X IPC JAN 2008
PART I MACRO COMP

Instructions. Answer the following questions. Each question consists of several steps. Complete each step to the best your ability. Good luck.

1. Consider a closed economy with many identical representative agents. At date t there are $N(t)$ persons devoted to production. The growth rate of N is λ. Real per-capita consumption is $c(t)$ units of a single good. Preferences are given by

$$\int_0^\infty e^{-\rho t}\frac{1}{1-\sigma}[c(t)^{1-\sigma}-1]N(t)dt,$$

where ρ and σ are both positive. Let $K(t)$ denote the total stock of capital. Total output is expressed as

$$N(t)c(t)+\dot{K}(t)=A(t)K(t)^{\beta}N(t)^{1-\beta}, \tag{1}$$

where $0<\beta<1$, and the rate of technology change, $\dot{A}/A$, is μ.

   (a) What is the economic meaning of equation (1)?

   (b) Write down the current-value Hamiltonian for the planning problem, and derive all necessary first order conditions. Explain the economic intuition for each condition.

   (c) We focus on the balanced growth path, on which all endogenous variables grow at constant rates. Denote the growth rate of $c(t)$ by π. Show that on such a path, the marginal product of capital must equal $\rho+\sigma\pi$. That is

$$\beta A N^{1-\beta}K^{\beta-1}=\rho+\sigma\pi$$

   (d) Further show that

$$\frac{\dot{K}}{K}=\frac{\dot{N}}{N}+\frac{\dot{c}}{c}=\pi+\lambda$$

   (e) In this economy, what is the decisive factor that determines the rate of economic growth π?

   (f) What is endogenous growth? Does this economy have endogenous growth? If so, explain why. If not, how would you modify this model to make endogenous growth possible?

2. In the standard RBC model capital is fully utilized. In real life there is evidence that capital is sometimes under-utilized. Next we consider a model in which only u_t percent of capital stock is utilized. A representative agent chooses sequences of consumption c, hours to work n, rate of capacity utilization u and capital stock k to solve:

$$\max E_t\sum_{t=0}^{\infty}\beta^t(\log c_t-\frac{n_t^{1+\gamma}}{1+\gamma}) \tag{2}$$

$$s.t.\ c_t+k_{t+1}-(1-\delta_t)k_t=y_t \tag{3}$$

$$\delta_t = \tau u_t^{\theta},\ 0<\tau<1,\ \theta>1 \tag{4}$$

$$y_t = A_t(u_tk_t)^{\alpha}n_t^{1-\alpha} \tag{5}$$

where $\gamma\geq 0$, $0<\alpha<1$, and A_t is technology, $u_t\in(0,1)$ is the rate of capacity utilization, $\delta_t\in(0,1)$ is the rate of depreciation defined as an increasing function of capacity utilization.

   (a) Provide the economic intuition for (4).

   (b) Linearize the resource constraint (3) around the steady state.

   (c) Derive all necessary first order conditions for this problem. Explain their economic meanings.

   (d) Assuming $\tau=1/\theta$. Show that

$$u_t=(\alpha\frac{y_t}{k_t})^{1/\theta} \tag{6}$$

   What happens to the utilization rate if capital increases?

   (e) What is an impulse mechanism and what is a propogation mechanism? What possible effects does the inclusion of u_t have on the these mechanisms?

PART 2 ANSWER ALL

January 2007
Macro comprehensive exam
Hanes' questions

1) Consider the Ramsey (Ramsey-Cass-Koopmans) model, with a rate of growth g in the labor-efficiency parameter A, and a rate of population growth n.

Assume that the economy is initially in a long-run steady state. Define c and k in the usual way.

a) Suppose there is an unexpected, one-time upward jump (increase) in the population, with no change in the steady growth rate n.

i) Recall the phase diagram with k on the horizontal axis and c on the vertical axis. Using such a graph, show what happens to the economy in response to the event. Use the following symbols to label points:

(1) to label the point that is the initial LRSS before the event
(2) to label the point that is the combination of c and k immediately after the event
(3) to label a combination of c and k some time after the event, but before the new LRSS
(4) to label the point that is the new LRSS after the event.

ii) Consider what happens over time to consumption per person C/L. Draw a graph with the log of (C/L) on the vertical axis and time on the horizontal axis. Use t_0 to denote the point in time that the event occurs. Show what happens.

b) Suppose there is an unexpected, one-time upward jump (increase) in A, with no change in its steady growth rate g. Answer as for part a). That is:

i) Using a phase diagram, show what happens to the economy in response to the event, with:
(1) to label the point that is the initial LRSS before the event
(2) to label the point that is the combination of c and k immediately after the event
(3) to label a combination of c and k some time after the event, but before the new LRSS
(4) to label the point that is the new LRSS after the event.

ii) Draw a graph with the log of (C/L) on the vertical axis and time on the horizontal axis. Use t_0 to denote the point in time that the event occurs. Show what happens.

2) Consider a closed-economy IS-LM model with a fixed price level and a fixed money supply M, described by the following expressions:

$$\frac{M}{P} = L(i, Y) \quad where \quad L_i<0,\ L_Y>0$$

$$Y = E(Y, r, G, T) \quad where \ 0 < E_Y < 1\ ,\ E_r < 0\ ,\ E_G>0,\ E_T<0$$

As usual, r denotes the real interest rate. Consider the effect of an exogenous change in expected inflation π^e. Derive an expression showing the effect of a change in π^e on output Y, and an expression showing the effect of a change in π^e on the nominal interest rate i.

3) Consider the Malthusian model of economic growth, with the rate of population growth described by:

$$n = G(w - \sigma) \qquad G'(\,) > 0$$

where w is the real wage received by peasants.
Assume the economy is initially in its long-run steady state.

a) Describe what happens to the population and the real wage if there is a permanent decrease in the parameter σ. Use *words in full sentences* and a graph with the real wage on the vertical axis, population on the horizontal axis.

b) Consider an economy where the population's birth rate b (births per capita, per unit time) and mortality rate d (deaths per capita, per unit time) are given by:

$$b = c + fw \qquad\qquad d = h - jw$$

where c, f, h and j are all positive parameters.

i) Is this economy a special case of the Malthusian model, or is it fundamentally different from the Malthusian model? Explain, using *words in full sentences* and an equation or equations.

ii) Suppose that improvements in public order *reduce* the death rate that would occur at any given real wage. Explain how this would affect the peasants' standard of living in the long run, using *words in full sentences* and an equation or equations.

4) Consider an economy where

$$y_t = -\beta r_t$$

$$\pi_t = {}_{t-1}\pi_t^e + \alpha y_t + \varepsilon_t$$

In these expressions, y is the output gap and r_t is the *difference* between the real interest rate and the natural rate of interest. The real interest rate is $i_t - {}_t\pi_{t+1}^e$.

ε_t is an i.i.d. random variable. As of period (t-1), the expected value of ε_t is zero.

The central bank sets the nominal interest rate i_t. When it sets i_t, it knows the value of ε_t and the public's inflation expectations. The central bank acts to minimize a loss function:

$L = \frac{1}{2} E[y_t^2] + \frac{1}{2} E[(\pi_t - \pi^*)^2]$ where y is the output gap.

a) Derive the value of r_t that the central bank sets taking ${}_{t-1}\pi_t^e$ as given.

b) Now suppose the economy is in rational expectations equilibrium. Derive the value of r_t that the central bank sets in rational expectations equilibrium, and the values of y_t and π_t that will result.

c) Write down an equation that gives the nominal interest rate i_t as a function of the same period's inflation rate π_t, denoting the natural rate of interest by $\bar{r}$.

5) In all of the following models, the outcome of the market economy is or can be "inefficient," that is not Pareto optimal. For each model, explain why the market economy can be inefficient.

a) Diamond OLG model.

b) Romer's baseline static monopolistic-competition model in a *flexible*-price equilibrium, with the price level fully adjusted to aggregate demand m.

c) Romer's baseline static monopolistic-competition model with menu costs, in a *fixed*-price equilibrium following a downward shock to aggregate demand (m turned out to be surprisingly low).

6) Consider an economy like Romer's baseline real business cycle model. For simplicity, assume that the population is fixed and there is one person per household. The immortal representative-agent person-household acts to maximize:

$$E\left[\sum_{t=0}^{\infty} e^{-\rho t}\left(\frac{c_t^{1-\theta}}{1-\theta} - z l_t^2\right)\right]$$

where z is a parameter and notation is as usual: l is the fraction of his time that a household-person supplies as labor and c is his consumption.The technology parameter A is has a long-run trend growth rate g. Let r denote the real interest rate, equal to the return to capital after depreciation, and w denote the real wage per unit of labor (not per efficiency-unit of labor).

a) Write down the "intratemporal first-order condition" that relates a person's consumption c in a period to the same period's real wage w and labor-supply fraction l.

b) In a nonstochastic long-run steady state, both the real wage and consumption must be growing at rate g. Using this fact and your answer to a), demonstrate that the value of the felicity-function parameter θ must be one, so that the utility function is equivalent to:

$$E\left[\sum_{t=0}^{\infty} e^{-\rho t}\left(\ln(c_t) - z l_t^2\right)\right]$$

c) Now consider a model with "habit formation" in the utility function, so that:

$$E\left[\sum_{t=0}^{\infty} e^{-\rho t}\left(\frac{(c_t - b c_{t-1})^{1-\theta}}{1-\theta} - z l_t^2\right)\right]$$

i) Using appropriate mathematical expressions and words in full sentences, explain why this change to the felicity function tends to create "persistence" in consumption.

ii) In order for this habit-formation model to have a long-run steady state, the value of the felicity-function parameter θ must be one, so that the utility function is equivalent to:

$$E\left[\sum_{t=0}^{\infty} e^{-\rho t}\left(\ln(c_t - b c_{t-1}) - z l_t^2\right)\right]$$

Using appropriate mathematical expressions and words in full sentences, explain why this is true.

7) Consider the old-Keynesian Friedman-Phelps Phillips curve $\pi_t = {}_t{}_1\pi_t^e + \beta y_t$

versus the new-Keynesian Phillips curve $\pi_t = {}_t\pi_{t+1}^e + \beta y_t$

and their implications for the time-series behavior of the output gap y and inflation π under rational expectations.

a) In what way is the new-Keynesian Phillips curve (plus rational expectations) *more* consistent with the actual time-series behavior of the output gap y in reality? Use words in full sentences *and* an equation or equations in your answer.

b) In what way is the new-Keynesian Phillips curve (plus rational expectations) *less* consistent with the actual time-series behavior of inflation π in reality? Use words in full sentences *and* an equation or equations in your answer.

8) In the "new Keynesian" model that Clarida, Gali and Gertler use to analyse monetary policy,

$$x_t = -\varphi r_t + {}_t\pi_{t+1}^e + g_t$$

$$\pi_t = {}_t\pi_{t-1}^e + \lambda x_t + u_t$$

where x is the output gap, g is the "spending shock," u is the "cost-push" shock, and the central bank can observe the values of both g and u when setting the real interest rate.
Consider the case where the central bank has "discretion" (it cannot "pre-commit" in its policy actions).

a) Describe what the central bank will do, and the results for the economy, if the economy is subject *only* to spending shocks - that is, if u is always zero.

b) Describe what the central bank will do, and the results for the economy, if the economy is subject *only* to cost-push shocks - that is, if g is always zero.

9) In some models, there is a welfare cost - a loss of profit or utility - associated with an increase in the long-run, predictable, trend inflation rate. In other models, there is no such welfare cost. For each of the models listed below, state whether there is such a welfare cost of higher trend inflation, and *explain* why or why not.

a) Lucas supply function model.

b) Rotemberg's model of a price adjustment cost that increases quadratically with the size of the adjustment.

c) Mankiw-Reis sticky-information model.