi_t = \beta y_t + {}_{t-1}\pi^{e}_{t}$

c) $\pi_t = \beta y_t + {}_{t}\pi^{e}_{t+1}$

d) $\pi_t = \beta\left(y_t + y_{t-1} + {}_{t}y^{e}_{t+1} + {}_{t-1}y^{e}_{t}\right) + \frac{1}{2}\left({}_{t-1}\pi^{e}_{t} + {}_{t}\pi^{e}_{t+1}\right)$

8) Suppose an economy's representative household acts to maximize:

$$E\left[\sum_{t=0}^{\infty} \beta^t \left(\ln(C_t) - \theta L_t^2 - \frac{1}{1-\sigma}(M_t/P_t)^{1-\sigma}\right)\right]$$

$$\text{subject to } Z_{t+1} = \frac{P_t}{P_{t+1}}\left[\frac{M_t}{P_t} + (1+i_t)\left(Z_t - \frac{M_t}{P_t}\right) + w_t L_t - C_t\right]$$

where Z_{t+1} is real wealth entering period $(t+1)$, M_t is the nominal money balance held across period t, and i_t is the nominal interest rate paid on nonmoney assets held across period t. w_t is the real wage. Derive the quantity of real money balance M_t/P_t that a household will choose to hold, as a function of consumption C_t and the nominal interest rate i_t.

9) In "Resuscitating Real Business Cycles," King and Rebelo present an RBC model in which *slowdowns* in productivity growth can generate absolute *decreases* in output. Explain how.

PART 2 XIAO

1. Consider an economy with endogenous technological process and no depreciation of capital stock. There are a large number of firms and a large number of consumers. There is no population growth and the aggregate supply of labor is fixed. Each firm, say firm i, uses the following production technology to produce a homogeneous consumption good:

$$Y_i = AK_i^{\alpha}L_i^{1-\alpha}, 0 < \alpha < 1. \tag{1}$$

Suppose the technology level is equally accessible to all firms, and depends on the average level of capital stock, K:

$$A = BK^{1-\alpha}. \tag{2}$$

   (a) Solve a typical firm's profit maximization problem and derive expressions for the equilibrium rate of interest (rental rate for capital) and wage. If all firms are identical, show that the interest rate is

$$r = \alpha BL^{1-\alpha}. \tag{3}$$

   (b) Suppose the life-time utility of the representative consumer is

$$\int_0^{\infty} \frac{C^{1-\gamma}}{1-\gamma} e^{-\rho t} dt,$$

   where both ρ and γ are positive parameters. The consumer's income comes from wage and rental income (consumers rent their capital to firms). Write down the budget constraint for the consumer.

   (c) Set up the current value Hamiltonian and solve for the first order optimality conditions for the consumer.

   (d) Show that the growth rate of consumption is given by

$$\frac{\dot{C}}{C} = \frac{\alpha BL^{1-\alpha} - \rho}{\gamma}. \tag{4}$$

   (e) Solve the social planner's problem for this economy, and also derive the growth rate of consumption. Is the growth rate the same as that in equation (4)? If yes, explain why they are the same. If not, explain why they are not the same.

2. In this question, you will be solving an asset pricing model with production. Consider the problem faced by a representative consumer who seeks to maximize expected lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\theta}}{1-\theta} - Al_t\right), \tag{5}$$

where c_t is consumption and l_t is work hours. There are n firms, each produces a dividend stream d_{it}. The consumer owns 1 share of stock from each firm at time 0. The time t budget constraint is

$$c_t + \sum_{i=1}^{n} p_{it} z_{it+1} = \sum_{i=1}^{n} z_{it}(p_{it} + d_{it}) + w_t l_t \tag{6}$$

where w_t is the wage rate, p_{it} is the price of a share in firm i, and z_{it} is the quantity of shares of firm i held by the consumer at time t.

(a) Solve the consumer's problem by deriving all necessary conditions. Derive the pricing equation for the stock return R_{it}.

(b) A typical firm, say firm i, tries to maximize the expression

$$E_t \sum_{t=0}^{\infty} \beta^t \frac{u'(c_{t+j})}{u'(c_t)} d_{it+j},$$

where j goes from 0 to infinity. Explain the intuition of this objective function.

(c) Dividends are defined as

$$d_{it} = y_{it} - w_t l_t - I_{it}, \tag{7}$$

where y_{it} is the production function defined as

$$y_{it} = A_t f(k_{it}, l_t), \tag{8}$$

where A_t is the usual technology shock, and k_{it} is firm i's capital stock. Assume f is h.d.1. I_t is the firm's investment defined as

$$I_{it} = k_{it+1} - (1 - \delta) k_{it}, \tag{9}$$

where δ is the depreciation rate. Explain the intuition for equation (7).

(d) Plug the three equations in part c into the firm's objective function. The firm chooses k_{it+1} and l_t to maximize the objective. Obtain the first order conditions. Provide some intuitions for these conditions.

(e) Combine the first order condition for capital you obtained here and the first order condition for assets in the consumer's problem. What can you say about the relationship between expected asset returns and expected (net) real return to (or marginal product of) capital?