

## Micro comp questions for Jan 2010

### Part A (answer all 5)

1. (Khanna) Part 1:

Suppose you are going to the new all-you-can-eat Star Buffet restaurant on Old Vestal Road for dinner. You are in the mood for dumplings and lo-mien, which are the only two foods that you will eat. Explain how you will decide how much of each dish to eat?

Part 2:

Now suppose you are going to eat ice cream after the dinner. Each ice cream costs \$3.50. How does the answer to the question in part 1 change?

2. (Pape) Suppose that the profit function of a firm is described by

$$\pi(p_1, p_2, p_3) = (p_1 - p_2)^2 / p_3 \text{ for } p_1 > 0, p_2 > 0, p_3 > 0.$$

Then both the following production plans are efficient:

$(2, -3, -1/2)$ , AND  $(1, -1, -1/4)$ . True/False/Uncertain and justify.

3. (Pape) An agent who chooses consumption bundles  $(x, y)$  to maximize  $u(x, y) = x^2 + y^2$  satisfies WARP. True/False/Uncertain and Justify.

4. (Pape) Find *all* NE of the following game:

↖	L	R
U	<b>5,5</b>	<b>0,0</b>
D	<b>0,0</b>	<b>0,0</b>

5. (Pape) Suppose good G is a public good. Claim: If a social planner can assign each agent a personalized tax rate for G, then the socially optimal level of G can be achieved. (T/F/U and Justify).

Part B (answer all 6)

1. (Khanna): Every year, the University gives each graduate exactly four tickets for friends and family to attend the Commencement ceremonies. These tickets are provided to the graduating students for free and they cannot obtain tickets by any other means (i.e., there is no black market for Commencement tickets and students do not exchange tickets between themselves). For this academic year, the University is considering a new system under which students will be charged a positive price for the tickets and each student can purchase any quantity she wants at that price. From the point of view of the students and their families, which system is better? (You may assume that all students have identical preferences and budget constraints.)

2. (Khanna): Consider a perfectly competitive firm in a constant cost industry that is at its long run equilibrium. Now suppose a tax of \$ $t$  per unit is imposed on the firm. Carefully outline the short run and long run response of the firm and industry. That is, carefully explain how the SR and LR equilibrium is reached, highlighting the difference between the SR and the LR.

3. (Pape): Jack values only one good: widgets. The nature gives him two options. Option (i) provides him with  $x$  units of widgets for sure. Option (ii) provides him with  $y$  units in the event that he is fortunate. If he is not fortunate, option (ii) leaves him with 0 units of widgets. Jack is an expected utility maximizer.

Part 1: Suppose Jack is risk neutral and that he is indifferent between options (i) and (ii). What is the subjective probability that he attributes to the event of being fortunate?

Part 2: Jill is an expected utility maximizer but is risk averse. He is indifferent between options (i) and (ii). Introduce option (iii), which provides Jill with half of what he gets under option (i) together with half of what he gets under option (ii). That is,  $x+y/2$  if he is fortunate and  $x/2$  if he is not. Show that he strictly prefers option (iii) to options (i) and (ii). Hint: Jill's belief may be different from those of Jack.

4. (Pape): In a paper discussed by Pape at this year's AEA meetings, the author defines a game in which two agents, A and B, are selling a good to a representative agent. The author finds the equilibrium quantities  $q_A$ ,  $q_B$  for the game in the following way:

1. An initial price  $p$  for the good is exogenously given.
2. The two agents A and B each choose a quantity  $q_A$ ,  $q_B$  to produce. The quantity each produces follows the Cournot duopoly solution, given the agents' decreasing returns to scale production functions and the given price  $p$ .
3. Given the quantities that are defined in the previous step, the representative agent's utility maximization problem determines the new price  $p'$ .
4. Payoffs for agents A and B are determined according to:  $q_i$  times  $p'$ , minus costs of production.

Pape objects to this means of finding equilibrium quantities  $q_A$ ,  $q_B$ . What is his objection?

5. (Pape) Consider the Prisoner's Dilemma below. Suppose two agents play this game followed by the game listed in part A, number 4. Find one SPNE in which (C,C) is played in the PD game or explain why this is impossible.

\	C	D
C	<b>3,3</b>	<b>0,4</b>
D	<b>4,0</b>	<b>1,1</b>

Suppose the second game was replaced with the following game. Find one SPNE in which (C,C) is played in the PD game or explain why this is impossible.

\	L	R
U	<b>0,0</b>	<b>-1,5</b>
D	<b>5,-1</b>	<b>0,0</b>

6. (Pape) Suppose there are two agents bidding for a single object. The auction is an *all-pay auction*, in which all agents must pay their bid, regardless of whether they win (but, naturally, it still holds that the agent with the highest bid wins the object). Suppose that all valuations are i.i.d., and distributed uniform over the interval  $[0,1]$ . Find the symmetric Nash equilibrium bidding strategies for the players.

Part C (answer 2 of 3)

1. (Pape) Consider two consumers and a single firm economy with two goods. Consumers' preferences are represented by the utility functions:

$$u_1(c, l) = cl^2$$

$$\text{and } u_2(c, l) = c^2 l$$

where  $c$  is consumption,  $l$  is leisure. Consumers are endowed with  $\omega_1 = (0, 9)$  and  $\omega_2 = (0, 3)$  of consumption and total possible leisure.

The firm produces  $c$  from labor  $L$ ;  $c = f(L) = \theta L$ ,  $\theta > 0$ . As usual, one unit of leisure can be converted into one unit of labor, by working. Each consumer owns 50% of the firm.

a) Determine the competitive equilibrium prices and quantities.

b) Consider an increase in  $\theta$ . What happens to the labor supplied by each consumer? The consumption of  $c$ ? Be specific.

c) Consider the following change in technology, from  $f$  above to:  $c = g(L) = \theta \ln(L + 1)$ .

i. Claim: The CE of this new economy (i.e. the old economy, but with this new technology) may Pareto dominate the old equilibrium. (T/F/Uncertain and justify).

ii. Claim: Moreover, the ratio of total consumption to total leisure will be lower in the new equilibrium, relative to the old equilibrium. (T/F/Uncertain and justify).

2. (Pape) Valhalla & Company is a monopolist in the robotics industry. Its factory is capable of producing robots of different quality levels to suit various niches of the market. However, it faces scale economies that make it costly to engage in such model proliferation. More precisely, it may choose to produce a robot of quality  $q$  for a cost of

$$c(q, n) = n^2 q^2$$

where  $n$  is the number of different qualities of robots the firm produces.

Consumers in the market are of two types, 1 and 2. The utility function of the  $i$ th type is given by  $u_i = \beta_i q - p$ , where  $p$  is the price of the robot. Consumers who don't purchase get  $u = 0$ . Assume  $\beta_1 < \beta_2$ . The proportion of consumers of the second type is given by  $\lambda$ .

(a) Suppose that Valhalla exploits its scale economies to the full and only produces one model of robot. Find its profit function  $\pi_1(\lambda)$ .

(b) Now suppose that Valhalla chooses to produce two models of robot (such that each will be bought by a different consumer and both types will be served). Find the quality levels and prices of the two models.

(c) TRUE/FALSE: The firm is indifferent between ordering one model or two when the proportion of consumers of the second type is given by  $\lambda = \beta_1/\beta_2$

3. (Pape) An economy consists of two consumers, R, J, two goods,  $x, y$ , and two states, 1, 2. Both consumers have the same probability assessment about two states; each thing the first state is exactly as likely as the second.

The utility functions are as follows:

$$u_R(x, y) = 2 \ln x + \ln y$$

$$u_J(x, y) = \ln x + 2 \ln y.$$

And their endowments are

	State 1	State 2
R	(1,1)	(2,2)
J	(2,2)	(1,1)

Suppose that consumers have to trade before the actual state is revealed and the markets for both contingent goods exist.

(a) Define the Arrow-Debreu equilibrium for this economy. That is, what set of things comprise the equilibrium? What conditions must they satisfy?

(b) Compute the Arrow-Debreu equilibrium for this economy.

(c) Suppose that instead of all state-contingent commodities, only two assets were available: asset A which delivers one unit of  $x$  regardless of state, and asset B which delivers one unit of  $Y$  regardless of state. Then, after the state is realized, agents can trade in spot markets. Can the final allocation from b be achieved? Explain.

(d) Suppose that instead of all state-contingent commodities, only two assets were available: asset C which delivers *one unit of each good* in state 1, and asset D which delivers *one unit of each good* in state 2. Then, after the state is realized, agents can trade in spot markets. Can the final allocation from part b be achieved? Explain.