February 2009
Macro comprehensive exam RETAKE
Hanes' questions

PART I
ANSWER ALL

1) Consider a closed-economy IS-LM model with a fixed price level and a fixed money supply M, described by the following expressions:

$$\frac{M}{P} = L(i,Y) \quad where \quad L_i<0,\ L_Y>0$$

$$Y = E(Y,r,G,T) \quad where\ 0 < E_Y < 1\ ,\ E_r < 0\ ,\ E_G>0,\ E_T<0$$

As usual, r denotes the real interest rate.

a) Derive $\partial Y/\partial P$ assuming that people believe the price level will return to a long-run steady-state value $\bar{P}$, so that $\pi^e = (\bar{P}-P)/P$.

b) Derive $\partial Y/\partial P$ assuming that people believe the current change in the price level will continue in the future, so that $\pi^e = \partial P/P$.

c) Is the AD curve steeper in case a) or in case b)?

2) Consider an efficiency-wage model where labor effort is $e = \left(\frac{w_i - X}{X}\right)^{\beta}$.

a) What is the value of the wage that maximizes a firm's profit, taking X as given?

b) Assume $X=(1-bu)w$, where w is the wage paid by all other firms and u is the unemployment rate. Assuming all firms are identical, what is the NAIRU (or u_{EQ}) in this economy?

3) Consider Romer's Model of Imperfect Competition, in which each household operates a monopoly firm, supplies labor to a perfectly competitive labor market, and acts to maximize:

$$U_i = C_i - \frac{1}{\gamma} L_i^{\gamma} \qquad where \qquad \gamma > 1$$

where C_i is a household's consumption (money income divided by the price index) and L_i is the quantity of labor supplied by the household to the perfectly competitive labor market. A firm/household gets one unit of output Q_i for each unit of labor input H_i hired from the perfectly competitive labor market: $Q_i = H_i$

Demand for one producer's product can be described by: $Q_i = (P_i/P)^{-\eta}\ Y$ where Y is per capita real income.

a) From the utility function, derive an expression that gives one household's labor supply L_i as a function of the real wage W/P.

b) What is the value of L_i in the flexible-price equilibrium, as a function of the model's parameters?

4) Consider an economy that can be described by the Diamond OLG model. There is no depreciation or technological improvement (*g* is zero). The rate of growth of population is *n*. A person's lifetime utility function (lifetime utility as a function of first-period consumption C_1 and second-period consumption C_2) is:

$$U = \ln(C_1) + \frac{1}{1+\rho} \ln(C_2)$$

Suppose the government introduces an American-style Social Security program, under which a person pays a tax *T* when he is young and receives a payment $(1 + n)T$ when he is old.

a) Derive utility-maximizing first-period consumption C_1 as a function of the real interest rate *r*, the real wage *W*, the tax *T*, and the parameters of the economy.

b) From your answer to a), write an expression for *s*, the fraction of first-period wage income that a young person saves.

c) According to your answer to b), does the introduction of the Social Security program increase *s*, decrease *s*, or have no effect on *s*?

d) Is the introduction of the Social Security program a good thing? Explain.

5) The country of Morea is an open economy with a floating exchange rate, static exchange-rate expectations, a fixed price level, and a central bank that fixes the money supply. Suppose that foreign investors suddenly come to believe there is a chance (a probability less than 100 percent) that, sometime in the future, the Morean government will confiscate any foreign investments in Morea (seize any assets in Morea held by foreigners). What is the effect of this change in beliefs on the exchange rate, the interest rate, and output in Morea, assuming that:

a) There is "imperfect capital mobility" between Morea and the rest of the world

b) There is "perfect capital mobility" between Morea and the outside world (that is, uncovered interest-rate parity holds).

6) Consider a central bank that minimizes a loss function of the ordinary type:

$$L = \frac{1}{2} E\left[y_t^2 + a(\pi_t - \pi^*)^2\right]$$

where *y* is the output gap and π^* is the target inflation rate. Olivier Blanchard has said that under some conditions there is a "divine coincidence": there is no conflict between the goals of stabilizing output and stabilizing inflation. That is, by attempting to keep inflation at the target, a central bank will automatically be acting to keep the output gap equal to zero.

Describe a situation where the divine coincidence does *not* hold.

7) Consider the "New Keynesian IS-LM" models we discussed, with "IS shocks" in the equation relating the output gap to the real interest rate, and "aggregate supply" or "cost-push" shocks in the Phillips curve equation.

a) In the models presented by King and Clarida-Gali-Gertler, what causes the IS shocks?
b) In the model presented by Clarida-Gali-Gertler, what causes cost-push shocks?

8) Consider an economy where

$$y_t = -(\beta + \varepsilon_t) r_t + e_t$$ where y is the output gap, r denotes the *gap* between the real interest rate and the natural rate of interest

and $$\pi_t = {}_t\pi^e_{t+1} + \alpha y_t$$

The central bank sets the real interest rate gap r_t to minimize a loss function of the ordinary type, with a desired inflation rate equal to *zero*:

$$L = \frac{1}{2} E\left[y_t^2 + \pi^2\right]$$

e_t and ε_t are uncorrelated mean-zero random variables with variances σ_e^2 and σ_ε^2. Before the central bank sets r_t it *can* observe the realized value of e_t, but it *cannot* observe the realized value of ε_t. At time t, *no one* knows what will be the realized values of e_{t+1} and ε_{t+1}.

a) Assume that ${}_t\pi^e_{t+1}$ is always equal to zero. Derive the relation between the central bank's observation of e_t and the value of r_t that it sets. What will be the resulting values of y_t and π_t?

b) Is ${}_t\pi^e_{t+1} = 0$ a rational expectation? Explain why or why not.

c) In this economy, will the observable relation between r and y reveal the slope of the IS curve? Explain why or why not.

d) In this economy, will the observable relation between π and y reveal the Phillips-curve coefficient α? Explain why or why not.

PART II

ANSWER ALL

1. Growth Theory Questions

   (a) Consider a version of the Optimal Growth model. The representative agent has an objective function

   $$\int_0^\infty e^{-\beta t}\frac{c_t^\gamma}{\gamma}dt,$$

   where $1 > \beta > 0$ and $\gamma < 1$ are parameters. As usual we take advantage of the First Welfare Theorem to solve a social planner's problem. The planner's resource constraint is

   $$\begin{aligned}\dot{k}_t &= k_t^\alpha - (n+g+\delta)k_t - c_t,\\ &k_0 \textit{ is given}\end{aligned}$$

   where k_t^α is the production function, n is the growth rate of population ($\frac{\dot{L}}{L} = n$), g is the growth rate of technology ($\frac{\dot{A}}{A} = g$), and δ is depreciation rate of capital. Note that all lower-case letters represent variables in per effective labor terms. For example, $k_t = K_t/(A_t L_t)$, where K_t is total stock of capital, A_t is technology, and L_t is total labor.

   (b) Derive the first order conditions. Don't forget the transversality condition.

   (c) Carefully explain the economic intuition of each condition.

   (d) Does this economy have a unique steady state? If so, express steady-state levels of k and c as functions of parameters. If not, explain why not.

   (e) What is the golden-rule level of consumption at the steady state? Is it equal to the level of consumption you derived in part (c)?

   (f) At the steady state, the growth rates of k and c are, by definition, 0. What does this imply about the long-run growth rate of the aggregate level of capital, K_t? Derive this growth rate mathematically.

2. Follow the following steps to write down a discrete-time real business cycle model, solve it, and answer the questions given below. Note that you have some discretion here, but you must follow each instruction carefully.

   (a) Specify the preferences of a representative agent. The utility function must be concave. Then specify a production function that utilizes both labor and capital to produce a single good. The function must have constant returns to scale. Finally, define gross investment as net new capital plus depreciation. Be careful to limit the values of each parameter properly.

(b) Next, solve the consumer's and the competitive firm's optimization problems separately. To do this, you need to make clear what are the objective function and the budget constraint for each problem. Note that we are solving a *decentralized* competitive equilibrium problem instead of a planner's problem. Define prices such as wages and interest rates whenever necessary.

(c) Explain carefully the economic meanings and intuition for each first order condition you derived in part c.

(d) Briefly define what a competitive equilibrium is for this economy. List the equilibrium conditions for each market you defined.

(e) What is an impulse mechanism? In the model you have just constructed, which variable corresponds to it? Can you specify the law of motion for this variable?

(f) What is a propagation mechanism? Describe the propagation mechanisms of the model you have just constructed.

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