1 Part A: Short answer or explanation. 5 questions. Make sure to explain your answers throughly.

1. (Khanna) Consider an economy with only two goods, X_1 and X_2. Let $P_2 = 1$. Use revealed preference to answer the following questions.

   (a) Person 1 behaves as follows in situations A and B. Is his behavior consistent?

Situation	Income	P1	X1 purchased
A	40	1	20
B	60	2	25

   (b) Person 2 behaves as follows in situations A and B. Is she consistent? In which situation is she better off, and why?

Situation	Income	P1	X1 purchased
A	40	1	20
B	61	2	15

2. (Jones) If a consumer has convex preferences, then his utility function must be concave. [True/-False/Uncertain and Justify.]

3. (Jones) A competitive firm prefers uncertain prices to certain prices. [True/False/Uncertain and Justify.]

4. (Pape) There is no mixed strategy Nash equilibrium of the Prisoner's Dilemma. [True/-False/Uncertain and Justify.]

5. (Pape) Walrasian Equilibria and Nash Equilbria are Pareto optimal. [True/False/Uncertain and Justify.]

2 Part B: Mathematical or advanced questions. 6 questions.

6. (Khanna) Suppose a consumer has the standard smoothly convex indifference curves. With the aid of a diagram, show how this person's optimal consumption bundle might still result in a corner solution where she spends all her income on a single good. Explain intuitively why she ends up at the corner.

7. (Khanna) Suppose that there are two types of users of fireworks, Careless and Careful. Careful users never get hurt, but Careless users sometimes injure not only themselves but also the innocent Careful users. The short run marginal cost curves for each of the 1000 firms in the fireworks industry are given by $MC = 10 + Q_i$, where Q_i is the number of fireworks produced by each firm i. The demand curve for fireworks by Careful users is given by $P = 50 - 0.001Q$. To avoid the externality of injury to Careful users, policy makers would like to ban fireworks altogether. What is the minimum compensation that Careful users would need in order to accept the ban? Explain how you arrive at your answer.

8. (Jones) A consumer consumes 2 goods and her expenditure function is $e(p, u) = \sqrt{p_1 p_2} u$. Use two different methods to find her Marshallian demand function for Good 1, i.e., $x_1(p, y)$.

9. (Jones) A function $f : \mathbb{R}^n_{++} \to \mathbb{R}$ is called superadditive if $f(z^1 + z^2) \geq f(z^1) + f(z^2)$ for all z^1 and z^2 in $\mathbb{R}^n_{++}$. Show that the cost function of a competitive firm $c(w, y)$ is superadditive in input prices w. Use this to prove that the cost function is nondecreasing in w without requiring it to be differentiable.

10. (Pape) Consider a duopoly setting, in which A and B are two firms. They each may choose their output q_A and q_B. The price at which they sell their goods are determined by the following demand curve:

$$p = 2 - (q_A + q_B) \tag{1}$$

Suppose for simplicity that the marginal cost of production is zero.

Suppose that EITHER the quantities are determined simultaneously, OR that A chooses his quantity first, announces it, and then B chooses a quantity. Which would A prefer? Explain!

11. (Pape) Suppose there are two consumers $i = 1, 2$ in an economy with two private goods: x and g. Suppose the utility of the consumers are given by:

$$u_i(x, g) = x + i \ln(g) \tag{2}$$

Suppose that each agent starts with an allocation of 3 units of x.

Suppose that there is a single firm which can manufacture g at a constant marginal cost of $c > 0$; suppose that each consumer owns a 50% share of that firm.

Suppose that you had a magic wand which could change g into a *public good*, and the amount of g would be determined by the private equilibrium for a public good. Question: would changing g into a public good be a Pareto improvement? Explain!

3 Part C: Longer questions. Answer all questions.

12. (Jones) Suppose that a consumer's utility function $u(\mathbf{x})$ is homogeneous of degree 1.
    (a) Let $\mathbf{p}$ denote the price vector and u utility. Show that the expenditure function is multiplicatively separable in $\mathbf{p}$ and u and can be written in the form $e(\mathbf{p}, u) = e(\mathbf{p}, 1)u$.
    (b) Let y denote income. Show that the marginal utility of income depends on $\mathbf{p}$, but is independent of y.

13. (Jones) Consider the quadratic von Neumann-Morgenstern (VNM) utility function $u(w) = a + bw + cw^2$.
    (a) What restrictions if any must be placed on parameters a, b and c for this function to display risk aversion?
    (b) Over what domain of wealth can a quadratic VNM utility function be defined?
    (c) Given the gamble

$$g = \left(\frac{1}{2} \circ (w + h), \frac{1}{2} \circ (w - h)\right) \tag{3}$$

    where $h > 0$, show that the certainty equivalent of g is less than the expected value of g, i.e., $CE < E(g)$.
    (d) Show that this function, satisfying the restrictions in part (a), can *not* represent preferences that display *decreasing* absolute risk aversion.

14. (Pape) Suppose that there are two entrepreneurs, A and B, who are racing to bring a similar invention to market. The probability that each agent brings the invention to market first is a function of how much *effort* $e_i, i = A, B$ that the agents expend, according to the following function:

$$Prob(\text{Agent } i \text{ is first}) = \frac{e_i}{e_A + e_B} \tag{4}$$

For simplicity, let's assume there are only two possible values of e_i: 1 and 2.

The cost of expending effort e_i is e_i, and the entrepreneur who brings the invention to market first wins the value $V > 2$, while the entrepreneur who fails wins a value of zero. Assume these agents are risk-neutral.

(a) Write the normal form representation of the game, with the payoffs as functions of V.

(b) Consider the requirement that $V > 2$. What would be peculiar if $V < 2$?

(c) Find all Nash Equilibria of this game, as a function of V.

(d) Is/are the Nash Equilibria Pareto Optimal? Answer carefully.

15. (Pape) Agents A and B have the following identical utility function over x and y:

$$u(x, y) = \ln(x) + \ln(y) \tag{5}$$

Right now, A has 1 unit of x and no units of y. B has no units of x and 1 unit of y.

Also, A has a .25 probability of receiving 3 units of x as a gift from his uncle. Assume that B knows this.

Suppose A can choose whether they trade BEFORE or AFTER he learns whether he will get the gift. Which is better for him? Which is better for B? Answer carefully, and make sure you show all your claims.