ANSWERS to Macroeconomics Comprehensive Examination January 2010 Hanes' questions

1) According to a recent article in the *Washington Post,* poor girls in India have begun to demand that a potential husband provide their future home with toilet facilities before they will consider marriage: "In India, more women demand toilets before marriage...'I won't let my daughter near a boy who doesn't have a latrine,' said Usha Pagdi...'No loo? No I do!'" Assuming that the Indian economy can be described by the Malthusian model of economic growth, how would you expect this development to affect real wages and population in India? Illustrate your answer with a graph. *10 pts. This is an increase in the subsistence wage, what I called* σ *is class. The event tends to raise LRSS real wages and reduce population.*

2) Suppose an economy's representative household is infinitely-lived and acts to maximize a utility function subject to *no* uncertainty - future values of variables are known with certainty. Felicity is increasing in consumption C_t , decreasing in the quantity of labor supplied L_t , and increasing in holdings of real money balance M_t/P_t , as follows:

$$U = \sum_{t=0}^{\infty} \beta^t \left(\ln(C_t) - \tfrac{1}{2}\theta L_t^2 + \frac{1}{1-\sigma} (M_t/P_t)^{1-\sigma} \right) \qquad where \quad 0<\beta<1$$

$$\text{subject to } Z_{t+1} = \frac{P_t}{P_{t+1}} \left[\frac{M_t}{P_t} + (1+i_t)\,(Z_t - \frac{M_t}{P_t}) + W_t L_t - C_t \right]$$

where Z_{t+1} is real wealth entering period *(t+1)*, M_t is the nominal money balance held across period *t*, i_t is the nominal interest rate paid on nonmoney assets held across period *t*, and W_t is the real wage. At time *t*, the household takes Z_t as given and chooses consumption, labor supply, and real money balances to maximize this lifetime utility function.

a) Write down the value function for the household's problem. Hint: *Z* is a state variable.

b) *Using the value function*, derive the quantity of real money balance M_t/P_t that a household will choose to hold, as a function of consumption C_t and the nominal interest rate i_t .

c) *Using the value function*, derive labor supply L_t as a function of W_t and C_t .

d) *Using the value function*, derive C_t as a function of C_{t+1} , i_{t+1} and $\frac{P_t}{P_{t+1}}$

e) If the inflation rate is denoted π , then $P_{t+1} = (1+\pi)P_t$. Using this notation and a common approximation, rewrite your answer to d) in terms of C_{t+1} and the *real* interest rate.

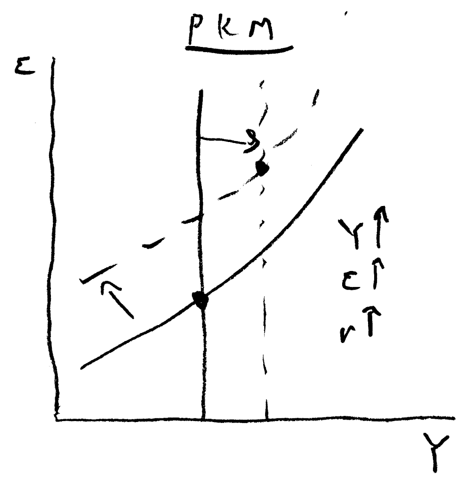
This question is exactly the same as Question 1 on the fall 2009 final exam for Econ 614, except that here I told you to derive each answer from the value function.

3) Consider an economy that can be described by the Solow model of economic growth with a fixed savings rate *s*, a rate of population growth *n*, and a rate of growth g_0 in the productivity parameter *A*. Assume the economy is in a LRSS. Then, at time t_0, the rate of growth in the productivity parameter increases to a *higher* rate g_1. Illustrate this event on two graphs. The first graph has *k* (capital per efficiency-unit of labor) on the horizontal axis and *y* (output per efficiency-unit of labor) on the vertical axis. The second graph has time on the horizontal axis and the log of output per worker on the vertical axis. On the second graph clearly denote time t_0. *10 pts. On the second graph, it must be clear that on the transition path to the new LRSS the rate of growth (the slope) is greater than* g_0, *less than* g_1.

4) Consider an open economy with a floating exchange rate and a central bank that sets interest rates (not the money supply). The central bank sets the real interest rate r as a function of output Y and the inflation rate π, along the lines of a "Taylor rule." The country's net exports NX are determined *only* by the real exchange rate ε . Exchange-rate expectations are *static*, that is $\varepsilon^e/\varepsilon \ = \ 0$. Treat the inflation rate π as an *exogenous* variable. Using graphs, describe how the country's output Y, real interest rate r and real exchange rate ε are affected by the following events.
a) An increase in the foreign interest rate r^*.
b) An increase in the domestic inflation rate π.

15 pts. I did not say whether the economy was characterized by perfect capital mobility or imperfect capital mobility. If you answered for just one case, you had to tell me which case you were assuming. That is, you had to say something like "I am assuming imperfect capital mobility."

a) $r^*\uparrow$?



Imperfect

$CF(r-r^*)+NX(\varepsilon)=0$

At given r, $CF\downarrow$, $NX\uparrow$, $\varepsilon\uparrow$

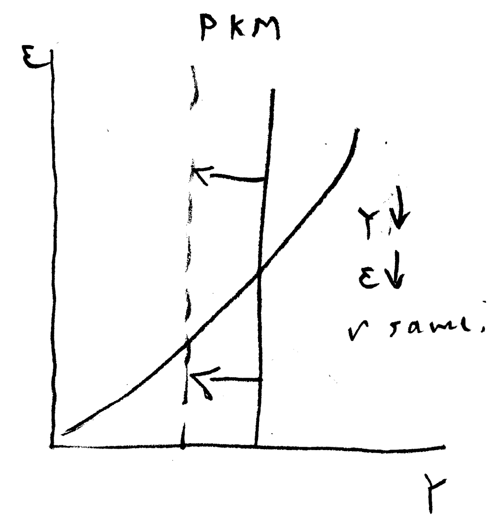
so IS^{**} shifts out.

$Y\uparrow$

$\varepsilon\uparrow$

$r\uparrow$

b) $\pi\uparrow$



Imperfect

$CF(r-r^*)+NX(\varepsilon)=0$

MP shifts up/back, so $Y\downarrow$, $r\uparrow$.

As $r\uparrow$, $CF\uparrow$, $NX\downarrow$, $\varepsilon\downarrow$.

5) Consider an economy that can be described by the Diamond OLG model. The aggregate production function is CES:

$$Y = [L^{\alpha} + K^{\alpha}]^{\frac{1}{\alpha}} \quad where \quad 0 < \alpha < 1$$

Note that there is *no* improvement over time in technology. Also, there is no depreciation. The rate of growth of population is n. A person's lifetime utility function (lifetime utility as a function of first-period consumption C_1 and second-period consumption C_2) is:

$$U = \ln(C_1) + \frac{1}{1+\rho}\ln(C_2)$$

Let r denote the real interest rate and w denote the real wage.

a) Write down the budget constraint that describes a necessary relation between $w_t, C_{1t}, C_{2,t+1}, and\ r_{t+1}$.

2 pts. $C_2 = (1+r)(w - C_1)$ or $w = C_1 + \frac{1}{1+r}C_2$ or...

b) Using Lagrangians, derive an expression for s, that is the fraction of first-period labor income that a young person saves. 6 pts.

$$\mathcal{L} = \ln C_1 + \frac{1}{1+\rho}\ln C_2 + \lambda\left[w - C_1 - \frac{1}{1+r}C_2\right]$$

$$\frac{\partial \mathcal{L}}{\partial C_1} = \frac{1}{C_1} - \lambda = 0 \Rightarrow \lambda = \frac{1}{C_1}$$

$$\frac{\partial \mathcal{L}}{\partial C_2} = \frac{1}{1+\rho}\frac{1}{C_2} - \lambda\frac{1}{1+r} = 0 \Rightarrow \lambda = \frac{1+r}{1+\rho}\frac{1}{C_2}$$

$$\frac{1}{C_1} = \frac{1+r}{1+\rho}\frac{1}{C_2} \Rightarrow C_2 = C_1\frac{1+r}{1+\rho}$$

Substitute into budget constraint, solve for C_1

$$C_1 = w\frac{1+\rho}{2+\rho}$$

$$s = \frac{w - C_1}{w} = 1 - \frac{1+\rho}{2+\rho} = \frac{1}{2+\rho}$$

c) Using your answer to **b)** and the CES production function, derive an expression that I called $G(k_t)$, that is an expression that gives k_{t+1} as a function of k_t.

6 pts. I have you 4 pts if you got correct answer but did not simplify at all, left a jumble of k's at the end.

$$k_{t+1} = \frac{1}{1+n} s \underbrace{w_t}_{MPL}$$

$$\frac{\partial Y}{\partial L} = \frac{1}{\alpha}\left[L^\alpha + k^\alpha\right]^{\frac{1}{\alpha}-1} \alpha L^{\alpha-1} = \left[L^\alpha + k^\alpha\right]^{\frac{1-\alpha}{\alpha}} \underbrace{L^{\alpha-1}}_{\left[L^{-\alpha}\right]^{\frac{1-\alpha}{\alpha}}}$$

$$= \left[(L^\alpha + k^\alpha)L^{-\alpha}\right]^{\frac{1-\alpha}{\alpha}} = \left[1+(k/L)^\alpha\right]^{\frac{1-\alpha}{\alpha}}$$

so

$$k_{t+1} = \frac{1}{1+n}\frac{1}{2+\rho}\left[1+k^\alpha\right]^{\frac{1-\alpha}{\alpha}}$$

6) Consider a central bank that cares about inflation only, not about the output gap. Its target inflation rate is zero. Thus, it acts to minimize a loss function

$$L = \frac{1}{2} E_t[\pi_t^2]$$

The economy has an "old Keynesian" expectations-augmented Phillips curve $\pi_t = {}_{t-1}\pi_t^e + (\alpha+\varepsilon_t)\, y_t$

where ε is a mean-zero, i.i.d. random variable with variance σ^2. The central bank sets the level of output y_t to minimize its loss function. (It really sets the real interest rate, which in turn determines output, but we can ignore that intermediate step.) At the time that the central bank sets y_t it cannot observe or forecast the realized value of ε_t, though it does know the distribution for ε.

a) Taking ${}_{t-1}\pi_t^e$ and y_t as given, what is the value of loss L before the value of ε_t is realized? (Hint: use the formula for the expected value of a random-variable-squared, which is just a rearrangement of the definition of a variance.) *3 pts.*

$$E[\pi] = \pi^e + \alpha y \qquad \sigma_\pi^2 = \sigma^2 y^2 \qquad \text{so } L = \tfrac{1}{2}\left[(\pi^e + \alpha y)^2 + \sigma^2 y^2\right]$$

b) Taking ${}_{t-1}\pi_t^e$ as given,

i) what value will the central bank choose for y_t to minimize loss?

ii) what will be the realized value of π_t?

6 pts.

$$0 = \frac{\partial L}{\partial y} = (\pi^e + \alpha y^*)\alpha + \sigma^2 y^* \Rightarrow y^* = -\frac{\alpha}{\alpha^2+\sigma^2}\pi^e$$

$$\pi = \pi^e + (\alpha+\varepsilon)y^* = \pi^e - (\alpha+\varepsilon)\frac{\alpha}{\alpha^2+\sigma^2}\pi^e$$

$$= \left(1 - (\alpha+\varepsilon)\frac{\alpha}{\alpha^2+\sigma^2}\right)\pi^e$$

c) Using your answer from b), derive the values of $_{t-1}\pi_t^e$, y_t and realized π_t in *rational expectations equilibrium. 3 pts.*

In REE,

$$\pi^e = E[\pi] = \left(1 - \alpha \frac{\alpha}{\alpha^2 + \sigma^2}\right)\pi^e$$

$$\Rightarrow \pi^e = 0, \text{ so } y^* = 0$$

so $\pi = 0$

7) In the simplest "New Keynesian IS-LM" models, the capital stock is held fixed (or there is no capital), so that the slope of the NK IS curve reflects just one thing: the representative agent's intertemporal consumption preferences as expressed by the Euler equation. In slightly more complicated New Keynesian models, such as that of Christiano, Eichenbaum and Evans, investment in fixed capital is a variable, so that the slope of the NK IS curve *also* reflects the parameters of the aggregate production function and the assumed costs of changing the rate of investment. Suppose you found that, in the actual American economy, real GDP appears much *more* sensitive to changes in real market interest rates (such as the fed funds rate or the Treasury bill rate) than can be accounted for by plausible calibrations of the Euler equation, production functions and costs of changing the rate of investment. That is, real GDP appears to be excessively sensitive to changes in these real interest rates. We have learned (at least) *two* possible reasons that real GDP might be "excessively" sensitive to the real interest rate, in this sense. Explain both. If you can only think of one, explain that one.

5 pts for each (separate) explanation.

Financial-market imperfections as illustrated in BGG model boost the sensitivity of spending to chahges in the real interest rate.

*In an open economy with floating exchange rates, like actual US economy, the effect of an interest-rate change on the real exchange rate strengthens the effect on output - "the IS** curve is flatter than the IS curve."*

Note: "wealth effects" is not a separate answer from "financial market imperfections."

c) Using your answer from b), derive the values of $_{t-1}\pi_t^e$, y_t and realized π_t in *rational expectations equilibrium. 3 pts.*

7) In the simplest "New Keynesian IS-LM" models, the capital stock is held fixed (or there is no capital), so that the slope of the NK IS curve reflects just one thing: the representative agent's intertemporal consumption preferences as expressed by the Euler equation. In slightly more complicated New Keynesian models, such as that of Christiano, Eichenbaum and Evans, investment in fixed capital is a variable, so that the slope of the NK IS curve *also* reflects the parameters of the aggregate production function and the assumed costs of changing the rate of investment. Suppose you found that, in the actual American economy, real GDP appears much *more* sensitive to changes in real market interest rates (such as the fed funds rate or the Treasury bill rate) than can be accounted for by plausible calibrations of the Euler equation, production functions and costs of changing the rate of investment. That is, real GDP appears to be excessively sensitive to changes in these real interest rates. We have learned (at least) *two* possible reasons that real GDP might be "excessively" sensitive to the real interest rate, in this sense. Explain both. If you can only think of one, explain that one.

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8) Consider an open economy where net exports depend *only* on the real exchange rate. The exchange rate is flexible. You don't know whether international capital mobility is perfect or imperfect. You have time-series data on the real exchange rate (and changes in the real exchange rate $\dot{\varepsilon}/\varepsilon$), the domestic real interest rate *r*, the foreign real interest rate *r**, and the volume of net exports *NX*. Using these data, you can run OLS regressions that will reveal whether the economy has perfect or imperfect international capital mobility, *assuming* that expectations are rational. Explain what these regressions would be, and how they would reveal the nature of capital mobility.

10 pts. Under rational expectations, the realized change in exchange rates $\dot{\varepsilon}/\varepsilon$ will equal the expected change in exchange rates $\dot{\varepsilon}^e/\varepsilon$ plus a random error, that is an error uncorrelated with any variables observable at the time expectations are being formed. Think what should happen if you regress realized $\dot{\varepsilon}^e/\varepsilon$ on the real interest-rate spread (r - r) and net exports.*

Under PKM,

$$\frac{\dot{\varepsilon}^e}{\varepsilon} = (r - r^*)$$

uncorrelated with anything

$$\frac{\dot{\varepsilon}}{\varepsilon} = (r - r^*) + \nu$$

Under imperfect KM,

$$CF\left(r - r^* - \frac{\dot{\varepsilon}^e}{\varepsilon}\right) + NX = 0$$

$$\frac{\dot{\varepsilon}^e}{\varepsilon} \approx (r - r^*) + \frac{1}{CF'(\;)} NX$$

uncorrelated with anything.

$$\frac{\dot{\varepsilon}}{\varepsilon} \approx (r - r^*) + \frac{1}{CF'(\;)} NX + \nu$$

So if coeff on $(r+r^*)$ is one & coeff on NX is zero (or multicollinearity because $(r-r^*) \to \varepsilon \to NX$) then perfect KM.

if coeff on $(r-r^*)$ is one & coeff on NX is negative, then imperfect KM.

9) Consider Romer's discussion of menu costs and a "fixed price equilibrium" in the context of his single-period, nondynamic model. Romer concludes that plausibly small menu costs cannot support a fixed-price equilibrium if one assumes the labor market is a simple, perfectly competitive market. What if you assume the labor market is characterized by "indivisible labor" and "consumption insurance"? Would this make it harder to maintain a fixed-price equilbrium, or easier, or have no effect on the model's ability to maintain a fixed-price equilibrium? Explain.

10 pts. Recall two things:

- Romer says that to maintain FPE you need elastic labor supply, that is a weak relation between the real wage and employment, so that when you hold the price level fixed an increase in output/employment can occur without a big inrcease in the nominal wage level.

- indivisible labor + consumption insurance makes aggregate labor supply very elastic.

So it makes it easier to maintain FPE.

10) Recall the notion of "habit formation" in consumption.

a) Write down a felicity function (the component of lifetime utility generated in a single period) that has two components, consumption c_t and the fraction of time devoted to labor, denoted l_t . Do *not* include real money balances - we don't need to fool with that in this question. Make sure your felicity function has the following characteristics:

- it is of the type conventionally used to generate habit formation, using c_{t-1} to denote consumption in period t-1.
- it is consistent with the existence of a long-run steady state where the trend rate of growth in consumption is the same as the trend rate of growth in the productivity parameter.
- it is consistent with increasing marginal disutility of labor or decreasing marginal utility of leisure.
- it is additively separable between consumption and labor or leisure.

4 pts. On the consuption side, you need the natural log of $c_t - bc_{t-1}$; on the labor/leisure side, you need increasing marginal disutility of labor or decreasing marginal utility of leisure. Here's one that would do it:

$$u = \ln(c_t - b\,c_{t-1}) - l_t^2$$

b) Suppose that the representative agent is maximizing a lifetime utility function of form:

$$U = \sum_{t=0}^{\infty} \beta^t \,(u(.......)) \qquad where \qquad 0<\beta<1$$

where u(..) denotes your felicity function from a), and there is *no uncertainty*. The budget constraint is:

$$Z_{t+1} = (1+r_t)Z_t + W_t l_t - c_t$$

where Z_t denotes real wealth entering period t, r_t is the real interest rate, and W_t is the real wage. Using your felicity function together with this budget constraint and lifetime utility function, derive an expression that shows the intertemporal relation that will hold between the variables c_{t+1} , c_t , c_{t-1}, and the real interest rate r_{t+1}, as a result of utility maximization. Hint: this relation does *not* incorporate the trend rate of growth in the productivity parameter; it *does* incorporate the subjective time-discount parameter β . *6 pts.*

Take derivative of value fn w.r.t. c_t

$$\frac{1}{c_t - b\,c_{t-1}} + \beta\, V_Z(Z_{t+1})\frac{\partial Z_{t+1}}{\partial c_t} = 0$$

BS condition says $V_Z(Z_t) = \dfrac{\partial u}{\partial c_t}\cdot\dfrac{\partial c_t}{\partial Z_t}$ (holding fixed Z_{t+1}

Budget constraint says $\dfrac{\partial c}{\partial Z_t} = 1 + r_t$;

$$\frac{\partial Z_{t+1}}{\partial c_t} = -1$$

so

$$\frac{1}{c_t - b\,c_{t-1}} + \beta \frac{1}{c_{t+1} - b\,c_t}(1+r_{t+1})(-1) = 0$$

$$\Rightarrow \frac{c_{t+1} - b\,c_t}{c_t - b\,c_{t-1}} = \beta(1+r_{t+1})$$