MICROECONOMICS COMPREHENSIVE EXAMINATION

Binghamton University
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Please answer all questions

PART A

1. (Khanna) Recently, I was waiting for the elevator on the 10th floor of the Library Tower. As is not unusual, only one elevator was working on this particular day. I had already waited for about 4 minutes but the working elevator had not arrived. I was getting late for class and my goal was to reach my classroom as fast as possible. What should I have done – continued waiting for the working elevator to arrive, or run down the stairs? (Note: once the elevator arrives, it faster to travel down to the first floor by riding in the elevator compared to running down the stairs.) Provide a logical explanation for your answer.

2. (Pape) Consider two allocations, A and B. Suppose allocation A is Pareto optimal. Then allocation A is either:
   i. a Pareto improvement over allocation B, OR
   ii. all agents are indifferent between allocation A and allocation B.

   True/False/Uncertain? Justify.

3. (Pape) Explain what the First Theorem of Welfare Economics "tells us" as economists; that is, what policy prescriptions does it recommend? Make sure to mention any caveats to these recommendations.

4. (Jones) A utility function does not exist if a consumer's preference relation fails to be transitive. True/False/Uncertain? Justify.

5. (Jones) When income elasticities are all constant and equal, they must all be equal to 1. True/False/Uncertain? Justify.

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PART B

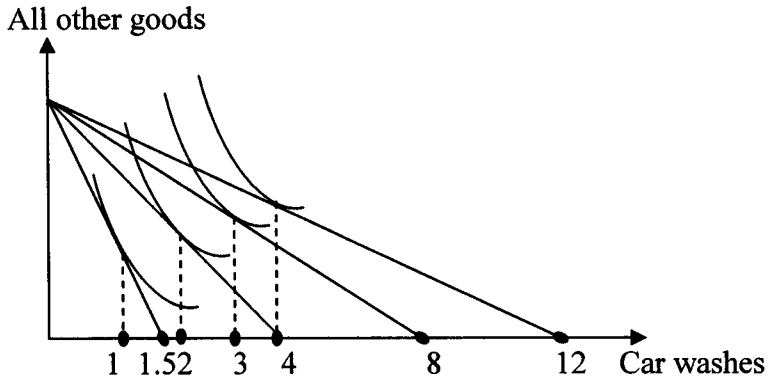
6. (Khanna) Consider a firm that uses two inputs, capital (K) and labor (L), and whose production technology can be described with a *constant returns to scale Cobb-Douglas* production function. This firm operates in a *perfectly competitive market* where it faces input prices w and v and output price P.

   a. Find this firm's cost minimizing combination of capital and labor.

   b. Now find this firm's profit maximizing level of output. Provide an intuitive explanation for the answer that you get.

7. (Khanna) Suppose that the city of Binghamton has exactly 100 identical consumers and 50 identical car washes. Each consumer has an income of $24. The diagram and table below show the indifference curve for a typical consumer and the marginal cost curve for a typical car wash.

Quantity	MC ($)
1	3
2	4
3	5
4	6
5	7
6	8
7	9
8	10



   (a) What is the price of a car wash in Binghamton in the short run?

   (b) Suppose that in the long run there is no entry or exit from the car wash industry in Binghamton and that the price of a car wash remains the same as in the short run. How is that possible? What can you conclude about the fixed costs at an individual car wash?

8. (Jones) Let $\pi(p, \mathbf{w})$ be competitive firm's profit function given the output price p and input price vector **w**. Prove: $\pi(p, \mathbf{w})$ is convex in (p, **w**).

9. (Jones) Prove: If a choice function x(p, y) satisfies WARP (weak axiom of revealed preference) and budget-balancedness, then it is homogeneous of degree 0 in (p, y).

10. (Pape) There is a firm A which makes good x with a technology that involves a constant marginal cost c. The demand curve $q_D(p)$ has the properties that $q_D(c)>0$ and $q_D'(p)<0$. The game proceeds as follows:

    (i) Firm A which chooses a quantity q_A.

    (ii) Firm B decides whether to pay a fixed cost $F>0$, and, if so, select a quantity q_B. If it chooses not to pay the fixed cost, it does not enter and receives a payoff of zero.

    Consider subgame perfect Nash equilibria of this game. Under what relationships among parameter values will B choose not to enter in equilibrium? Under what will B choose to enter, and how much will he produce in this case?

11. (Pape) Suppose there is an economy with two goods: money m, and the good x. x is produced by J competitive firms with production functions $f(m) = 2\,m^{1/2}$. Suppose each of N agents has an endowment of 16 units of m and owns exactly $1/N$ of each company. x has a byproduct, g; the amount of which is given by $g = X/N$, where X indicates the total amount of x produced and g the total amount of g in the economy. Each agent has the following utility function over goods and g:

$$u_i(x_i, m_i, g) = m_i + \ln(x_i) - g$$

    (a) What is the Walrasian equilibrium of this economy?

    (b) What is the socially optimal allocation of x in this economy? Compare it with (a). Explain.

    (c) Suppose a social planner had the power to tax x at some level t. Find the socially optimal level of t. Explain.

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PART C

12. (Jones) A consumer has a fixed time endowment of T. Let w denote wage rate and L his leisure. His income is then $w(T\text{-}L)$. His utility function is $u(x, L)$ where x is a vector of n consumption goods. Let p be the vector of the prices of the n consumption goods. The consumer's problems is to maximize $u(x, L)$ subject to the budget constraint.

    a. Show that his budget constraint can be expressed as $p \cdot x + wL = wT$.

    b. Let $L(p, w, wT)$ denote the consumer's optimal choice of leisure. Derive the Slutsky equation for the term $\frac{\partial L(p,w,wT)}{\partial w}$. (You can make use of the Slutsky equation from the conventional setting.)

    c. Analyze the equation you got in (b). Which part is the substitution effect and which is the income effect? Can the total effect be signed?

    d. Assuming that there is a single consumption good x. Draw a graph to illustrate the income and substitution effects.

13. (Jones) A consumer consumes 3 goods (goods denoted x_1, x_2 and x_3; prices denoted p_1, p_2 and p_3) with income level $y > 0$. His demand functions for goods 1 and 2 are given by

$$x_1 = 100 - 5\frac{p_1}{p_3} + \beta\frac{p_2}{p_3} + \delta\frac{y}{p_3}$$

$$x_2 = \alpha + \beta\frac{p_1}{p_3} + \gamma\frac{p_2}{p_3} + \delta\frac{y}{p_3}$$

    where Greek letters are nonzero constants.

    a. Indicate how to calculate the demand for good 3 (but do not actually do it).

    b. Are the demand functions for x_1 and x_2 appropriately homogeneous?

    c. Calculate the restrictions on the numerical values of α, β, γ and δ implied by utility maximization.

    d. Given your results in part (c), for a fixed level of x_3, draw the consumer's indifference curve in the x_1, x_2 plane.

    e. What does your answer to (d) imply about the form of the consumer's utility function $u(x_1, x_2, x_3)$?

14. (Pape) Consider the following game between two players:

First, Player One makes an offer which is either generous or mean. Then, Player Two then must decide to accept or reject this offer. If player two accepts a generous offer, then the payoffs are (1/2,1/2). If player two accepts a mean offer, then the payoffs are (3/ 4 , 1/4). If player two chooses reject in either case, then they play the following game (obviously, player One is row and player Two is column):

	L	**R**
U	1/9, 1/3	0, 1
D	1/6, 0	1/8, 1/8

(a) Draw the extensive form of the *complete* game.

(b) Find the Nash equilibrium that yields the highest payoff for player two.

(c) Find the Subgame Perfect Nash Equilibrium that yields the highest payoff for player two. Is the equilibrium you found in the previous part also subgame perfect? Explain.

(d) Find the Nash equilibrium that yields the lowest payoff for player two.

(e) Find the Subgame Perfect Nash Equilibrium that yields the lowest payoff for player two. Is the equilibrium you found in the previous part also subgame perfect? Explain.

15. (Pape) Consider an economy with two consumers: John (consumer 1) and Melissa (consumer 2), they both consume Oranges X and Leisure L. In the production of oranges each of them needs to work; more precisely, the production function is given by : $X = F(L_{1p,}, L_{2p})$, where L_{1p} is the number of hours that John works and L_{2p} is the number of hours that Melissa works in this production process.

The initial endowment of John is: 0 oranges and 44 hours to be allocated to work and leisure; and that of Melissa is: 0 oranges and 14 hours to be allocated between work and leisure.

The utility function of John is $U_1(X_1, L_1)$ and that of Melissa is $U_2(X_2, L_2)$, where L_1 , L_2 represent the leisure consumed by consumers 1 and 2.

Claim 1: "Any interior Pareto Optimal allocation *must* satisfy the following condition (*)"

$$(*) \qquad \frac{\partial U_1 / \partial L_1}{\partial U_1 / \partial X_1} = \frac{\partial F}{\partial L_{1p}} \quad \text{and} \quad \frac{\partial U_2 / \partial L_2}{\partial U_2 / \partial X_2} = \frac{\partial F}{\partial L_{2p}}$$

If this is true explain why in this case the Marginal Rates of Substitution of the two consumers are not necessarily equal?

Assume now that we have:

$$U_1(X_1, L_1) = X_1 L_1 \ ; U_2(X_2, L_2) = X_2 L_2 \ ; \quad F(L_{1p}, L_{2p}) = 2(L_{1p})^{0.5} + 2(L_{2p})^{0.5}$$

Denote by P the price of X, by W_1 the wage rate of John, and by W_2 the wage rate of Melissa. Assume that each one owns 50% of the firm producing X. Note that wages in equilibrium are not necessarily equal.

Claim 2: "The prices P=1 , W_1=0.25 , W_2=0.5 are competitive equilibrium prices". True/False/Uncertain? And justify

Claim 3: "If the Government *imposes* equal wage rate for this type of work, it will result in a non-optimal equilibrium allocation". True/False/Uncertain? And justify.