MACROECONOMIC THEORY COMPREHENSIVE EXAMINATION

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Each question consists of several parts. Complete each part to the best of your ability. Be sure to manage your time efficiently. Show all your work. Good luck!

Question 1

Consider the problem faced by a representative agent who seek to maximize expected life-time utility

$$max E_0 \sum_{t=0}^{\infty} \beta^t (ln c_t - \frac{l_t^{1-\eta}}{1-\eta}),$$

subject to the constraint

$$c_t + i_t = y_t - p_t e_t,$$

and a law of motion for the aggregate capital stock

$$k_{t+1} = (1-\delta)k_t + i_t - \phi(i_t/k_t)k_t,$$

where the variables and parameters are defined as follows.

- c_t : consumption

- i_t: investment

- y_t: output

- k_t : capital stock

- l_t: labor hours

- e_t: energy usage

- p_t : price of energy relative to consumption goods

- β: discount factor

- δ: depreciation rate of capital

- k_0: given

Define the production function as

$$y_t = G[F(k_t, e_t), l_t],$$

where the function F() aggregates capital and energy into capital services and G() aggregates capital services and labor into output. They are defined as

$$
\begin{aligned}
F(k_t, e_t) &= k_t^{\alpha} e_t^{1-\alpha}, \\
G[F(k_t, e_t), l_t] &= [F(k_t, e_t), l_t]^{\gamma} l_t^{1-\gamma},
\end{aligned}
$$

where $\gamma > 0$ and $\alpha > 0$.

1. What variables are endogenous, state and exogenous variables? Make a list of each.

2. The function $\phi(i_t/k_t)$ represents the per-unit cost of adjusting the aggregate stock of capital. It satisfies $\phi' > 0, \phi'' > 0$. Suggest a specific functional form that satisfies these requirements. Explain the economic meaning of these requirements.

3. **Derive all first order conditions of a social planner's problem. Carefully explain the economic meaning of each individual equation.**

4. In a standard real business cycle model, what is the impulse mechanism? In this particular model, which variable can serve as an impulse mechanism? Why?

5. Why can you solve a social planner's problem to represent the outcome of an competitive economic equilibrium? Using equations and words, construct the free market version of this economy. Set up the consumer's problem, the producer's problem, and define a market equilibrium. This version of the economy should have the same solution as the social planner's problem I defined above. You do not need to solve the problem. Just set it up.

Question 2

Consider a version of the Optimal Growth model. The representative agent has an objective function

$$\int_0^\infty e^{-\beta t} \ln c_t dt$$

As usual we take advantage of the First Welfare Theorem to solve a social planner's problem. The planner's resource constraint is

$$\begin{aligned}
\dot{k} &= y - c - \theta k \\
y &= k^a \\
a > 0, \qquad & k_0 \text{ given.}
\end{aligned}$$

Note that all variables have been properly detrended, so that all variables are in per effective labor terms (for example, $k = K/AL$).

1. Form the current value Hamiltonian and write down the first order conditions.

2. Find a system of two equations that characterize the equilibrium of the economy.

3. Find the steady state values for k and c.

4. Using phase diagrams, discuss the system dynamics around the steady state. Make sure you cover all possible scenarios.

5. Predict the long run growth rate of this economy. Again, make sure you cover all possibilities.

Question 3

In an attempt to stimulate aggregate demand, the federal government is considering two different tax-cut policies:

- A tax cut that goes into effect immediately

- An announced future tax cut

1. According to the infinite horizon, rational expectations version of consumption theory you learned in class, what is the effect of each policy on consumption and aggregate demand? Why?

2. Suppose now that the consumers' borrowing ability is severely constrained. That is, they are unable to borrow loans at any interest rates. Carefully explain the effect of each policy on consumption and aggregate demand. Which policy is a more powerful policy in terms of stimulating consumption and aggregate demand?

4) Consider the following model:

$$y_t = {}_t y_{t+1}^e - sr_t + u_t \quad where \ u_t = \rho u_{t-1} + \epsilon_t$$

$$\pi_t = {}_t \pi_{t+1}^e + \beta y_t$$

$$r_t = \phi \pi_t$$

where π is the inflation rate, y is the output gap, r is the gap between the real interest rate and the natural rate of interest, and ϵ is a mean-zero i.i.d. random variable.

a) Assume the economy must eventually converge to a long-run steady state with $y = 0$, $\pi = \pi^e = 0$. Solve for y_t and π_t in terms of u_t.

b) In this model, is the correlation between y_t and π_t positive or negative?

c) What is a *simple* way to change the model so that the correlation between y_t and π_t would be the opposite of what it is in this model?

5) Consider the old-fashioned IS/LM and IS/MP models - not the new-Keynesian models with the forward-looking ${}_t \pi_{t+1}^e$ and ${}_t y_{t+1}^e$, but the old-fashioned ones. Answer the following two questions in words (though you can use graphs and/or equations too, if you want).
a) In the IS/LM model, why does the "aggregate demand" curve slope down?
b) In the IS/MP model, why does the "aggregate demand" curve slope down?

6) Consider the Phillips curve, where π is the inflation rate and y is the output gap.

a) Describe the assumptions of *two* different models that imply $\pi_t = {}_{t-1}\pi_t^e + \beta y_t$

b) Describe the assumptions of *two* different models that imply $\pi_t = {}_t\pi_{t+1}^e + \beta y_t$

7) Suppose the elasticity of demand for a product tends to *increase* when employment is especially high. Would this make it *harder* or *easier* to maintain a "fixed price equilibrium" in a single-period "menu cost" model such as Mankiw's, or make no difference? Explain!

8) Consider an IS curve equation in which $y_t = {}_t y_{t+1}^e - sr_t + u_t$

where y is output and r is the real interest rate on short-term, riskless assets (such as government bonds). In this expression, u represents "shocks" to the IS curve. Think about all the models we learned this semester. What could u represent, other than changes in taxes (which wouldn't matter anyway if Ricardian equivalence holds)? That is, what factors can cause short-term variations in aggregate demand for goods and services at a given real interest rate for riskless assets? Tell me about *two* different factors.

9) In two of the models we studied, variable "capacity utilization" plays an important role.
a) What does the phrase "capacity utilization" mean, in terms of macroeconomic models?
b) For each of the two models, explain the role of variable capacity utilization. How did variable capacity utilization, as opposed to constant capacity utilization, help the model generate realistic outcomes?