

PART 1. Hanes' questions. Answer all

1) According to a recent article in the *Washington Post*, poor girls in India have begun to demand that a potential husband provide their future home with toilet facilities before they will consider marriage: "In India, more women demand toilets before marriage... 'I won't let my daughter near a boy who doesn't have a latrine,' said Usha Pagdi... 'No loo? No I do!'" Assuming that the Indian economy can be described by the Malthusian model of economic growth, how would you expect this development to affect real wages and population in India? Illustrate your answer with a graph.

2) Suppose an economy's representative household is infinitely-lived and acts to maximize a utility function subject to *no* uncertainty - future values of variables are known with certainty. Felicity is increasing in consumption C_t , decreasing in the quantity of labor supplied L_t , and increasing in holdings of real money balance M/P_t , as follows:

$$U = \sum_{t=0}^{\infty} \beta^t \left(\ln(C_t) - \frac{1}{2} \theta L_t^2 + \frac{1}{1-\sigma} (M_t/P_t)^{1-\sigma} \right) \quad \text{where } 0 < \beta < 1$$

$$\text{subject to } Z_{t+1} = \frac{P_t}{P_{t+1}} \left[\frac{M_t}{P_t} + (1+i_t) \left(Z_t - \frac{M_t}{P_t} \right) + W_t L_t - C_t \right]$$

where Z_{t+1} is real wealth entering period $(t+1)$, M_t is the nominal money balance held across period t , i_t is the nominal interest rate paid on nonmoney assets held across period t , and W_t is the real wage. At time t , the household takes Z_t as given and chooses consumption, labor supply, and real money balances to maximize this lifetime utility function.

a) Write down the value function for the household's problem. Hint: Z is a state variable.

b) Using the value function, derive the quantity of real money balance M_t/P_t that a household will choose to hold, as a function of consumption C_t and the nominal interest rate i_t .

c) Using the value function, derive labor supply L_t as a function of W_t and C_t .

d) Using the value function, derive C_t as a function of C_{t+1} , i_{t+1} and $\frac{P_t}{P_{t+1}}$.

e) If the inflation rate is denoted π , then $P_{t+1} = (1+\pi)P_t$. Using this notation and a common approximation, rewrite your answer to d) in terms of C_{t+1} and the *real* interest rate.

3) Consider an economy that can be described by the Solow model of economic growth with a fixed savings rate s , a rate of population growth n , and a rate of growth g_0 in the productivity parameter A . Assume the economy is in a LRSS. Then, at time t_0 , the rate of growth in the productivity parameter increases to a *higher* rate g_1 .

Illustrate this event on two graphs. The first graph has k (capital per efficiency-unit of labor) on the horizontal axis and y (output per efficiency-unit of labor) on the vertical axis. The second graph has time on the horizontal axis and the log of output per worker on the vertical axis. On the second graph clearly denote time t_0 .

4) Consider an open economy with a floating exchange rate and a central bank that sets interest rates (not the money supply). The central bank sets the real interest rate r as a function of output Y and the inflation rate π , along the lines of a "Taylor rule." The country's net exports NX are determined *only* by the real exchange rate ϵ . Exchange-rate expectations are *static*, that is $\epsilon^e/\epsilon = 0$. Treat the inflation rate π as an *exogenous* variable. Using graphs, describe how the country's output Y , real interest rate r and real exchange rate ϵ are affected by the following events.

a) An increase in the foreign interest rate r^* .

b) An increase in the domestic inflation rate π .

5) Consider an economy that can be described by the Diamond OLG model. The aggregate production function is CES:

$$Y = [L^\alpha + K^\alpha]^\frac{1}{\alpha} \quad \text{where } 0 < \alpha < 1$$

Note that there is *no* improvement over time in technology. Also, there is no depreciation. The rate of growth of population is n . A person's lifetime utility function (lifetime utility as a function of first-period consumption C_1 and second-period consumption C_2) is:

$$U = \ln(C_1) + \frac{1}{1+\rho} \ln(C_2)$$

Let r denote the real interest rate and w denote the real wage.

a) Write down the budget constraint that describes a necessary relation between $w_t, C_{1t}, C_{2,t+1}$, and r_{t+1} .

b) Using Lagrangians, derive an expression for s , that is the fraction of first-period labor income that a young person saves.

c) Using your answer to b) and the CES production function, derive an expression that I called $G(k_t)$, that is an expression that gives k_{t+1} as a function of k_t .

6) Consider a central bank that cares about inflation only, not about the output gap. Its target inflation rate is zero. Thus, it acts to minimize a loss function

$$L = \frac{1}{2} E_t[\pi_t^2]$$

The economy has an "old Keynesian" expectations-augmented Phillips curve $\pi_t = {}_{t-1}\pi_t^e + (\alpha + \varepsilon_t) y_t$,

where ε is a mean-zero, i.i.d. random variable with variance σ^2 . The central bank sets the level of output y_t to minimize its loss function. (It really sets the real interest rate, which in turn determines output, but we can ignore that intermediate step.) At the time that the central bank sets y_t it cannot observe or forecast the realized value of ε_t , though it does know the distribution for ε .

a) Taking ${}_{t-1}\pi_t^e$ and y_t as given, what is the value of loss L before the value of ε_t is realized? (Hint: use the formula for the expected value of a random-variable-squared, which is just a rearrangement of the definition of a variance.)

b) Taking ${}_{t-1}\pi_t^e$ as given,

i) what value will the central bank choose for y_t to minimize loss?

ii) what will be the realized value of π_t ?

c) Using your answer from b), derive the values of ${}_{t-1}\pi_t^e$, y_t and realized π_t in *rational expectations equilibrium*.

7) In the simplest "New Keynesian IS-LM" models, the capital stock is held fixed (or there is no capital), so that the slope of the NK IS curve reflects just one thing: the representative agent's intertemporal consumption preferences as expressed by the Euler equation. In slightly more complicated New Keynesian models, such as that of Christiano, Eichenbaum and Evans, investment in fixed capital is a variable, so that the slope of the NK IS curve *also* reflects the parameters of the aggregate production function and the assumed costs of changing the rate of investment.

Suppose you found that, in the actual American economy, real GDP appears much *more* sensitive to changes in real market interest rates (such as the fed funds rate or the Treasury bill rate) than can be accounted for by plausible calibrations of the Euler equation, production functions and costs of changing the rate of investment. That is, real GDP appears to be excessively sensitive to changes in these real interest rates.

We have learned (at least) *two* possible reasons that real GDP might be "excessively" sensitive to the real interest rate, in this sense. Explain both. If you can only think of one, explain that one.

8) Consider an open economy where net exports depend *only* on the real exchange rate. The exchange rate is flexible. You don't know whether international capital mobility is perfect or imperfect. You have time-series data on the real exchange rate (and changes in the real exchange rate ε/ε), the domestic real interest rate r , the foreign real interest rate r^* , and the volume of net exports NX .

Using these data, you can run OLS regressions that will reveal whether the economy has perfect or imperfect international capital mobility, *assuming* that expectations are rational. Explain what these regressions would be, and how they would reveal the nature of capital mobility.

9) Consider Romer's discussion of menu costs and a "fixed price equilibrium" in the context of his single-period, nondynamic model. Romer concludes that plausibly small menu costs cannot support a fixed-price equilibrium if one assumes the labor market is a simple, perfectly competitive market.

What if you assume the labor market is characterized by "indivisible labor" and "consumption insurance"? Would this make it harder to maintain a fixed-price equilibrium, or easier, or have no effect on the model's ability to maintain a fixed-price equilibrium? Explain.

10) Recall the notion of "habit formation" in consumption.

a) Write down a felicity function (the component of lifetime utility generated in a single period) that has two components, consumption c_t and the fraction of time devoted to labor, denoted l_t . Do *not* include real money balances - we don't need to fool with that in this question. Make sure your felicity function has the following characteristics:

- it is of the type conventionally used to generate habit formation, using c_{t-1} to denote consumption in period $t-1$.
- it is consistent with the existence of a long-run steady state where the trend rate of growth in consumption is the same as the trend rate of growth in the productivity parameter.
- it is consistent with increasing marginal disutility of labor or decreasing marginal utility of leisure.
- it is additively separable between consumption and labor or leisure.

b) Suppose that the representative agent is maximizing a lifetime utility function of form:

$$U = \sum_{t=0}^{\infty} \beta^t (u(\dots)) \quad \text{where} \quad 0 < \beta < 1$$

where $u(\dots)$ denotes your felicity function from a), and there is *no uncertainty*. The budget constraint is:

$$Z_{t+1} = (1+r_t)Z_t + W_t l_t - c_t$$

where Z_t denotes real wealth entering period t , r_t is the real interest rate, and W_t is the real wage. Using your felicity function together with this budget constraint and lifetime utility function, derive an expression that shows the intertemporal relation that will hold between the variables c_{t+1} , c_t , c_{t-1} , and the real interest rate r_{t+1} , as a result of utility maximization. Hint: this relation does *not* incorporate the trend rate of growth in the productivity parameter; it *does* incorporate the subjective time-discount parameter β .

PART 2. Jones' questions. Answer all

11. Given a complete set of Arrow-Debreu securities markets, show how to replicate a risk free bond. What does this imply about the prices of the securities?
12. Given a complete set of Arrow-Debreu securities markets, show how to replicate a mutual fund for the output of a specific country m . What does this imply about the price of a country m mutual fund?
13. Consider the standard Ramsey-Cass-Koopmans model. Assuming that the economy starts from a steady state (in terms of per labor efficiency units), demonstrate what happens if there is an increase in subjective impatience.
14. Under what conditions would a monopolist set prices as a constant mark-up over marginal cost? Show this explicitly. Next, illustrate this finding in the context of an endogenous growth model of the Romer type showing all relevant details. Provide another macro relevant illustration.
15. The representative individual in a small open economy maximizes $U(C_1, C_2)$ subject to

$$C_1 + C_2/(1 + r) = Y_1 + Y_2/(1 + r)$$

where Y_1 and Y_2 are fixed.

- a) Derive the intertemporal Euler equation.
- b) Use the Euler equation together with the differentiated budget constraint to show that

$$\frac{dU(C_1, C_2)}{dr} = \frac{\partial U(C_1, C_2)}{\partial C_2} (Y_1 - C_1)$$

c) Explain why this implies that a country benefits from a rise in the world interest rate if and only if its terms of intertemporal trade improve.

16. Consider the following result:

$$CA_t = (Y_t - \tilde{Y}_t) - (I_t - \tilde{I}_t) - (G_t - \tilde{G}_t)$$

where “ $\tilde{\cdot}$ ” denotes the permanent level of the variable.

- a) Sketch how the result is obtained.
- b) Explain the economic significance of the result.