MACROECONOMIC THEORY COMPREHENSIVE EXAM

January 2009

PLEASE ANSWER ALL QUESTIONS

PART I

Macro comprehensive examination, Hanes' questions, January 2009

1) Consider an economy described by the Malthusian model of economics growth. The real wage is denoted w. The birth rate is: $b = \beta + \gamma w$
The death rate is: $d = \delta - \varphi w$

The aggregate production function is: $Y = Land^{\alpha} L^{1-\alpha}$
Derive the long-run equilibrium size of the peasant population L in terms of the parameters $\alpha, \beta, \gamma, \delta, \varphi$ and the quantity of *Land*.

2) Consider an economy that can be described by the Diamond OLG model.
The aggregate production function is Cobb-Douglas: $Y = K^{\alpha} (AL)^{1-\alpha}$ *where* $0 < \alpha < 1$

There is no depreciation. The rate of growth of population is n. The rate of growth of A is g. A person's lifetime utility function (lifetime utility as a function of first-period consumption C_1 and second-period consumption C_2) is:

$$U = C_1^{\beta}[(\frac{1}{1-\rho}) C_2]^{(1-\beta)} \quad where \quad 0 < \beta < 1$$

a) **Derive** s as defined for the OLG model,that is the fraction of a young person's income (labor income) devoted to saving.

b) Using your answer from a) and the Cobb-Douglas production function, write down the equation that gives k_{t+1} as a function of k_t.

c) In this economy, is there a long-run steady state? Is it stable? Is there more than one long-run steady state? Briefly, explain how you know.

3) Consider the old-Keynesian Friedman-Phelps Phillips curve $\pi_t = {}_{t-1}\pi_t^e + \alpha y_t$
and the new-Keynesian Phillips curve $\pi_t = {}_t\pi_{t+1}^e + \alpha y_t$
where y denotes the output gap.

a) The old-Keynesian Phillips curve *plus* rational expectations appears inconsistent with time-series data on output gaps and inflation. Explain.

b) The new-Keynesian Phillips curve *plus* rational expectations appears inconsistent with time-series data on output gaps and inflation. Explain.

4) Write down an equation describing the Phillips curve in the model of Christiano, Eichenbaum and Evans (2005). Explain what each letter denotes. Explain the assumptions that give this Phillips curve.

5) Consider an open economy with a floating exchange rate. Assume that:
- the central bank conducts monetary policy by choosing a value for the money supply M
- the price level is fixed; expected future inflation is always zero
- there is "static exchange-rate expectations," that is the expected future change in the exchange rate is always zero.

Using graphs and words, for each of the following cases, explain what happens to *output*, the *exchange rate*, and the *domestic interest rate* in response to this event: an increase in exports at any given value of the exchange rate.
a) A "large" economy with imperfect international capital mobility
b) A "small" open economy with perfect international capital mobility

6) Consider an economy where $y_t = -\beta_t(r_t - \bar{r}) + \varepsilon_t$

where y is the output gap. One can assume that ε is an i.i.d. random variable (mean zero, no serial correlation). Alternatively, one can assume that ε is a constant, always equal to zero. The parameter on the real interest rate is $\beta_t = \beta + \nu_t$

One can assume that ν is an i.i.d. random variable (mean zero, no serial correlation). Alternatively, one can assume that ν is a constant, always equal to zero.
Also, $\pi_t = {}_{t-1}\pi_t^e + \alpha y_t + e_t$

One can assume that e is an i.i.d. random variable. Alternatively, one can assume that it is a constant, always equal to zero.

The central bank sets r_t to minimize a loss function:

$$L = E\left[\frac{1}{2}y_t^2 + \frac{1}{2}(\pi - \pi^*)^2\right]$$

The public has rational expectations. It knows the central bank's loss function and desired inflation rate π^*. But the public *cannot* observe ε_t, ν_t or e_t at the time that it forms its inflation expectation ${}_{t-1}\pi_t^e$.

Note I have *not* assumed anything about the central bank's information about ε_t, ν_t and e_t.

Consider various possible assumptions about ε_t, ν_t and e_t, and the central bank's information about them.

a) Under what circumstances - that is, under what specific assumptions about ε, e, and the central bank's information - will this economy show *no* correlation between r_t and y_t?

b) Under what circumstances - that is, under what specific assumptions about ε, e, and the central bank's information - will this economy show a *positive* correlation between r_t and y_t?

c) Under what circumstances - that is, under what specific assumptions about ε, e, and the central bank's information - will this economy show a *negative* correlation between r_t and y_t?

7) Consider a small open economy with a floating exchange rate and perfect international capital mobility (uncovered interest-rate parity prevails). The central bank follows an interest-rate rule with random deviations:

$$i_t = \bar{r} + \pi^e_{t+1} + \tau(\pi_t - \pi^*) + e_t \quad \textit{where} \quad e_t = \rho e_{t-1} + f_t \quad \textit{and} \quad 0 < \rho < 1$$

where f is an i.i.d. random variable and π^* is the central bank's target inflation rate. Note that the parameter ρ determines the degree of serial correlation in the central bank's deviations from the rule. Suppose that in the last few periods, the value of e has happened to be zero, the output gap has been zero, and inflation has been equal to π^*. Then, at time t_0, there is a positive realization of f. Meanwhile, there is no change in foreign interest rates.

a) Draw a graph that shows how the real exchange rate ε responds. The graph should have the real exchange rate ε on the vertical axis, time on the horizontal axis, and t_0 marked on the horizontal axis.

b) Consider the *magnitudes* of the initial response of the real exchange rate ε at time t_0. Is the magnitude related to the value of ρ? That is, if ρ is large, does that tend to increase, decrease, or have no effect on the size of the immediate response of the exchange rate to the interest-rate shock?

8) In a model with an old-Keynesian IS curve, and old-Keynesian Phillips curve, and a central bank that acts to minimize a conventional loss function, there is no "dynamic inconsistency" problem as long as the central bank behaves "as if" the desired output level is the natural rate of output. Clarida, Gali and Gertler show that, in a "new Keynesian" model, there can be a dynamic inconsistency problem even if the central bank behaves as if the desired output level is the natural rate of output. Explain why this is true.

9) Suppose a firm can enter into a long-term contract with its single employee. The firm produces according to $Q = L^{\alpha}$ *where* $0 < \alpha < 1$ and L is the quantity of labor provided by the single employee. The firm sells in a competitive market at an uncertain price Z. The price Z has a discrete distribution with k possible values. The probability that the price takes a possible value Z_i is denoted p_i. That is, p_i is the probability that the price will be Z_i. The expected value of Z is thus:

$$E[Z] = \sum_{i=1}^{k} p_i Z_i$$

The contract specifies an employment level L_i and a payment to the single employee C_i for each possible realization of the price Z_i. The employee acts to maximize an expected utility function of consumption C and labor L: $E\ [\ \ln(C) - L^2\ ]$

a) Write down the expected value of the firm's profit in terms of p_i, Z_i, C_i and L_i.

b) The firm chooses the form of the contract to maximize its expected profit, subject to the constraint that the contract must give the work expected utility greater than or equal to u_0. Write down the Lagrangian that describes the firm's problem.

c) From the first-order conditions for maximizing the Lagrangian, derive the worker's consumption C_i and labor L_i for a realized Z_i, in terms of the Lagrangian multiplier λ.

d) Using your answer to c), explain how the worker's labor and consumption are related to the realized value of Z.

10) Consider an economy like Romer's baseline real business cycle model, but with "habit formation" in consumption. For simplicity, assume that the population is fixed and there is one person per household. The immortal representative-agent person-household acts to maximize:

$$E\left[\sum_{t=0}^{\infty} e^{\rho t}\left(\frac{(c_t - bc_{t-1})^{1-\theta}}{1-\theta} - zl_t^2\right)\right] \quad \text{where} \quad 0 < b < 1$$

where z is a parameter and notation is as usual: l is the fraction of his time that a household-person supplies as labor and c is his consumption.The technology parameter A is has a long-run trend growth rate g. Let r denote the real interest rate, equal to the return to capital after depreciation, and w denote the real wage per unit of labor (not per efficiency-unit of labor).

a) Write down the "intratemporal first-order condition" that relates a person's consumption c in a period to the same period's real wage w and labor-supply fraction l.

b) In a nonstochastic long-run steady state, both the real wage and consumption must be growing at rate g. That is to say,

$$\frac{c_{t+1}}{c_t} = \frac{w_{t+1}}{w_t} = e^g$$

Using this fact and your answer to a), demonstrate that the value of the felicity-function parameter θ must be one, so that the utility function is equivalent to:

$$E\left[\sum_{t=0}^{\infty} e^{-\rho t}\left(\ln(c_t - bc_{t-1}) - zl_t^2\right)\right]$$

PART II

1. Growth Theory Questions

   (a) A central question of growth theories is what account for the vast growth in output per person over time. According to the Solow model, what is the most important determinant of economic growth? To what extent does capital accumulation affect a country's growth rate?

   (b) The Ramsey-Cass-Koopmans model (optimal growth model) relaxes a major assumption of the Solow model. What is this assumption? What are the advantages of relaxing this assumption? What is the most important determinant of growth identified by this model?

   (c) What is an endogenous growth model? In order to have endogenous growth, what key assumption of the previous two models will have to be relaxed? List two driving forces of economic growth identified by endogenous growth theories.

   (d) Consider the following growth model with a representative agent. Consumer preferences are characterized by

$$U = \int_0^\infty \frac{c^{1-\theta}}{1-\theta} e^{-\rho t} dt.$$

   Output is produced with both physical and human capital:

$$Y = A\{a(bK)^\psi + (1-a)[(1-b)H]^\psi\}^{\frac{1}{\psi}},$$

   where 0<a<1, 0<b<1, $\psi < 1$ are parameters. A is exogenous.

      i. Assume this is a one-good economy, where output can be used on a one-for-one basis for consumption and investment in K and H (denoted by I_K and I_H). Write down the national accounting identity equation (or resource constraint) for this economy. This is a closed economy without a government sector.

      ii. Suppose both physical and human capital depreciate at the rate δ. Write down two equations that characterize the capital accumulation process for K and H, respectively: gross investments equal new investment (net change in capital) plus depreciations. Note that this is a *continuous time* model.

      iii. **Set up the Hamiltonian and carefully write down all the first order conditions.**

      iv. Using the first order conditions, derive the optimal K/H ratio. Solve this ratio (call it ϕ) as a combination of other *parameters*.

      v. Substitute $\phi = K/H$ into the production function so that Y becomes a function of K and parameters only. Based on this production function, do you think this economy will have endogenous growth? Why or why not? Explain.

2. There are a large number of infinitely lived identical individuals in the economy. The preferences of the representative agent (individual) are of the following form:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\gamma}}{1-\gamma} - B\frac{N_t^{1+\theta}}{1+\theta}\right),$$
$$\gamma, B, \theta > 0; 0 < \beta < 1,$$

where β is the discount factor, C_t and N_t are consumption and hours worked, γ and θ measure curvatures of the utility function, and B is the weight given to hours.

There are also a large number of identical competitive firms in the economy that produces a homogeneous good. Each household owns a firm. The household also supplies labor and productive capital to the firms, and earns a wage rate w_t for each unit of labor and a rental rate r_t for each unit of physical capital Kt supplied. The household pays taxes on all forms of income at various tax rates. The budget constraint of the household is

$$C_t + I_t = w_t N_t + r_t K_t - T_t, \tag{1}$$

where I_t is gross physical investment defined as

$$I_t = K_{t+1} - (1-\delta)K_t. \tag{2}$$

δ is the depreciation rate of capital. The tax bill, T_t, is defined as

$$T_t = \tau_t^p w_t N_t + \tau_t^b (r_t K_t - \phi I_t) + \tau_t^d (1-\tau_t^b)(r_t K_t - \phi I_t). \tag{3}$$

$\tau_t^p > 0$ is the tax rate on personal labor income $w_t N_t$. Note that the economic profits of competitive firms are zero, but their capital income, in the form of accounting profits, is $r_t K_t$. Capital income is taxed at the rate τ_t^b after deductions at the corporate level. The deductions are represented by the term ϕI_t, where the parameter ϕ is the fraction of gross investment that is deducted. This formulation captures an element of the U.S. tax code that provides incentives to private sector investment by allowing some new investment as well as capital depreciation to be deducted from taxable income. Firms return all their after-tax capital income to the individuals. This capital income, $(1-\tau_t^b)(r_t K_t - \phi I_t)$, is taxed for the second time at the rate τ_t^d at the individual level. This is called "double taxation."

The firm combines physical capital and labor, to produce the final good, using the following production technology:

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha}, \tag{4}$$

where α represents the share of capital in total output and A_t represents a random shock to productivity.

(a) For simplicity, we assume that the government budget is balanced each period. Now denote g_t as government spending, and write down the budget constraint of the government.

(b) Show that profit maximization by the firm will yield the following first order conditions:

$$w_t = (1-\alpha)\frac{Y_t}{N_t}, \tag{5}$$

$$r_t = \alpha\frac{Y_t}{K_t}. \tag{6}$$

(c) **Solve the representative agent's problem, and derive all first order conditions. Carefully explain the economic intuition of each condition.** (hint: do not ignore equations 2 and 3)

(d) If the government removes "double taxation," what will the tax equation (3) look like? What do you think will happen to the steady state level of investment if double taxation is removed? Why?