Macro comprehensive examination, Hanes' questions, January 2009

1) Consider an economy described by the Malthusian model of economics growth. The real wage is denoted w. The birth rate is: $b = \beta + \gamma w$

The death rate is: $d = \delta - \varphi w$

The aggregate production function is: $Y = Land^{\alpha} L^{1-\alpha}$

Derive the long-run equilibrium size of the peasant population L in terms of the parameters $\alpha, \beta, \gamma, \delta, \varphi$ and the quantity of $Land$.

5 pts. To find the long-run steady state wage σ, set b equal to d and solve for w. So:

$$e = \frac{\delta - \beta}{\gamma + \varphi}$$

To find long-run equialibrium L, find the value of L that causes the marginal product of labor to equal σ. So:

$$e = (1-\alpha) Land^{\alpha} L^{-\alpha}$$

$$\Rightarrow L = Land \left(\frac{(\gamma+\varphi)(1-\alpha)}{\delta - \beta} \right)^{\frac{1}{\alpha}}$$

2) Consider an economy that can be described by the Diamond OLG model.

The aggregate production function is Cobb-Douglas: $Y = K^{\alpha} (AL)^{1-\alpha}$ *where* $0 < \alpha < 1$

There is no depreciation. The rate of growth of population is n. The rate of growth of A is g. A person's lifetime utility function (lifetime utility as a function of first-period consumption C_1 and second-period consumption C_2) is:

$$U = C_1^{\beta} [(\tfrac{1}{1+\rho}) C_2]^{(1-\beta)} \quad where \quad 0 < \beta < 1$$

a) **Derive** s as defined for the OLG model,that is the fraction of a young person's income (labor income) devoted to saving. *4 pts.*

$$s = \frac{w - c_1}{w} = 1 - \frac{c_1}{w}$$

$$c_2 = (1+r)(w - c_1)$$

Chose c_1 to maximize

$$u = c_1^{\beta} \left[\frac{1}{1-\rho} (1+r)(w - c_1) \right]^{1-\beta}$$

From $\frac{\partial u}{\partial c_1} = 0$, get $c_1 = \beta w$ so $\frac{c_1}{w} = \beta$

so $s = 1 - \beta$

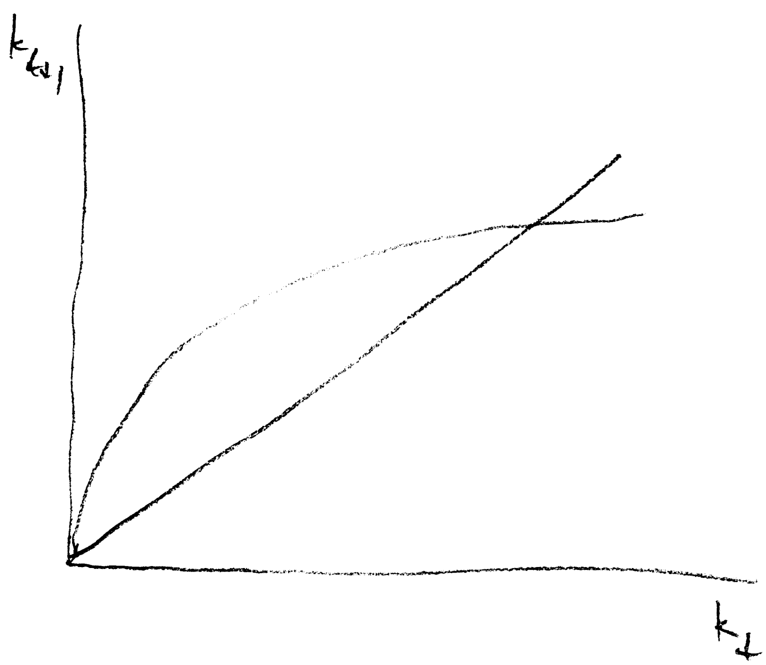
b) Using your answer from a) and the Cobb-Douglas production function, write down the equation that gives k_{t+1} as a function of k_t. *3 pts.*

$$k_{t+1} = \frac{1}{(1+\ )(1+\gamma)} s w = \frac{1}{(\ )(\ )} (1-\beta)(1-\alpha) k_t^{\alpha}$$

c) In this economy, is there a long-run steady state? Is it stable? Is there more than one long-run steady state? Briefly, explain how you know. *3 pts. There is one stable LRSS. You know this because* $G(k_t)$ *is concave. You know* $G(k_t)$ *is concave because its first derivative is positive, its second derivative is negative.*

$$\partial k_{t+1} / \partial k_t = \dots \alpha(1-\alpha) k_t^{\alpha-1}$$

$$\partial^2 k_{t+1} / \partial k_t^2 = \dots \alpha(1-\alpha)(\alpha-1) k_t^{\alpha-2}$$



3) Consider the old-Keynesian Friedman-Phelps Phillips curve $\pi_t = {}_{t-1}\pi_t^e + \alpha y_t$
and the new-Keynesian Phillips curve $\pi_t = {}_t\pi_{t+1}^e + \alpha y_t$
where y denotes the output gap.

a) The old-Keynesian Phillips curve *plus* rational expectations appears inconsistent with time-series data on output gaps and inflation. Explain. *4 pts. The old-Keynesian Phillips curve plus rational expectations implies that the expected value of the future output gap is always zero. This would mean that there is no serial correlation in the output gap, and that people never forecast an output gap other than zero. Not true. See class notes.*

b) The new-Keynesian Phillips curve *plus* rational expectations appears inconsistent with time-series data on output gaps and inflation. Explain.*4 pts. The new-Keynesian Phillips curve plus rational expectations implies that, when the output gap is negative (positive), the expect future inflation rate is higher (lower) than the current inflation rate. That means inflation should be rising in a recession, falling in a boom. Not true. See class notes.*

4) Write down an equation describing the Phillips curve in the model of Christiano, Eichenbaum and Evans (2005). Explain what each letter denotes. Explain the assumptions that give this Phillips curve. *4 pts. Their Phillips curve is, in effect,*

$$\pi_t = \pi^* + \frac{1}{2}(\pi_{t-1} + {}_{t-1}\pi_{t+1}^e) + stuff\ y_t$$

where y is the output gap and π^* is the long-run steady-state inflation rate. The key assumptions are that a firm is allowed to adjust its price randomly, as in the Calvo pricing model. If a firm is not allowed to adjust its price, its price does not remain fixed as in the Calvo model, but is scaled up by last period's general inflation rate, that is $\square_t = (1+\pi_{t-1})p_{t-1}$.

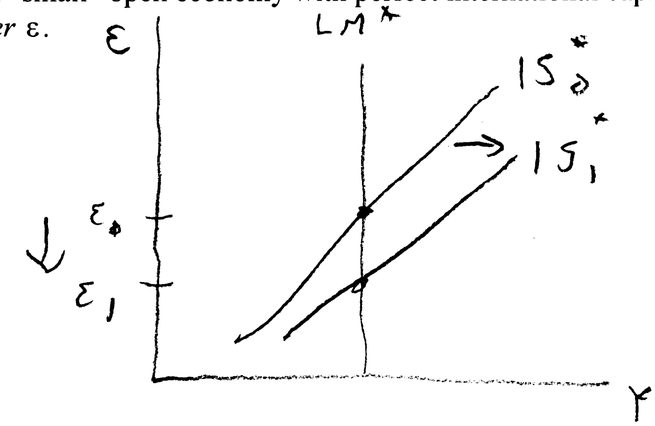
5) Consider an open economy with a floating exchange rate. Assume that:
- the central bank conducts monetary policy by choosing a value for the money supply M
- the price level is fixed; expected future inflation is always zero
- there is "static exchange-rate expectations," that is the expected future change in the exchange rate is always zero.

Using graphs and words, for each of the following cases, explain what happens to *output*, the *exchange rate*, and the *domestic interest rate* in response to this event: an increase in exports at any given value of the exchange rate.
a) A "large" economy with imperfect international capital mobility. *4 pts. First, figure out whether the IS** curve shifts. At any given interest rate, what happens to net exports? From the equation:*

$$CF(i-i^*) + NX(\varepsilon) = 0$$

see that there can be no change in net exports at a given interest rate. So if there is an increase in exports at any given exchange rate, the exchange rate must fall (appreciate) to counteract that and keep net exports unchanged. Result: output same, exchange rate falls, domestic interest rate same.

b) A "small" open economy with perfect international capital mobility. *4 pts. Same Y, same i, lower ε.*



6) Consider an economy where $y_t = -\beta_t(r_t - \bar{r}) + \varepsilon_t$

where y is the output gap. One can assume that ε is an i.i.d. random variable (mean zero, no serial correlation). Alternatively, one can assume that ε is a constant, always equal to zero. The parameter on the real interest rate is $\beta_t = \beta + \nu_t$

One can assume that ν is an i.i.d. random variable (mean zero, no serial correlation). Alternatively, one can assume that ν is a constant, always equal to zero.

Also, $\pi_t = {}_{t-1}\pi_t^e + \alpha y_t + e_t$

One can assume that e is an i.i.d. random variable. Alternatively, one can assume that it is a constant, always equal to zero.

The central bank sets r_t to minimize a loss function:

$$L = E\left[\frac{1}{2}y_t^2 + \frac{1}{2}(\pi - \pi^*)^2\right]$$

The public has rational expectations. It knows the central bank's loss function and desired inflation rate π^*. But the public *cannot* observe ε_t, ν_t or e_t at the time that it forms its inflation expectation ${}_{t-1}\pi_t^e$.

Note I have *not* assumed anything about the central bank's information about ε_t, $\nu_t$and e_t.

Consider various possible assumptions about ε_t, $\nu_t$and e_t, and the central bank's information about them.

I did not ask you to derive anything for this question. *It was meant to be intuitive. Each section 5 pts.*

a) Under what circumstances - that is, under what specific assumptions about ε, e, and the central bank's information - will this economy show *no* correlation between r_t and y_t? *In examples, we saw that there would be no relation between r and y if there were no Brainard uncertainty and the central bank was responding to foreseeable IS shocks: that gives stable y with varying r. Here, this means ν fixed (no Brainard uncertainty), e fixed, ε varies and is known to central bank. Alternatively, the central bank might be unable to foresee any shocks: then the central bank sets the same interest rate every period, and output varies.*

b) Under what circumstances - that is, under what specific assumptions about ε , *e*, and the central bank's information - will this economy show a *positive* correlation between r_t and y_t? *This occurs if there are foreseeable IS shocks, but the central bank under-responds to foreseen supply shocks because there is Brainard uncertainty. Here, this means e is fixed (no supply shocks),* ε *varies and is known to central bank (foreseeable IS shocks),* v *varies and unknown to central bank (Brainard uncertainty).*

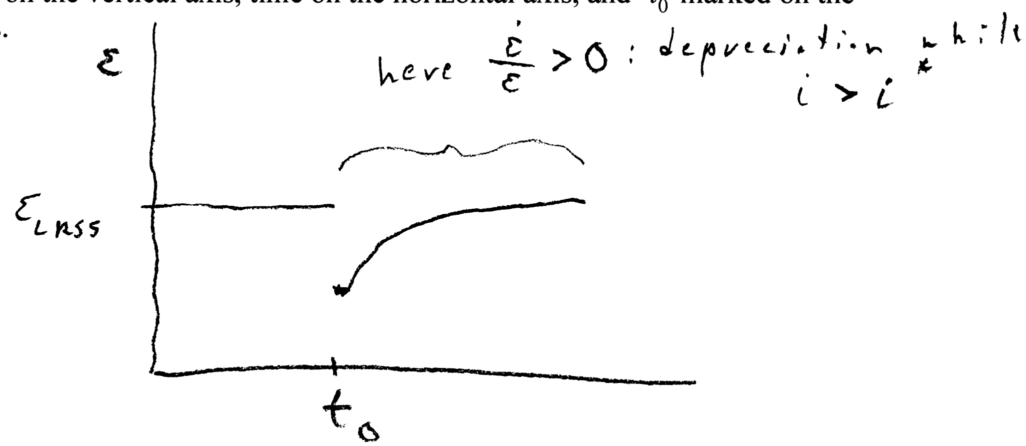
c) Under what circumstances - that is, under what specific assumptions about ε , *e*, and the central bank's information - will this economy show a *negative* correlation between r_t and y_t? *In response to supply shocks (cost-push shocks), , a central bank will partly counteract the effect on inflation by changing output. For example, when the central bank foresees a positive cost-push shock it raises r to depress y to keep inflation from rising so much. Here, that means* v *fixed (no Brainard uncertainty),* ε *fixed, e varies and is known to central bank.*

7) Consider a small open economy with a floating exchange rate and perfect international capital mobility (uncovered interest-rate parity prevails). The central bank follows an interest-rate rule with random deviations:

$$i_t = \bar{r} + {}_t\pi^e_{t+1} + \tau(\pi_t - \pi^*) + e_t \quad \textit{where} \quad e_t = \rho e_{t-1} + f_t \quad \textit{and} \quad 0 < \rho < 1$$

where f is an i.i.d. random variable and π^* is the central bank's target inflation rate. Note that the parameter ρ determines the degree of serial correlation in the central bank's deviations from the rule. Suppose that in the last few periods, the value of *e* has happened to be zero, the output gap has been zero, and inflation has been equal to π^*. Then, at time t_0, there is a positive realization of *f*. Meanwhile, there is no change in foreign interest rates.

a) Draw a graph that shows how the real exchange rate ε responds. The graph should have the real exchange rate ε on the vertical axis, time on the horizontal axis, and t_0 marked on the horizontal axis. 3 pts.



b) Consider the *magnitudes* of the initial response of the real exchange rate ε at time t_0. Is the magnitude related to the value of ρ? That is, if ρ is large, does that tend to increase, decrease, or have no effect on the size of the immediate response of the exchange rate to the interest-rate shock? *3 pts. If* ρ *is big, we expect r to remain high for longer. So we must have depreciation for a longer span of time. So there must be a bigger appreciation in the exchnage rate right now - bigger response.*

8) In a model with an old-Keynesian IS curve, and old-Keynesian Phillips curve, and a central bank that acts to minimize a conventional loss function, there is no "dynamic inconsistency" problem as long as the central bank behaves "as if" the desired output level is the natural rate of output. Clarida, Gali and Gertler show that, in a "new Keynesian" model, there can be a dynamic inconsistency problem even if the central bank behaves as if the desired output level is the natural rate of output. Explain why this is true. *5 pts. The new-Keynesian dynamic inconsistency problem is about* ***supply shocks*** *(or cost-push shocks) specifically. In response to a supply shock, a central bank that is minimizing a loss function will partly counteract the effect on inflation by changing output (for a positive supply shock, raise r to reduce output), with or without precommitment. With precommitment, the central bank will respond strongly (for a positive supply shock, raise r to reduce output a lot, though not enough to keep inflation unaffected): this reduces variance in both output and inflation. Without precommitment, the central bank will choose to respond more weakly, though it will still respond.*

9) Suppose a firm can enter into a long-term contract with its single employee. The firm produces according to $Q = L^{\alpha}$ *where* $0 < \alpha < 1$ and L is the quantity of labor provided by the single employee. The firm sells in a competitive market at an uncertain price Z. The price Z has a discrete distribution with k possible values. The probability that the price takes a possible value Z_i is denoted p_i. That is, p_i is the probability that the price will be Z_i. The expected value of Z is thus:

$$E[Z] = \sum_{i=1}^{k} p_i Z_i$$

The contract specifies an employment level L_i and a payment to the single employee C_i for each possible realization of the price Z_i. The employee acts to maximize an expected utility function of consumption C and labor L: $E\ [\ \ln(C)\ -\ L^2\ ]$

This problem is exactly the same as Econ 614 final exam fall 2008 number 3).

a) Write down the expected value of the firm's profit in terms of p_i, Z_i, C_i and L_i. *3 pts.*

b) The firm chooses the form of the contract to maximize its expected profit, subject to the constraint that the contract must give the work expected utility greater than or equal to u_0. Write down the Lagrangian that describes the firm's problem. *3 pts.*

c) From the first-order conditions for maximizing the Lagrangian, derive the worker's consumption C_i and labor L_i for a realized Z_i, in terms of the Lagrangian multiplier λ. *4 pts.*

d) Using your answer to c), explain how the worker's labor and consumption are related to the realized value of Z. *4 pts.*

10) Consider an economy like Romer's baseline real business cycle model, but with "habit formation" in consumption. For simplicity, assume that the population is fixed and there is one person per household. The immortal representative-agent person-household acts to maximize:

$$E\left[\sum_{t=0}^{\infty} e^{-\rho t}\left(\frac{(c_t - b c_{t-1})^{1-\theta}}{1-\theta} - z l_t^2\right)\right] \quad \textit{where} \quad 0 < b < 1$$

where z is a parameter and notation is as usual: l is the fraction of his time that a household-person supplies as labor and c is his consumption.The technology parameter A is has a long-run trend growth rate g. Let r denote the real interest rate, equal to the return to capital after depreciation, and w denote the real wage per unit of labor (not per efficiency-unit of labor).

This problem is the same as *as Econ 614 final exam fall 2008 number 7).*

a) Write down the "intratemporal first-order condition" that relates a person's consumption c in a period to the same period's real wage w and labor-supply fraction l. *3 pts.*

b) In a nonstochastic long-run steady state, both the real wage and consumption must be growing at rate g. That is to say,

$$\frac{c_{t+1}}{c_t} = \frac{w_{t+1}}{w_t} = e^{g}$$

Using this fact and your answer to a), demonstrate that the value of the felicity-function parameter θ must be one, so that the utility function is equivalent to:

$$E\left[\sum_{t=0}^{\infty} e^{-\rho t}\left(\ln(c_t - bc_{t-1}) - zl_t^2\right)\right]$$

5 pts.