MACROECONOMICS COMPREHENSIVE EXAM RE-TAKE
February 4, 2011

ANSWER ALL QUESTIONS

Part I   B. Jones

Comprehensive Exam Problems:

1. Consider a model in which there are two sectors: a non-tradable goods sector and a tradable goods sector. The real interest rate, r, is exogenous. The total amount of labor, L, is assumed to be fixed.

a) Construct the <u>GDP line</u>. Hint: the GDP line is constructed from the zero-profit conditions in the two sectors evaluated for a steady state in consumption). Let $k_T(r)$ and $k_N(r)$ denote the capital-labor ratios in the two sectors, $w(r)$ denote the real wage and $p(r)$ denote the relative price of non-tradables.

b) Construct the <u>GNP line</u>. Hint: the GNP line shows the economy's steady state budget constraint. Let $\bar{Q}$ denote wealth.

c) Under what assumptions is the <u>income expansion path</u> a line through the origin?

d) Show how to sign $w'(r)$ and $p'(r)$.

For the remainder of the question, assume that <u>preferences and production functions are Leontief</u>.

e) Under these assumptions, demonstrate that the GDP line is invariant to r.

f) Under these assumptions, demonstrate that the income expansion path is invariant to r.

g) Assuming that $w'(r)L + \bar{Q} > 0$, determine the effect of an increase in r on the GNP line.

h) What is the effect of an increase in r on the consumption of tradables and non-tradables.

2. Suppose we have a two-country, two-period, S state endowment setup. Both Home and Foreign have an exponential period utility function, $U(C) = -e^{-\gamma C}/\gamma$, where $\gamma > 0$.

a) Assuming complete Arrow-Debreu securities markets, calculate equilibrium prices and consumption levels.

b) Now, assume that trading is restricted to risk-free bonds and shares in Home and Foreign period 2 outputs. Is the resulting allocation efficient? Work everything out carefully.

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Instructions. Each question consists of several parts. Complete each part to the best of your ability. **Be sure to manage your time efficiently.** Show all your work. Good luck!

3. There are N different firms in the economy. Firm i's production function is

$$y_i = k_i^\varepsilon K^v, \varepsilon > 0, v > 0$$

where k_i is firm i's capital stock, K is the economy wide level of capital stock. Firm i's output is therefore affected by the size of the economy's aggregate capital level. We call this "positive externality". Note that from the point of view of each firm, the effect of K on its own production is exogenous (cannot be controlled). We also assume that initial capital k_{i0} is given, and $k_i \geq 0$ at all time. There are N agents in the economy, whose preferences are given by

$$\int_0^\infty e^{-rt} \ln c_i dt$$

where c is consumption, and r is the discount factor. We assume each agent owns one firm. The resource constraint of each agent is

$$\dot{k}_i = y_i - c_i$$

(a) Write down the Hamiltonian, and derive all necessary conditions for this firm. (20 points)

(b) Reduce the first order conditions to a system of two differential equations in c and k. (7 points)

(c) Assume that all agents are identical, that is, $k_i = k_j = k$ and $c_i = c_j = c$ for all i and j. Since K is the aggregate level of capital, it must be that

$$K = Nk$$

Use these relations to modify the necessary conditions you obtain in part a. (8 points)

(d) Use the first order conditions to draw phase diagrams and predict this economy's growth rate. Is there a balanced growth path? If so, is it stable? (hint: your answers will depend on the assumptions about parameter values. A complete answer requires an analysis of all possible cases.) (30 points)

4. The preferences of consumers are characterized by the utility function

$$E_0 \sum_{t=0}^\infty \rho^t [\log(c_t - d_t) - \frac{n_t^{1+\gamma}}{1+\gamma}], \tag{1}$$

where c_t and n_t are consumption and hours worked, respectively. d_t is an exogenous shock to consumer preferences that represent random fluctuations in consumption demand. $\gamma \geq 0$. The consumer's budget constraint is

$$c_t + i_t = w_t n_t + r_t K_t, \tag{2}$$

where w_t is the wage rate, r_t is the rental rate of capital, and K_t is the level of physical capital the consumers lend to a firm. i_t is defined by

$$K_{t+1} = i_t + (1 - \delta)K_t \tag{3}$$

Define the production technology as

$$Y_t = Z_t K_t^a n_t^{1-a}, \tag{4}$$

where Z_t is a stochastic shock to productivity. The competitive firm's objective is to maximize its profits at time t.

(a) Solve the consumer's problem and derive a set of first order conditions. Explain the intuitions of each equation.

(b) Solve the firm's profit maximization problem and express r_t and w_t as functions of K_t, n_t and Z_t.

(c) Given the assumption in (4) and competitive markets, what should $Y_t - w_t n_t - r_t K_t$ be equal to? Plug this into (2) to obtain the resource constraint for the economy.

(d) What is an "impulse mechanism"? In this model, what are the possible impulses of business cycles?

(e) What is a "propagation mechanism"? Describe one propagation mechanism for this model.

PART III

Macroeconomics Comp, January 2011, Re-take. Hanes' questions.

5) Consider an economy where the natural rate of interest $\bar{r}$ and the money supply M are fixed. Demand for high-powered money is $m^D - p = y - bi$
where m is the log of money, p is the log of the price level, and y is the log of output. Suppose $\bar{Y}$ has been growing at a stable, low rate g_0. Then, at time t_1, an unforeseen event occurs: $\bar{Y}$ starts growing at a *faster* rate $g_1 > g_0$. There is one path for the price level that would allow output and the real interest rate to remain equal to the natural rates and expected inflation to equal realized inflation (except just prior to the surprise increase in output growth). Draw this price-level path on a graph that has time on the horizontal axis and the log of the price level on the vertical axis. Mark t_1 on the horizontal axis.

6.) Consider an open economy where domestic inflation, domestic expected inflation and ROW expected inflation are all fixed (π, π^e and π^{e*} are all fixed). Net exports depends only on the exchange rate according to a function $NX(\epsilon)$. Exchange-rate expectations are "static." What happens to the economy if there is an increase in the level of net exports that would prevail at any given value of the exchange rate ϵ ? For each case below, *clearly state* what happens to domestic output, the domestic interest rate, the exchange rate ϵ , and capital flow CF.
Illustrate with appropriate graphs!
a) Perfect capital mobility, reserve gain zero, central bank fixes the money supply, exchange rate floats.
b) Perfect capital mobility, reserve gain zero, fixed exchange rate, central bank adjusts the money supply to maintain the fixed exchange rate. In this case, of course, nothing happens to the exchange rate.
c) Imperfect capital mobility, reserve gain zero, central bank follows an interest-rate rule $r(\pi, Y)$, exchange rate floats.

7) Consider an economy that can be described by the Solow model with an aggregate production function:
$$Y = (K^\rho + (AL)^\rho)^{1/\rho} \quad where \quad 0 < \rho < 1$$
with a rate of population growth n, a rate of "technological improvement" g , and a rate of depreciation δ Derive the long-run steady state value of capital per efficiency-unit of labor k^* that will prevail if the savings rate is equal to a value s_0.

8) Consider a central bank that acts to minimize a loss function $L = E\left[\frac{1}{2}y_t^2 + \frac{1}{2}(\pi - \pi^*)^2\right]$

a) Describe some circumstances under which the behavior of this central bank would correspond to an "interest rate rule" of the type assumed for the IS/MP model.
b) Describe some circumstances under which the behavior of this central bank would be different from an "interest rate rule" of the type assumed for the IS/MP model.
I am not looking for equations.

9) Consider a central bank in an economy where inflation is $\pi_t = {}_{t-1}\pi_t^e + \alpha y_t$
and $y_t = -\beta_t r_t + \epsilon_t$ where $\beta_t = \beta + v_t$

y is the output gap. r is the difference between the real interest rate and the natural rate of interest. v is a random variable with mean zero and variance σ^2. The central bank can set r_t . At the time that the central bank sets r_t it knows the public's inflation expectations and the value of ϵ_t. But it is *uncertain* about the effect on spending of a given value of r. Assume that the public's expected inflation is always equal to the central bank's target inflation rate.
a) The central bank's goal is to keep inflation for the period equal to a target rate of zero. It does not care about the level of output. Write down a loss function that describes the central bank's preferences, in terms of π and any other relevant variables.
b) Making appropriate substitutions into a) and accounting for any relevant forms of uncertainty, write down a function that gives the central bank's loss in terms of the value of r set by the central bank.
c) Derive the value of r that minimizes b).
d) Given c), what is *expected* inflation from the central bank's point of view at the time it sets r?
e) Does this central bank's behavior correspond to "inflation forecast targeting"? Explain.

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