Micro Comp (First attempt) June 3, 2013

1 Part A: Short answer or explanation. 4 questions.

1. (Jones) Show that a competitive firms cost function is superadditive in input prices, i.e., $c(\mathbf{w}^1 + \mathbf{w}^2, y) \geq c(\mathbf{w}^1, y) + c(\mathbf{w}^2, y)$.

2. (Jones) Show that the share of income that a consumer spends on good x_i can always be measured by $\frac{\partial \ln e(\mathbf{p}, u^\star)}{\partial \ln p_i}$, where $u^\star = v(\mathbf{p}, y)$.

3. (Pape) Find all Nash Equilibria of the following game:

	b_1	b_2	b_3
a_1	$-1, 2$	$5, 1$	$-1, 3$
a_2	$3, 1$	$4, 0$	$0, 3$
a_3	$0, 2$	$1, 0$	$1, 1$

4. (Pape) Suppose there is a fixedamountof orange juice concentrate to be made into juice for a party in Library Tower 907 for the end of the year. Suppose that the flavor of the orange juice is a differentiable function of the amount of water which is added to it: too little water, and it is too strong-tasting, and too much, and it tastes weak. Also, the amount of orange juice produced (and therefore that can be consumed) is of course an increasing function of the amount of water added. The person in charge of making the orange juice says "the best amount of water to use is that which maximizes flavor!" Is she right or wrong? Justify your answer.

2 Part B: Mathematical or advanced questions. 6 questions.

5. (Jones) A consumers utility function is given by $u(x_1, x_2) = x_1 + lnx_2$. His income is y, and the prices of the 2 goods are p_1 and p_2, respectively.
   (a) Find his Marshallian demand function.
   (b) Draw a few of his indifference curves. What feature of his indifference curves explains the special form of his Marshallian demand?

6. (Jones) Consider the following "Independence Axiom" on a consumer preferences, $\succsim$, over gambles: If
   $(p_1 \circ a_1, \cdots, p_n \circ a_n) \sim (q_1 \circ a_1, \cdots, q_n \circ a_n)$,
   Then for every $\alpha \in [0,1]$, and every simple gamble $(r_1 \circ a_1, \cdots, r_n \circ a_n)$,
   $$\big((\alpha p_1 + (1-\alpha)r_1) \circ a_1, \cdots, (\alpha p_n + (1-\alpha)r_n) \circ a_n\big) \sim \\ \big((\alpha q_1 + (1-\alpha)r_1) \circ a_1, \cdots, (\alpha q_n + (1-\alpha)r_n) \circ a_n\big)$$
   Show that this axiom follows from the axioms of Substitution and Reduction to Simple Gambles. Interpret this axiom.

7. (Jones) Given a production function $f(x)$, the **output elasticity of input** i is defined as $\mu_i(x) \equiv (f_i(x)x_i)f(x)$. The **elasticity of scale** at point x is defined as $\mu(x) \equiv \lim_{t\to 1} \frac{d\ln f(tx)}{d\ln t}$.
   (a) Show that $\mu(x) = \sum_{i=1}^{n} \mu_i(x)$.
   (b) Explain in words what elasticity of scale measures.
   (c) For the production function $y = k\Big(1+x_1^{-\alpha}x_2^{-\beta}\Big)^{-1}$, where $\alpha > 0, \beta > 0$, and k is an upper bound on the level of output, so that $0 \le y < k$, show that the elasticity of scale equals $(\alpha+\beta)(1-\frac{y}{k})$. What can we say as y approaches k?

8. (Pape) Consider a bargaining game between three players $i = 1, 2, 3$. There is a pie which starts at size 1, and in period t has shrunk to size s_t, which is given below. Assume t starts at 1. At the beginning of each period t, one player is making an offer and the other two must accept or reject that offer. The 'offerer' position rotates as you would expect: at $t = 1$, player 1 offers, $t = 2$ player 2 offers, at $t = 3$ player 3 offers, and at $t = 4$ player 1 offers again, et cetera. The offering player makes an offer (a, b, c), where $a + b + c \le s_t$, where an amount a goes to player 1, b to player 2, and c to player 3. Then the other two players must each choose to accept or reject this offer. If *both* accept, then the pie is split according to the offer. If *either one* rejects the offer, then time advances to the next period (i.e. the pie shrinks), and the next offering player has a chance to make an offer. This repeats until the pie disappears. Agents' payoffs are simply the amount of pie they receive (which might be zero).
   (a) Suppose $s_t = \max\{\big(1 - \frac{1}{8}(t-1)\big), 0\}$. Find all subgame perfect Nash equilibria.
   (b) Suppose $s_t = \delta^{t-1}$ for some $\delta \in (0,1)$. Find all subgame perfect Nash equilibria.

9. (Pape) There is a technology which makes good x with a technology that involves a constant marginal cost c. The demand curve for x is $q_D(p) = M - kp$ where $M > c$ and $k > 0$. There are two firms: A and B. Firm A has the opportunity to spend a dollar amount H for the privilege of choosing his quantity before firm B: if he chooses not to pay H, then they choose simultaneously. Consider the subgame perfect Nash equilibrium/equilibria for this game. For what values of H does A pay it, if any?

10. (Pape) Suppose there is a one dimensional city, which stretches from 0 to 1. There are two firms which sell pencils; one is located at 0 and one is located at 1, and we will identify firm j by its location, so $j = 0, 1$. Each firm j can choose a price p_j at which to sell pencils. Suppose each firm's marginal cost of producing pencils is $c \geq 0$.

    Suppose that there are an infinite number of agents $i = [0, 1]$, each at a location equal to their index number i. Each agent wants to buy at most one pencil. The utility of agent i who buys a pencil from a firm at location j for price p is:

    $$u_i(j, p) = 2 - (j - i)^2 - p \tag{1}$$

    The utility of an agent who does not buy a pencil is 0.

    Suppose the game proceeds as follows: First, firms choose their prices. Second, agents chose which firm they will buy a pencil from, if any.

    (a) Suppose $c = 0$. What is/are the subgame-perfect nash equilibria of this game? A picture may be helpful.

    (b) Now suppose $c > 0$. How does the SPNE change? A picture may be helpful.

3 Part C: Longer questions. 4 questions.

11. (Jones) There are many sellers of used cars. Each seller has exactly one used car to sell and is characterized by the quality of the used car he wishes to sell. Let $\theta \in [0, 1]$ index the quality of a used car and assume that θ is uniformly distributed on $[0, 1]$. If a seller sells his car (of quality θ) for a price of p, his utility is $u_s = p - \theta$. If he does not sell his car, then his utility is 0. Buyers of used cars receive utility $\sqrt{\theta} - 2p$ if they buy a car of quality θ at price p and receive utility 0 if they do not buy a car. Sellers know the quality of the car they are selling, but buyers do not know it. There is an infinite number of buyers.
    (a) Show that if information was symmetric, then all cars would be traded.
    (b) Show that $E(\sqrt{\theta}|p) = 2p$ in a competitive equilibrium under asymmetric information.
    (c) Find all the competitive equilibria under asymmetric information. How many equilibria are there? Describe what cars are traded and at what price.

12. (Jones)
    (a) Write down the usual Slutsky equation.
    (b) Suppose, instead of a fixed income, the consumer only has an initial *endowment* of goods $\omega = (\omega_1, , \omega_n) >> 0$, where ω_i is the quantity of good i that he is endowed with. (E.g, my initial endowment could be 4 apples, 2 bananas and 7 coffees.) His "income" is the market value of his endowment, i.e., $y = p \cdot \omega$. Use your answer in (a) as the basis, modify whatever is necessary to derive the Slutsky equation for this new setting, i.e., an expression for $\frac{\partial x_i(p, p \cdot \omega)}{\partial p_j}$ which shows the income and substitution effects.
    (c) Use a graph (assuming $n = 2$) to illustrate the income and substitution effects in this setting. Explain how they are different from the conventional setting in which the consumer's income is fixed.

13. (Pape) Consider the one-consumer, one-producer economy; i.e. the Robinson Crusoe economy. Suppose the production function is $f(z) = \sqrt{z}$ and the utility function is $u(l, x) = \frac{2}{3}ln(l) + \frac{1}{3}ln(x)$, where l is **leisure** and z is the input of **labor** into production. Assume Robinson has an endowment of zero food and his endowment of leisure is 1 unit. Assume that he fully owns the firm. Suppose that the price of x is 1.

    Suppose that tax revenues must be raised in this economy. It can either be (a) *an income tax* t_i, which means that t_i dollars are paid to the government for each unit of labor that Robinson sells to the firm or (b) *a profit tax* t_π, which means that t_π dollars are paid to the government for each dollar of profit raised by the firm.
    (a) Find the Walrasian Equilibrium of this economy under the income tax t_i.
    (b) Find the Walrasian Equilibrium of this economy under the profit tax t_π.
    (c) Suppose that T dollars total must be raised. Given the level of tax revenue T, which tax policy inflicts the least pain on Robinson? Explain.

14. (Pape) Consider a two person economy with an externality. Let the identities of the two agents be $i = 1, 2$. Let there be two goods: a private consumption good x and h is a good that 1 purchases and consumes, which produces an externality for 2.

$$u_i(x_i, h) = x_i + \phi_i(h) \tag{2}$$
$$\text{where } \phi_1(h) = h^\alpha, \alpha \in (0,1) \tag{3}$$
$$\text{and } \phi_2(h) = -h^\beta, \beta > 1 \tag{4}$$

Suppose that there is a single firm which produces h with the following production function: $h = f(x) = 2\sqrt{x}$.

(a) Find the Walrasian equilibrium for this economy.

(b) Find the optimal level of h in this economy. Find a Pigouvian tax or subsidy to bring about this level of h.

(c) Draw a graph for the market for h which describes the private and optimal levels of h as well as the policy solution.

(d) Suppose that 2 can make an effort e that changes her experienced externality. In particular, replace her utility function with:

$$\tilde{u}_2(x_2, h, e) = x_i + \phi_2(h - e) \tag{5}$$

What is the new Walrasian equilibrium? Comment on the policy implications.