Micro Comp (First attempt) January, 2012

1 Part A: Short answer or explanation. 5 questions. Make sure to explain your answers throughly.

1. (Khanna) Consider a perfectly competitive industry with identical firms, each of which have the following total cost function

$$TC(q) = 0.01q^2 + 100 \tag{1}$$

   (a) Does the typical firm in this industry experience economies of scale or diseconomies of scale, or both? Explain how you arrive at your answer.

   Suppose this industry has the following market demand curve

$$q(p) = 10,000 - 100p \tag{2}$$

   (b) Compute the long run equilibrium price, quantity, and the number of firms in this industry.

2. (Jones) If a consumer has convex preferences, then his optimal consumption choice is unique. [True/False/Uncertain and Justify]

3. (Jones) If a good is normal, it will satisfy the law of demand. [True/False/Uncertain and Justify]

4. (Pape) In Walrasian equibrium, if a bundle $\vec{x}$ is affordable by some agent, then it is feasible. [True/False/Uncertain and Justify.]

5. (Pape) The process of Iterated Elimination of Strictly Dominated Strategies never eliminates a Nash Equilibrium. [True/False/Uncertain and Justify.]

2 Part B: Mathematical or advanced questions. 6 questions.

6. (Khanna) Keiko spends all her income on wireless minutes (W) for her cellphone and gasoline (G) for her car. Her marginal rate of substitution for gasoline with wireless minutes is $MRS_{GW} = \frac{10}{\sqrt{G}}$. The price of gasoline is $P_G = 1$ and the price of wireless minutes is $P_W = 0.5$. Draw Keiko's income-consumption curve and her Engel curves for gasoline and wireless minutes.

7. (Khanna) Suppose Sasha consumes two goods, X and Y, and has the following CES utility function

$$U(X,Y;\tilde{X},\tilde{Y}) = \left[\alpha\left(\frac{X}{\tilde{X}}\right)^{\rho} + (1-\alpha)\left(\frac{Y}{\tilde{Y}}\right)^{\rho}\right]^{\frac{1}{\rho}} \tag{3}$$

where $0 < \alpha < 1$, and $\rho < 1$ and $\rho \neq 0$. $\tilde{X}, \tilde{Y} > 0$ stand for Sasha's friend, Alexi's consumption of goods X and Y. That is, Sasha's utility depends not only on his own consumption of X and Y but also on his friend's consumption of these goods, even though he cannot control what his friend consumes.

   (a) Derive Sasha's marginal rate of substitution between X and Y.

   (b) Derive an expression for Sasha's optimal consumption of good X, $X^\star$. You may assume that $P_X = P_Y = 1$.

   (c) Show that MRS_{XY} is an increasing function of $\tilde{X}$ when $\rho < 0$ and that it is a declining function of $\tilde{X}$ when $0 < \rho < 1$.

   (d) With the help of a diagram (or two) provide an intuitive explanation for how Sasha's consumption of X changes with $\tilde{X}$ when (a) $\rho < 0$ and (b) $0 < \rho < 1$.

8. (Jones) An individual consumes 2 goods. His utility function is $u(x_1, x_2) = \min\{ax_1, bx_2\}$ where $a > 0$ and $b > 0$. Find his indirect utility function, expenditure function, Hicksian demand functions, and Marshallian demand functions.

9. (Jones) Let $c(w, y)$ be the cost function generated by the production function f and suppose that (1) $\max_{y\geq 0} py - c(w,y)$ and (2) $\max_{x\in\mathbb{R}^n_+} pf(x) - w \cdot x$ have solutions $y^\star \geq 0$ and $x^\star \geq 0$, respectively.

   (a) Show that $\hat{y} = f(x^\star)$ solves (1).

   (b) Show that if $c(w, y^\star) = w \cdot \hat{x}$ and $y^\star = f(\hat{x})$, then $\hat{x}$ solves (2).

   (c) Use parts a and b to show that $py^\star - c(w, y^\star) = pf(x^\star) - w \cdot x^\star$.

10. (Pape) **A Sequential Artwork Selling Mechanism.** Mr. 1 and Mr. 2 both have independent valuations $\theta_i \sim U[0,1]$ for a piece of art. There is a hidden price p for the art, which is distributed according to some distribution G, also on the interval $[0,1]$. First, it is Mr. 1's turn. Mr. 1 is allowed to observe p, and then Mr. 1 has the opportunity to purchase the art at the price $\$p$ if he wants, or he can pass. If he chooses to purchase, he receives the artwork and pays p, and the game is over. If he passes, then it is Mr. 2's turn. Mr. 2 must then decide whether to pay an entrance fee of $\$x$, $0 < x < 1$ for the privilege of seeing the price p, or to pass. If he sees the price p, he can then decide whether to purchase the artwork at price p or to pass again. If he passes at any point, the game is over. Find the Subgame Perfect Nash equilibrium of this game. Make sure Mr 2's strategy is a function of x.

11. (Pape) An exchange economy has three consumers and three goods. Consumer's utilities and initial endowments are:

$$u^1(x,y,z) = \min\{x,y\} \quad \omega_1 = (1,0,0)$$
$$u^2(x,y,z) = \min\{y,z\} \quad \omega_2 = (0,1,0)$$
$$u^3(x,y,z) = \min\{x,z\} \quad \omega_3 = (0,0,1)$$

Find the competitive equilibrium in this economy.

3 Part C: Longer questions. Answer all questions.

12. (Jones) An individual's true taxable income is m. The tax rate is $t(0 < t < 1)$. The government does not know what the individual's true income is, so the individual must report that information to the government. Let x be the income that he chooses to report. However, the government can audit the individual afterwards. Auditing is random, and it occurs with probability P. We assume that an audit will reveal the individual's income perfectly. If the audit finds that the individual has been honest, then he simply pays tm. If it finds that he has reported less than his true income, then he pays tm plus a fine equal to $st(m - x)$ where $0 < s < 1$ (so the fine is proportional to under-reported tax liabilities). If the individual is not audited, then he just pays tx. Finally, the individual's utility-over-wealth function is $u(\cdot)$ is strictly increasing and strictly concave.

    (a) Find the condition under which the individual will choose to report honestly. Offer an intuitive explanation for why this result is true, and also interpret what it implies about tax enforcement policies.

    (b) Find the condition under which the individual will choose to report less than his true income.

    (c) Assume that the condition you found in part b holds. Show that if the individual's utility function exhibits constant absolute risk aversion, then the amount by which he under-reports his income will be a constant.

13. (Jones) A consumer lives 2 periods. His incomes in Period 1 and Period 2 are m_1 and m_2, respectively. There is a single consumption good in each period whose price is 1. The consumers utility function is $u(x_1, x_2)$ where x_1 and x_2 denote the quantity of his consumption good in Period 1 and Period 2, respectively. Borrowing and lending between the 2 periods are possible at the interest rate r. The consumers problems is to maximize $u(x_1, x_2)$ subject to his intertemporal budget constraint.

    (a) Show that his intertemporal budget constraint can be expressed as $(1 + r)x_1 + x_2 = (1 + r)m_1 + m_2$.

    (b) Let $x_1(r, m_1, m_2)$ denote the consumers optimal choice of consumption in Period 1. Derive the Slutsky equation for the term $\frac{\partial x_1(r,m_1,m_2)}{\partial r}$. (You can make use of the Slutsky equation for the conventional setting.)

    (c) Analyze the equation you got in b. Assuming that consumption in Period 1 is normal, can you sign the term $\frac{\partial x_1(r,m_1,m_2)}{\partial r}$? If your answer is "no," what will the sign depend on?

    (d) Draw a graph to illustrate how an increase in the interest rate r affects the consumers optimal choice.

14. (Pape) Suppose there are two goods, x and y, and two agents, A and B. Agent i's utility over x and y ($i \in \{A, B\}$) is given by:

$$u(x, y) = \ln(x) + 2\ln(y) \quad (4)$$

Also suppose that there are two states of the world, numbered 1 and 2. Suppose that the probability of state 1 is given by π_1 and the probability of state 2 is given by $\pi_2 = 1 - \pi_1$.

Finally, suppose that endowments are given by:

$$\omega_A = \big((1, 0), (2, 0) \big) \quad (5)$$
$$\omega_B = \big((0, 1), (0, 4) \big) \quad (6)$$

(a) True/False/Uncertain and Justify. In the Arrow-Debreu equilibrium of this economy,

$$\frac{p_x(1)}{p_y(1)} = \frac{p_x(2)}{p_y(2)} \quad (7)$$

(b) True/False/Uncertain and Justify. Agent A would be willing to pay to increase π_1 (and lower π_2).

(c) True/False/Uncertain and Justify. In the Radner equilibrium of this economy, no assets will be traded at time zero.

15. (Pape) There are two players, Rowland (the row player) and Collette (the column player) play a game with two rounds, and each round is a 'stage game.' First, they play Stage Game a, and, immediately afterwards, they play Stage Game b, which is a version of the Battle of the Sexes. Rowland and Collette's payoffs for the overall game is the sum of their payoffs in each stage game, with no discounting.

	L	R
U	5, 5	0, 10
D	6, 1	1, 0

(a) Stage Game a.

	F	O
F	3, 1	0, 0
O	0, 0	1, 3

(b) Stage Game b.

(a) How many strategies does Rowland have? List them.

(b) Propose a subgame-perfect Nash equilibrium which gives Collette the highest possible payoff. Explain.

(c) Now suppose that there is a π chance that Collette is *crazy*. After the first stage game, whether Collette is crazy is revealed to both players. Crazy Collette has the following payoffs for the second stage game: For what π does the possibility of crazy Collette make

	F	O
F	3, −1	0, 3
O	0, −1	1, 3

Figure 1: Stage Game b'.

sane Collette *worse off*, or are there no such π? For what π does the possibility of crazy Collette make sane Collette *better off*, or are there no such π? Explain!