

Hanes' comp questions January 2012 ANSWERS

1) Nowadays, the central bank policy committee in nearly every large country sets a value for the interest rate rather than the money supply. Explain why, using graphs *not* equations. **10 pts.** *This is about the Poole (QJE, 1970) paper we did in class, and the class discussion of it. Consider a central bank that wants to stabilize output. (If you add a Phillips curve to the model, stabilizing inflation is usually also a matter of stabilizing output, so the argument can also hold for a central bank that has preferences over output AND inflation.) In an IS/LM model, compare the effects of fixing the money supply versus fixing the interest rate when there are shocks to spending/IS versus money demand/LM. Fixing the interest rate gives more stable output in the face of LM shocks. Experience has shown that central banks are very bad at forecasting money demand, which is to say the position of the LM curve for a given money supply. Thus, central banks fix  $i$  not  $M$ .*

2) Consider a closed economy where the aggregate production function is Cobb-Douglas with two inputs: labor (practically equivalent to the population), and a *fixed* quantity of land. There is no technological improvement. The birth rate is a positive linear function of the real wage. The death rate is an exogenous parameter, not a function of the real wage. Using appropriate *equations*, explain what will happen in the long run to the population, the real wage, *and* the real rent received by the owner of a unit of land if there is a *decrease* in the exogenous death rate. **10 pts.**

*First, use equations for the birth rate and death rate to determine the equilibrium "subsistence" real wage*

$$\sigma: \quad d = \delta \quad b = \kappa + \lambda w$$

*In LRSS,  $w$  must take the value that makes  $b = d$ . That value of  $w$  is  $\sigma = 1/\lambda (\delta - \kappa)$ .*

*So a decrease in the exogenous death rate  $\delta$  causes the LR real wage  $\sigma$  to fall.*

*Second, use the production function to define what happens to the population if  $\sigma$  falls.*

*Production function:  $Y = Land^\alpha L^{1-\alpha}$*

*MPL, which is real wage, is  $(1-\alpha)Land^\alpha L^{-\alpha}$*

*To see effect on  $L$  of a decrease in  $\sigma$ , set MPL equal to  $\sigma$  and solve for  $L$ :*

$$L = \left( \frac{(1-\alpha)}{\sigma} \right)^{\frac{1}{\alpha}} Land = \left( \frac{(1-\alpha)}{(1/\lambda (\delta - \kappa))} \right)^{\frac{1}{\alpha}} Land$$

*so decrease in  $\sigma$  (decrease in  $\delta$ ) causes  $L$  to increase.*

*Finally, use the production function to define what happens to MPL and if  $L$  increases. Real rent received by owner of a unit of land is MPL and, which is  $\alpha Land^{\alpha-1} L^{1-\alpha}$ . So the increase in  $L$  causes real rent to increase.*

3) Consider a small open economy with perfect international capital mobility. The foreign interest rate is  $r^*$ . The exchange rate floats; exchange-rate expectations are static. The central bank fixes the money supply. The price level is fixed. Expected inflation is zero. This economy is subject to *financial-market imperfections* as described by the model in Romer's textbook. Each potential business project in the economy requires one unit of capital (one unit of output invested as capital in the business). The expected return to a business project (the expected return to the required unit of capital) is  $\gamma$ , where  $\gamma$  varies across potential entrepreneurs/projects. Information is "asymmetric:" a potential investor in a project can observe its realized return only after paying a cost  $c$ . The total number of projects undertaken is equal to total investment spending in this economy. (There is no other type of investment spending in this economy.) Each potential entrepreneur owns some *land*. What happens to the exchange rate, output and capital flow in the economy if there is a general, exogenous *decline* in the price of land in the economy?

*10 pts. This question is about the asymmetric-information imperfect-capital market model. Recall that in that model the number of investment projects undertaken is positively related to entrepreneurs' wealth: a decrease in entrepreneurs' wealth decreases spending on business projects at any given value of the required expected return to investment  $r$ . In this economy, the required expected return for an investor or entrepreneur in a business project is  $r^*$  because exchange-rate expectations are static (hence  $\dot{\epsilon}/\epsilon = 0$ ). A decline in the price of land means a decline in entrepreneurs' wealth, hence a decline in spending on*

investment projects at any given  $r^*$ .

The question is also about the Mundell-Fleming model. In that model, with static  $e$ -rate expectations, a decrease in  $(C+I)$  at any given real interest rate (a backward shift in the  $IS^*$  curve) results in no change in output, but an increase (depreciation) in the exchange rate. The depreciation in the exchange rate spurs an increase in  $NX$  which allows  $Y$  to remain the same in the face of a decrease in domestic spending. As  $CF+NX=0$ , the increase in  $NX$  is associated with a decrease in  $CF$ .

So the outcome here is an increase (depreciation) in the exchange rate, a decrease in net capital inflow, no change in output.

4) Suppose an economy can be described by three equations:

- A Friedman-Phelps Phillips curve:  $\pi_t = {}_{t-1}\pi_t^e + fy_t$

- An IS curve:  $y_t = -ar_t$

- An interest-rate rule followed by the central bank:  $r_t = by_t + c(\pi_t - \bar{\pi})$

where  $y$  denotes the log of the output gap ( $y$  is zero if output is equal to the natural rate of output),  $r$  denotes the difference between the real interest rate and the natural rate of interest, and  $\bar{\pi}$  is a fixed value.

Any of these equations may also be subject to an additive mean-zero disturbance. The value of any realized disturbance  $\epsilon_t$  is unknown as of time  $(t-1)$ . Expectations are model-consistent rational expectations.

a) Suppose there are disturbances to the IS curve equation only.

i) Derive  ${}_{t-1}\pi_t^e$  in rational expectations equilibrium.

ii) Given your answer to i), derive  $\pi_t, y_t, r_t, i_t$  in terms of  $\epsilon$  and  $\bar{\pi}$ .

$\epsilon_t$  4 pts.  $y_t = -ar_t + \epsilon_t$

i) From IS & IR

$$y = -a(by + c(\pi - \bar{\pi})) + \epsilon$$

$$y = -\frac{ac}{1+ab}(\pi - \bar{\pi}) + \frac{1}{1+ab}\epsilon$$

gives  $y = -\frac{ac}{1+ab}(\pi - \bar{\pi}) + \frac{1}{1+ab}\epsilon$

put into Phillips curve, solve for  $\pi$

$$\pi = \frac{1}{1+ab+fc} \left[ (1+ab)\pi^e + f - c\bar{\pi} + f\epsilon \right]$$

Apply REE

$$\pi^e = E[\pi] = \frac{1}{1+ab+fc} \left[ (1+ab)\pi^e + f - c\bar{\pi} \right]$$

Solve for  $\pi^e$ :

$$\pi^e = \bar{\pi}$$

ii) Put  $\pi^e = \bar{\pi}$  into

$$\text{get } \pi = \bar{\pi} + \frac{f}{1+ab+fc} \epsilon \quad \text{put into}$$

$$\text{get } y = \frac{1}{1+ab+fc} \epsilon$$

put  $\pi$  &  $y$  into interest rate rule

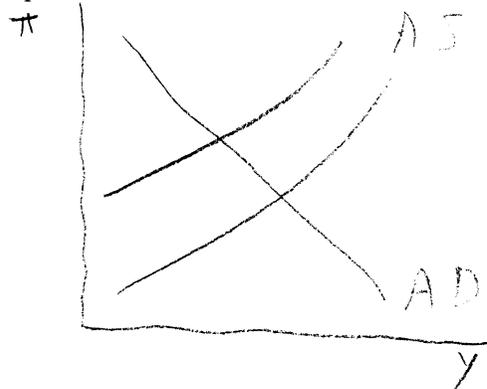
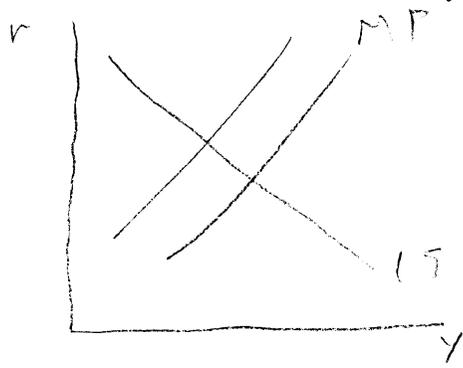
$$i_t = \bar{r} + \frac{b + cf}{1 + b + cf} \varepsilon$$

$$i_t = \bar{r} + \pi_t + \frac{b + cf}{1 + b + cf} (\pi_t - \bar{\pi})$$

Assume  $\varepsilon = \pi_t - \bar{\pi}$

$$i_t = \bar{r} + \frac{b + cf}{1 + b + cf} \varepsilon$$

b) What type or types of disturbance - to the IS curve, to the interest-rate rule, or to the Phillips curve - will create the fluctuations in the economy described by the graphs below?



4 pts. This must be a disturbance to the Phillips curve - that is, a supply shock.

c) Which type or types of disturbance will create a correlation between  $r$  and  $y$  that matches the slope of the IS curve? 4 pts. A disturbance to the interest-rate rule or the Phillips curve. (Either will trace out the IS curve.)

d) Which type or types of disturbance will create a correlation between  $\pi$  and  $y$  that matches the coefficient  $f$  in the Phillips curve? 4 pts. A disturbance to the interest-rate rule or the IS curve. (Either will trace out the AS curve.)