MACROECONOMICS COMPREHENSIVE EXAM

JANUARY 20, 2012

ANSWER ALL QUESTIONS

PART 1 XIAO

Each question consists of several parts. Complete each part to the best of your ability. Be sure to manage your time efficiently. Show all your work. Good luck!

1 A growth model

This is a two sector growth model. The two sectors will be referred to as the goods sector, X, and the education sector, Y. There are two reproducible factors of production, physical capital (K) and human capital (H). The goods are produced with a constant returns to scale technology

$$X = F(sK, uH) = uHf(k_x), \tag{1}$$

where s and u are the shares of physical and human capital, respectively, allocated to the goods sector. $k_x = \frac{sK}{uH}$ is the capital/labor ratio. Goods may be consumed or added to the capital stock. Capital stock evolves according to

$$\dot{K} = uHf(k_x) - \delta K - C, \tag{2}$$

where δ is the depreciation rate of capital.

In the education sector, Y, the production function, is also assumed to exhibit constant returns to scale, with

$$Y = G[(1-s)K, (1-u)H] = (1-u)Hg(k_y), \tag{3}$$

where $k = \frac{(1-s)K}{(1-u)H}$. Human capital evolves according to

$$\dot{H} = (1-u)Hg(k_y) - \eta H, \tag{4}$$

where η is the depreciation rate of human capital. f and g are strictly increasing and strictly concave functions.

A representative agent's optimization problem is given as

$$max_{C,s,u,K,H} \int_0^\infty \frac{C^{1-\sigma}}{1-\sigma} e^{-\rho t} dt$$

subject to (2) and (4), H(0)>0, K(0)>0, where ρ is a constant rate of discount. $\sigma \in (0,1)$.

Questions:

1. Show that the second identity in (1) is true, given our assumptions of the production function.
2. Explain the economic meanings of equations (2) and (4).
3. Set up the maximization problem by defining a current value Hamiltonian. Which are state variables? Which are control variables?
4. Obtain all first order conditions of the maximization problem.
5. What is a balanced growth path? Outline the steps you would take to obtain the growth rate of this economy when it is on the balanced growth path. You do not need to actually solve it. Just make an outline of how you are going to solve for it.

2 A business cycle model

We will consider a version of the real business cycle model. As usual, the agent in our model faces a work-leisure decision. But unlike in a standard model, the agent may not be able to find a job. In other words, in each period the agent faces a lottery which determines whether he will work or not. The probability of working is N_t. The agent's time endowment is T hours. If he works, he works for a fixed length of time, f hours. If he goes to work, he incurs a fixed cost of ξ hours (commute, for example). The agent can choose how much effort he puts into his work. The level of effort is denoted by e_t. So, the effective leisure of the agent is T-$\xi - fe_t$ if he works. If he does not work, his leisure is T.

So the agent's utility function is

$$maxE_0 \sum_{t=0}^{\infty} \beta^t \{ln(C_t) + \theta[N_t ln(T - \xi - fe_t) + (1 - N_t)ln(T)]\}, \tag{5}$$

where C is consumption, β is the discount factor, θ is a positive parameter, and $E_0$is the expectation operator.

Output is produced via the production function

$$Y_t = A_t K_t^{1-\alpha} (N_t f e_t)^{\alpha}, \tag{6}$$

where $0 < \alpha < 1$, K_t is capital stock, and A_t is a productivity shock that follows an exogenous process. The aggregate resource constraint is

$$C_t + K_{t+1} - (1 - \delta)K_t + G_t = Y_t, \tag{7}$$

where G_t is government consumption, which follows an exogenous process.

We will solve a social planner's problem. An important assumption we are going to make is that the social planner will choose the work probability N_{t+1} at time t. This captures the fact that employment usually responds slowly to market conditions. Essentially, N_t is a state variable like K_t.

Questions:

1. Using the Lagrangian method, solve for all first order conditions of the social planner's problem.
2. Explain the economic intuition of each equation you obtained in question 1.
3. Show that you can obtain the same first order conditions using Bellman's equation.
4. How does work effort respond to a rise in productivity? What about a rise in government spending?
5. (Bonus) How is the Solow residual typically calculated? If someone uses the typical approach to calculate the Solow residual, how is the calculated residual inaccurate when used to estimate productivity changes?

PART 2 HANES

3) Nowadays, the central bank policy committee in nearly every large country sets a value for the interest rate rather than the money supply. Explain why, using graphs *not* equations.

4) Consider a closed economy where the aggregate production function is Cobb-Douglas with two inputs: labor (practically equivalent to the population), and a *fixed* quantity of land. There is no technological improvement. The birth rate is a positive linear function of the real wage. The death rate is an exogenous parameter, not a function of the real wage. Using appropriate *equations*, explain what will happen in the long run to the population, the real wage, *and* the real rent received by the owner of a unit of land if there is a *decrease* in the exogenous death rate.

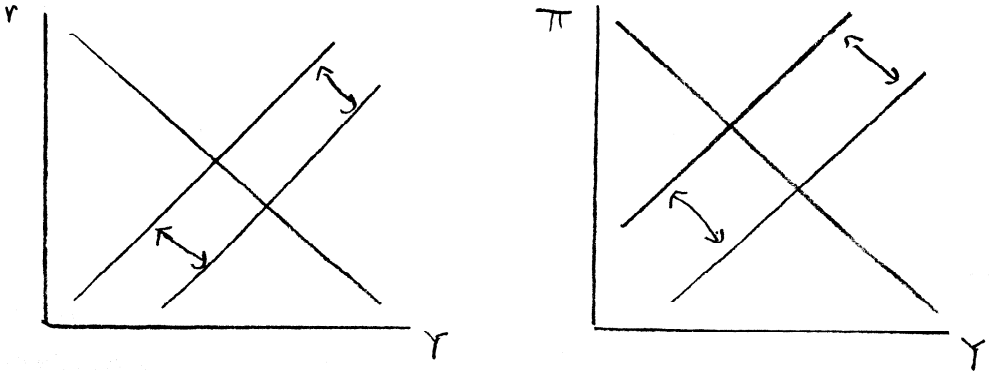
5) Consider a small open economy with perfect international capital mobility. The foreign interest rate is r^*. The exchange rate floats; exchange-rate expectations are static. The central bank fixes the money supply. The price level is fixed. Expected inflation is zero. This economy is subject to *financial-market imperfections* as described by the model in Romer's textbook. Each potential business project in the economy requires one unit of capital (one unit of output invested as capital in the business). The expected return to a business project (the expected return to the required unit of capital) is γ, where γ varies across potential entrepreneurs/projects. Information is "asymmetric:" a potential investor in a project can observe its realized return only after paying a cost c. The total number of projects undertaken is equal to total investment spending in this economy. (There is no other type of investment spending in this economy.) Each potential entrepreneur owns some *land*. What happens to the exchange rate, output and capital flow in the economy if there is a general, exogenous *decline* in the price of land in the economy?

6) Suppose an economy can be described by three equations:
- A Friedman-Phelps Phillips curve: $\pi_t = {}_{t-1}\pi_t^e + fy_t$
- An IS curve: $y_t = -ar_t$
- An interest-rate rule followed by the central bank: $r_t = by_t + c(\pi_t - \bar{\pi})$

where y denotes the log of the output gap (y is zero if output is equal to the natural rate of output), r denotes the difference between the real interest rate and the natural rate of interest, and $\bar{\pi}$ is a fixed value.
Any of these equations may also be subject to an *additive* mean-zero disturbance. The value of any realized disturbance ϵ_t is unknown as of time *(t-1)*. Expectations are model-consistent rational expectations.

a) Suppose there are disturbances to the IS curve equation *only*.
i) *Derive* ${}_{t-1}\pi_t^e$ in rational expectations equilibrium.
ii) Given your answer to i), derive π_t, y_t, r_t, i_t in terms of ϵ and $\bar{\pi}$.

b) What type or types of disturbance - to the IS curve, to the interest-rate rule, or to the Phillips curve - will create the fluctuations in the economy described by the graphs below?



c) Which type or types of disturbance will create a correlation between r and y that matches the slope of the IS curve?
d) Which type or types of disturbance will create a correlation between π and y that matches the coefficient f in the Phillips curve?

PART 3 JONES

7) A consumer's instantaneous utility function has the following form: $u(\Omega(c_1, \dots, c_N))$, where there are *N* commodities consumed. The function, Ω, is assumed to be linearly homogeneous. The function, *u*, is twice continuously differentiable and concave. Let $(p_1, \dots, p_N)$ denote the prices of the *N* commodities in domestic money terms (*i.e.* in \$, €, ¥ or whatever).

A) Define the price, *P*, corresponding to aggregate consumption, $C = \Omega(c_1, \dots, c_N)$, in domestic money terms.

B) Prove:

$$\frac{\partial C}{\partial c_n} = \frac{p_n}{P}$$

for all *n* and interpret.

C) Let $(1 + i_{t+1})$ be the gross nominal rate of return on a domestic bond. Write down the corresponding gross real return.

D) Now, write down the gross real return on a foreign bond letting $(1 + i^*_{t+1})$ be the gross nominal rate of return and P^*_t the foreign consumption price corresponding to P_t. Assuming PPP (in terms of consumption-based price levels) and covered interest parity, obtain an expression for the difference between the real rates of return on domestic and foreign bonds in terms of current and future domestic consumption prices (P_t and P_{t+1}), the domestic nominal interest rate (i_{t+1}), the forward rate ($\mathcal{F}_t$) and the future exchange rate (ε_{t+1}).

E) Home consumers may hold both domestic and foreign bonds. Using CCAPM type logic and previous results (you may just write down the optimality conditions, you do not need to derive them), show that

$$E_t\left[\left(\frac{\mathcal{F}_t - \varepsilon_{t+1}}{P_{t+1}}\right)\frac{u'(C_{t+1})}{u'(C_t)}\right] = 0$$

Note: i_{t+1} is known at time *t*.

F) Assuming $u(C)$ is linear, prove that

$$E_t\left[\frac{\mathcal{F}_t - \varepsilon_{t+1}}{P_{t+1}}\right] = E_t\left[\frac{(1/\mathcal{F}_t) - (1/\varepsilon_{t+1})}{P^*_{t+1}}\right] = 0$$

What does the assumption mean (in economic terms)? Why is this result important?

8) Let $A(z) = a^*(z)/a(z)$ and let $B(z) = zL^*/\big((1-z)L\big)$, where L and L^* are the home and foreign labor forces respectively. $a(z)$ denotes the number of home labor units per unit of good z produced by home with $a^*(z)$ defined in the corresponding way for foreign.

A) What does the function *A* look like? Explain why home would produce up to point $\tilde{z}$, where $A(\tilde{z}) = w/w^*$.

B) What does the function *B* look like? Why would you assume that $B(\tilde{z}) = w/w^*$ (your argument must be focused on economic logic)?

C) Let $\tilde{z}$ denote the point of intersection between *A* and *B* in a graph. Starting from this initial point, assume that foreign experiences a proportional increase in the labor required to produce all goods. Show how this affects your graph.

D) What is the effect of this on Home's real wage? Does Home gain or lose industries and why (just stating what the graph shows will earn you no points here)? Is home better or worse off?

E) Compare the changes in the prices of goods that are consistently produced by Home (*i.e.* before and after the shock) with the changes in the prices of goods that are consistently produced by Foreign.

F) Consider changes in the prices of goods that switch from one country to another and draw conclusions about Home's terms of trade.