Macro Comp Questions Retake, February 19, 2010

Answer all

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Question 1

a) Use the linear-quadratic stochastic consumption model with $(1+r)\beta = 1$. For simplicity, assume that $G = I = 0$ on all dates.

Use the current account identity and the permanent income consumption function to show that

$$C_{t+1} - C_t = \frac{r}{1+r} \sum_{s=t+1}^{\infty} \left(\frac{1}{1+r}\right)^{s-(t+1)} (E_{t+1}Y_s - E_t Y_s)$$

b) Suppose that output follows a stochastic process like this:

$$Y_{t+1} - Y_t = \rho(Y_t - Y_{t-1}) + \varepsilon_{t+1}$$

where $0 < \rho < 1$. Show that for $s > t$, $E_{t+1}Y_s - E_t Y_s = (1 + \rho + \cdots + \rho^{s-(t+1)})\varepsilon_{t+1}$.

After you have completed all other questions, you may answer the following question for **bonus points**:

Prove that combining results from a) and b) leads to the conclusion that consumption innovations are *more* variable than output innovations.

Question 2

Using the Ramsey-Cass-Koopmans model, describe the effect of an increase in the rate of population growth on consumption and capital both in terms of *per labor efficiency units*.

Question 3

Consider the following equation:

$$V_1^m = \frac{E_1[Y_2^m]}{1+r} + Cov_1\left(\beta \frac{u'(C_2)}{u'(C_1)}, Y_2^m\right)$$

Provide a sketch of how you obtain this result and provide a detailed interpretation of its meaning. Then, use it to provide an expression for the corresponding risk-premium.

Question 4

Using the Metzler diagram, consider the effects of an increase in future Home productivity (A_2) on saving and investment for both Home and Foreign. Compare and contrast this with an increase in *current* home productivity (A_1).

Question 5

Prove that a country trading in complete asset markets will fully insure against *all* future consumption fluctuations at actuarially fair prices.

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Macroeconomics Retake Comprehensive Examination February 2010 Hanes' questions Answer all

1)Recall the notion of "habit formation" in consumption.

a) Write down a felicity function (the component of lifetime utility generated in a single period) that has two components, consumption c_t and the fraction of time devoted to labor, denoted l_t . Do *not* include real money balances - we don't need to fool with that in this question. Make sure your felicity function has the following characteristics:
- it is of the type conventionally used to generate habit formation, using c_{t-1} to denote consumption in period *t-1*.
- it is consistent with the existence of a long-run steady state where the trend rate of growth in consumption is the same as the trend rate of growth in the productivity parameter.
- it is consistent with increasing marginal disutility of labor or decreasing marginal utility of leisure.
- it is additively separable between consumption and labor or leisure.

b) Suppose that the representative agent is maximizing a lifetime utility function of form:

$$U = \sum_{t=0}^{\infty} \beta^t \left(u(.....)\right) \quad \textit{where} \quad 0<\beta<1$$

where u(..) denotes your felicity function from a), and there is *no uncertainty*. The budget constraint is:

$$Z_{t+1} = (1+r_t)Z_t + W_t l_t - c_t$$

where Z_t denotes real wealth entering period *t*, r_t is the real interest rate, and W_t is the real wage. Using your felicity function together with this budget constraint and lifetime utility function, derive an expression that shows the intertemporal relation that will hold between the variables c_{t+1} , c_t , c_{t-1}, and the real interest rate r_{t+1}, as a result of utility maximization. Hint: this relation does *not* incorporate the trend rate of growth in the productivity parameter; it *does* incorporate the subjective time-discount parameter β .

2) Consider an economy described by the Solow model, with a fixed savings rate *s*, population growth *n,* and a Cobb-Douglas production function

$$Y = K^{\alpha}(AL)^{1-\alpha} \quad \textit{where} \quad 0 < \alpha < 1$$

where the parameter *A* grows at rate *g*.The depreciation rate is δ.

a) Write down the "intensive form production function" that gives output per efficiency-unit of labor *y* as a function of capital per efficiency-unit of labor *k*.

b) Using your answer to a), derive an expression that gives long-run steady state output per *worker* (not output per efficiency-unit of labor) as a function of *s, n, g,* and δ.

c) Using your answer to a) derive an expression that gives long-run steady state output per *worker* (not output per efficiency-unit of labor) in the "golden rule" long-run steady state.

3) Explain how the "aggregate demand" curve associated with the IS/LM model is different from the agrgegate demand curve associated with the IS/MP model.

4) Consider an economy that can be described by the Diamond OLG model. The aggregate production function is Cobb-Douglas:

$$Y = K^{\alpha}(AL)^{1-\alpha} \quad \textit{where} \quad 0 < \alpha < 1$$

where the "technology" parameter grows at rate *g*. There is no depreciation. The rate of growth of population is *n*. A person's lifetime utility function (lifetime utility as a function of first-period consumption C_1 and second-period consumption C_2) is:

$$U = C_1^{\beta} C_2^{1-\beta}$$

Let *r* denote the real interest rate and *w* denote the real wage per efficiency-unit of labor.

a) Write down the budget constraint that describes a necessary relation between $w_t, C_{1t}, C_{2,t+1}, and\ r_{t+1}$.

b) Using Lagrangians, derive an expression for *s*, that is the fraction of first-period labor income that a young person saves.

c) Using your answer to **b)** and the production function, derive an expression that I called $G(k_t)$, that is an expression that gives k_{t+1} as a function of k_t .

d) Does this economy have a long-run steady state? Does it have more than one steady state? Explain how you know.

5) Recently, Olivier Blanchard (now chief economist at the IMF) wrote that central banks in countries with "floating" exchange rates often have objectives for exchange rates as well as inflation - they do not try to *fix* the exchange rate but they do care about the level of the exchange rate, in addition to inflation and output. "This does not imply that inflation targeting should be abandoned. Indeed, at least in the short term, imperfect capital mobility endows central banks with a second instrument in the form of reserve accumulation and sterilized intervention. This tool can help control the external target [the exchange rate target] while domestic objectives [the inflation target and output] are left to the policy rate [the domestic interest rate]."

a) Why does "imperfect capital mobility" matter for Blanchard's claim? That is, why can a central bank use this "second instrument" only if capital mobility is imperfect?

b) Why does Blanchard specify that this second instrument can be used "at least in the short term"? Is there some reason to believe it could not be used in the long term? Explain.

6) Consider an agent who must set a price subject to the possibility that he will be unable to adjust the price again for a number of periods. We derived the following condition. An agent *i* who is able to set a price at time *t* will set the price equal to:

$$z_{it} = \sum_{\tau=0}^{\infty} \omega_{t+\tau} E_t[p^*_{i,t+\tau}]$$

where $\omega_{t+\tau}$ is a weight and $p^*_{i,t+\tau}$ is the price the pricesetter would choose in a future period $(t+\tau)$) if he were always free to adjust his price, with no nominal rigidities. We applied this condition in a number of different specific models.

a) In the case of the Taylor "overlapping contracts" model, where a price must remain fixed for exactly two periods, what are the values of ω_t, ω_{t+1} and ω_{t+2}?

b) In words, explain how we derived the condition. For the derivation, we had to make some very restrictive assumptions. Make sure you describe these restrictive assumptions.

7) In many simple New Keynesian models, price inflation was determined by this type of Phillips curve:

$$(1) \qquad \pi_t = E_t[\pi_{t+1}] + \alpha y_t$$

where y_t is the current output gap and $E_t[\pi_{t+1}]$ is a rational expectation of next period's inflation rate.

a) Rudd and Whelan (2007) say that this Phillips curve implies another description of inflation:

$$(2) \qquad \pi_t = \alpha \sum_{\tau=0}^{\infty} E_t[y_{t+\tau}]$$

Explain how (2) can be derived from (1). Hint: Rudd and Whelan assume the long-run steady state inflation rate is zero.

b) Rudd and Whelan say that "the empirical case against the NKPC ..is quite strong." What do they mean? Explain how (1) or (2) is inconsistent with the actual behavior of inflation and output gaps.

c) Gali and Gertler and Sbordone argue that a New Keynesian Phillips curve in terms of "real marginal cost" *is* consistent with the actual behavior of inflation. What do they mean? What is their evidence?

8) What is the "information asymmetry" that is at the heart of Bernanke, Gali and Gertler's model of the "financial accelerator"?

9) Consider an old-Keynesian Friedman-Phelps Phillips curve $\pi_t = {}_{t-1}\pi_t^e + \alpha y_t + \varepsilon_t$

where y denotes the output gap. A nonzero realization of the disturbance term ε_t is a "supply shock." Show that a central bank should partially, but not completely counteract the effect of a foreseeable supply shock on inflation, assuming that the central bank minimizes a standard loss function of inflation and the output gap. To keep things simple, assume that the public's inflation expectation ${}_{t-1}\pi_t^e$ is zero, the central bank's target inflation rate is zero, and the desired output level in the central bank's loss function is the natural rate.