

ANSWER ALL QUESTIONS

1/7/2011

January 2011, Macroeconomics comprehensive exam, Hanes' questions
Look over the entire examination before you begin.

1) Consider two economies, A and B, that can be described by the Malthusian model. The level of the "subsistence" real wage is the same in A and B. In A, occasional plagues strike, killing off people until the surviving population develops immunity. In B, an efficient health service prevents plagues with vaccines. Consider the mean value of the real wage in each economy, averaged over a very long span of time - one or two hundred years. Is this long-run mean real wage higher in A, higher in B, or the same in both economies? Explain, using appropriate graphs.

2) Consider an open economy where domestic inflation, domestic expected inflation and ROW expected inflation are all fixed (π , π^e and π^{e*} are all fixed). Net exports depends only on the exchange rate according to a function $NX(\epsilon)$. What happens to the economy if there is a change in expectations of the future exchange rate: "before," people were expecting no future change in the exchange rate ("static exchange-rate expectations"); suddenly, people come to expect the exchange rate will *appreciate* in the future. Meanwhile, there is *no* change in the foreign interest rate i^* . For each case below, *clearly state* what happens to:

- domestic output
- the domestic interest rate
- the exchange rate ϵ
- capital flow CF

Illustrate with appropriate graphs!

- a) Perfect capital mobility, reserve gain zero, central bank fixes the money supply, exchange rate floats.
- b) Imperfect capital mobility, reserve gain zero, central bank follows an interest-rate rule $r(\pi, Y)$, exchange rate floats.
- c) Imperfect capital mobility, reserve gain zero, central bank follows an interest-rate rule $r(\pi, Y)$, exchange rate floats.

3) Consider two economies, Saxonia and Gallia. In both countries, some companies are funded by collateralized loans; others are funded by selling stock (equity). Stock is a claim to a share of the company's profits. In Saxonia, companies are required to publish detailed accounts of their operations' profit and loss, prepared by outside accountants, with severe penalties for firm managers who falsify information. In Gallia, companies are not required to publish such accounts. In which country would you expect to see a *larger* fraction of companies funded by collateralized loans (and hence a smaller fraction funded by stock)? *Explain why*.

4) Recall the lesson of the Kydland-Prescott model of "time-inconsistent" monetary policy: in rational expectations equilibrium, inflation will exceed society's desired inflation rate, if the central bank's policy expresses society's desire for output greater than the natural rate of output. *Prove* this result using appropriate equations. For simplicity, assume that the central bank can directly control the current inflation rate, taking expected inflation as given.

5) Consider an economy with an expectations-augmented Phillips curve: $\pi_t = \pi_t^e + \alpha y_t$

and an IS curve: $y_t = -\beta r_t$, where y is the output gap (the difference between output and the natural rate of output) and r is the gap between the real interest rate and the natural rate of interest. The central bank follows an "interest-rate rule" $r_t = \gamma y_t + \theta \pi_t$

Consider adding "additive disturbances" to the equations describing this economy.

- a) What kind of additive disturbances, in which equation(s), would generate a positive correlation between the output gap and the real interest rate, and a positive correlation between the output gap and inflation, in rational expectations equilibrium? Illustrate this situation using *two* graphs. Both graphs have output on the horizontal axis. On the vertical axis, one graph has the real interest rate; the other has the inflation rate.
- b) What kind of additive disturbances would generate a negative correlation between the output gap and the real interest rate, and a negative correlation between the output gap and inflation, in rational expectations equilibrium? Illustrate this situation using two graphs, as above.

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Comprehensive Exam Questions

January 2011

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6. Problem 1

For each part, identify a relevant model and formulate the corresponding argument. Be clear and concise and emphasize economic intuition. You must incorporate graphs or equations as necessary, but your answer will be graded largely on the quality of your written explanation.

1.a) For a large open endowment economy, an increase in a country's current output could actually make the country worse off.

1.b) The hypothesis that output growth is serially correlated is difficult to reject statistically. If so, then it is a puzzle that standard Keynesian consumption functions match the data. Propose some possible explanations of the puzzle.

1.c) There is a tendency for countries with high productivity in tradable goods compared with non-tradable goods to have higher price levels.

1.d) Under risk-neutrality, the only price vector consistent with general equilibrium is actuarially fair.

1.e) Hyper-inflationary equilibrium are ruled out if there is an absolute necessity for money.

7. Problem 2

Answer using the dynamic Ricardian model with a continuum of goods and no transport costs. Suppose that there is a proportional fall in Foreign's unit labor requirements for all goods: *i.e.* a proportional downwards shift in the $A(z)$ schedule. Prove that although Home loses some industries as a result, it actually gains because its worker's real wages increase.

8. Problem 3

Suppose that there are S states of nature. Also, assume that there are $N + 1$ assets. One of the assets (the 1st one) has a deterministic gross return of $1 + r$. The others have state dependent returns. Define the matrix of gross returns, R , as follows:

$$R = \begin{bmatrix} 1 + r & 1 + r^1(1) & \cdots & 1 + r^N(1) \\ 1 + r & 1 + r^1(2) & \cdots & 1 + r^N(2) \\ \vdots & \vdots & & \vdots \\ 1 + r & 1 + r^1(S) & \cdots & 1 + r^N(S) \end{bmatrix}$$

where $1 + r^n(s)$ is the gross return on the n^{th} asset in state s .

3.a) Prove that if $\text{Rank}(R) = S$, then you can synthesize a complete set of Arrow-Debreu securities.

3.b) Suppose that the N securities are mutual funds for each country's output (there are N countries). Ignoring the rank condition in the previous part, under what functional form assumptions would the resource allocation be the same as with a complete set of Arrow-Debreu securities? Explain the intuition behind this finding.

9. **Problem 4**

Consider a model in which the consumer's instantaneous utility function, $u(C, \bar{L} - L)$, includes both consumption and leisure. Assume that the instantaneous utility function is CRRA in an aggregator function, $\Omega(C, \bar{L} - L)$, which is CES. Derive the Euler equation for consumption as a function of the real interest rate and a price index for an aggregate of consumption and leisure. Also, explain what the price index depends on and derive an explicit form for it.

10. **Problem 5**

For the money-in-the-utility function model with a constant money supply, derive the dynamic equation for real balances (or, equivalently, for the price level) and show that the steady state is unstable. Why is this referred to as "saddle path" stability by Obstfeld and Rogoff? For bonus points, provide the condition that Obstfeld and Rogoff use to rule out speculative hyper-deflations and explain it.

WEI XIAD QUESTIONS

// Consider the following Growth model. The production function for the i th producer of goods is

$$Y_i = A(K_i)^\alpha (H_i)^\lambda H^\varepsilon, \quad (1)$$

where $0 < \alpha < 1, 0 < \lambda < 1, 0 \leq \varepsilon < 1$. The variables K_i and H_i are the inputs of physical and human capital used by firm i to produce goods, Y_i . The variable H is the economy's aggregate level of human capital. Output from the goods sector can be used as consumables, C , or as gross investment in physical capital, I_K . Physical capital depreciates at the rate δ . In other words,

$$\dot{K}_i = I_{K_i} - \delta K_i. \quad (2)$$

The production function for human capital is

$$I_{H_j} = B H_j, \quad (3)$$

where H_j is the human capital employed by the j th producer of human capital, and $\sum_j I_{H_j}$ is the gross investment in human capital. Human capital also depreciates at the rate δ . Households have the preferences

$$U = \int_0^\infty \frac{c^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt. \quad (4)$$

- Consider first a competitive equilibrium in which producers of Y and H act as perfect competitors. Derive the first order conditions for the decentralized competitive equilibrium.
- What is the steady-state growth rate of C , Y , and K ? How does the answer depend on the size of the human capital externality, ε ?
- What is the steady-state growth rate of H ? Under what conditions does H grow at the same rate as K in the steady state?
- Write down the social planner's problem, which maximizes the utility of the representative consumer. Write down his first order conditions.
- Redo parts b and c for the social planner's problem. Comment on the difference.

12. Consider the following real business cycle model with two countries. Each country is represented by a single agent. The preferences of the representative consumer in country i , for $i=1,2$, are characterized by an expected utility function of the form

$$u_i = E_0 \sum_{t=0}^{\infty} \beta^t \frac{[c_{it}^v (1 - n_{it})^{1-v}]^{1-\gamma}}{1-\gamma}, \quad (5)$$

where c_{it} and n_{it} are consumption and employment in country i . $0 < v < 1, \gamma > 0$. There is a single good produced in each country with inputs

(4)

of capital k and domestic labor n , and is influenced by the productivity shock z . Output in country i is

$$y_{it} = z_{it} k_{it}^{\theta} n_{it}^{1-\theta}. \quad (6)$$

We assume z_{it} is a stochastic process that follows

$$\log z_{it} = \rho \ln z_{it-1} + e_{it},$$

where e is normally distributed with mean 0 and variance σ_e^2 , and $0 < \rho < 1$. Capital accumulation is characterized by the equation

$$I_{it} = k_{it+1} - (1 - \delta)k_{it},$$

where δ is the depreciation rate. Our assumption is that the consumption good and investment can freely flow across countries, but labor cannot. Labor immobility means country i 's production can only be produced with country i 's labor. Consequently, the world-wide resource constraint is

$$\sum_i (c_{it} + I_{it}) = \sum_i y_{it}. \quad (7)$$

Note that unlike in a closed-economy model, in this economy $c_{it} + I_{it} \neq y_{it}$. Instead of setting up the world competitive market problem, we take advantage of the welfare theorems by solving a social planner's problem. The problem is to

$$\max u_1 + u_2$$

subject to the resource constraint (7).

- (a) Derive all first order conditions of the maximization problem. Explain the economic intuition of each equation.
- (b) Suppose there is a positive productivity shock in country 1. What will happen to consumption, investment, and labor in country 1? What will happen to these variables in country 2? Why?
- (c) How are the aggregate consumption of the two countries correlated? What about investment and output? Do you think these correlations match those of real international data? Why or why not?

5