Each question consists of several parts. Complete each part to the best of your ability. Be sure to manage your time efficiently. Show all your work. Good luck!

Question 1

Consider the following Real Business Cycle theory (RBC). The preferences of consumers are characterized by the utility function

$$E_0 \sum_{t=0}^{\infty} \rho^t [\log(c_t) - \frac{n_t^{1+\gamma}}{1+\gamma}], \tag{1}$$

where c_t and n_t are consumption and hours worked, respectively. $\gamma \geq 0$. The consumer's budget constraint is

$$c_t + i_t = w_t n_t + r_t K_t, \tag{2}$$

where w_t is the wage rate, r_t is the rental rate of capital, and K_t is the level of physical capital the consumers lend to a firm. i_t is defined by

$$K_{t+1} = i_t + (1-\delta)K_t \tag{3}$$

Define the production technology as

$$Y_t = Z_t K_t^a n_t^{1-a}, \tag{4}$$

where Z_t is a stochastic shock to productivity. The competitive firm's objective is to maximize its profits at time t.

1. Solve the consumer's problem and derive a set of first order conditions. Explain the intuitions of each equation.
2. Solve the firm's profit maximization problem and express r_t and w_t as functions of K_t, n_t and Z_t.
3. Given the assumption in (4) and competitive markets, what should $Y_t - w_t n_t - r_t K_t$ be equal to? Plug this into (2) to obtain the resource constraint for the economy.
4. Set up a social planner's problem that would give rise to an equilibrium that is identical to the one you derived above.
5. According to this RBC model, if there are no external disturbances, what is the long-run growth rate of the economy?
6. What is the external disturbance that causes the economy to fluctuate? AFTER an external disturbance hits, how does it propagate through the economy? Use an example, carefully describe two ways that the disturbance can propagate through the economy.

Question 2

Consider a version of the Optimal Growth model. The representative agent has an objective function

$$\int_0^{\infty} e^{-\beta t} \ln c_t dt$$

As usual we take advantage of the First Welfare Theorem to solve a social planner's problem. The planner's resource constraint is

$$\begin{aligned} \dot{k} &= y - c - \theta k \\ y &= Ak \\ A > 0, &\quad k_0 \text{ given.} \end{aligned}$$

Note that all variables have been properly detrended, so that all variables are in per effective labor terms (for example, $k = K/AL$).

1. Form the current value Hamiltonian and write down the first order conditions.
2. Find a system of two equations that characterize the equilibrium of the economy.
3. One of the equations is an autonomous differential equation. Solve to get the path for consumption.
4. Define "balanced growth path." Suppose the economy is on a balanced growth path. Solve for the path for k. That is, obtain an expression for k that characterizes its dynamic path in the long run.
5. Will the transversality condition always be satisfied? If yes, prove it. If not, derive the conditions under which it will be satisfied.

Question 3

Suppose the federal government decides to increase spending by $800 billion in 2013. Consider the following two economic policy scenarios:

- To finance the spending, the government raises taxes by $800 billion in 2013.
- Instead of raising taxes, the government decides to run a deficit of $800 billion. All the required funds will be financed by issuing 10 year Treasury bonds.

  According to the infinite horizon, rational expectations version of consumption theory you learned in class, do these two policy measures affect a rational consumer's consumption path differently? If yes, explain how the consumers' behavior would be different in each scenario. If not, explain why you do not think the economic outcomes would be different.

4) Consider the following model:

i) $y_t = {}_t y^e_{t+1} - sr_t$

ii) $\pi_t = {}_t\pi^e_{t+1} + \beta y_t + u_t \quad where\ u_t = \rho u_{t-1} + \epsilon_t \qquad 0<\rho<1$

iii) $r_t = \phi \pi_t$

where π is the inflation rate, y is the output gap, r is the gap between the real interest rate and the natural rate of interest, and ϵ is a mean-zero i.i.d. random variable. Equation i) is a "New Keynesian IS curve." ii) is a "New Keynesian Phillips curve." iii) is a simple interest-rate rule; it describes the behavior of the central bank.

a) Assume the economy must eventually converge to a long-run steady state with $y = 0$, $\pi = \pi^e = 0$. Solve for y_t and π_t in terms of u_t.

b) In this model, is the correlation between y_t and π_t positive or negative?

c) Now suppose the central bank does *not* follow the interest-rate rule described by equation iii). Instead, the central bank acts like the central bank in Clarida, Gali and Gertler (1999), "The Science of Monetary Policy: A New Keynesian Perspective." That is, the central bank acts to minimize the expected value of a loss function that embodies a desired inflation rate and a desired output gap. Assume specifically that the central bank's desired output gap is zero (the central bank acts as if it wants output to equal the natural rate of output). Also assume that the central bank can observe the value of u_t when it sets the interest rate for period t. In this setting,

- would the correlation between u_t and π_t be positive, negative or zero?
- would the correlation between u_t and y_t be positive, negative or zero?

Hint: you should not have to do any math here.

5) Demonstrate the "dynamic inconsistency of optimal monetary policy" proposition: in rational expectations equilibrium, inflation will exceed society's desired inflation rate, if the central bank's policy expresses society's desire for output greater than the natural rate of output. *Prove* this result in a model with a Friedman-Phelps (not new-Keynesian) Phillips curve $\pi_t = E_{t-1}\pi_t + \beta(y - \bar{y})_t$ where y is output (not the output gap) and $\bar{y}$ is the natural rate of output..

6) Consider two economies, A and B, where aggregate supply follows the "Lucas supply function" model. In each economy, the nominal money supply is known to follow a random walk:
$m_t = m_{t-1} + \epsilon_t$ where ϵ is mean-zero i.i.d.

The variance of ϵ is bigger in economy A.

For each economy, you have time-series data on inflation and the output gap y. For each economy, you run regressions to estimate an equation of this form: $\pi_t = \beta_y y_t + \beta_\pi \pi_{t-1}$

Explain what you will find. In each economy, will estimated β_y be positive, negative or zero?
will estimated β_π be positive, negative or zero?

Will the estimated coefficients be the same for the two countries, or different? If they are different, how will they differ?

7) Briefly describe *three* assumptions in the model of Christiano, Eichenbaum and Evans (2005), "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," that add to the degree of "real rigidity" in that model.

8) Consider an economy where prices are set as in the Calvo model, with a fixed probability z that a firm will be allowed to change its price in any given period, where a period is one quarter of a year. But at the beginning of each decade (ten years) a firm is allowed to *choose* the value of z. If a firm chooses a higher value of z at the beginning of a decade, it will gain a higher probability of being allowed to change its price in any given period within the decade, but it must also pay a higher per-period cost $C(z)$ (that is $C'(z) > 0$). The average rate of growth in aggregate demand m varies across decades, in a way that can be predicted by firms at the beginning of a decade.

a) Comparing across decades, what relation would you expect to observe between the average rate of price inflation in a decade and the average duration (lifetime) of a price in a decade? Explain.

b) Comparing across decades, what relation would you expect to observe between the average rate of price inflation in a decade and the coefficient on the output gap in the Phillips curve? Explain.

9) Consider an economy like, but not exactly like, Romer's baseline imperfect competition model. There are many identical households. A household's utility function is:

$$U_j = C_j - \frac{1}{\gamma} L_j^{\gamma} \qquad where \quad \gamma > 1$$

where C_j is a household's consumption (money income divided by the price index) and L_j is the quantity of labor supplied by the household. There are just as many firms as households. Each firm is a monopolist, selling one product. Note that labor input per firm equals labor supply per household, because the number of firms and households is equal.

Demand for one firm's product can be described by: $Y_i = (P_i/P)^{-\eta} Y$

Under these circumstances, each firm's profit-maximizing price is equal to a fixed markup times marginal cost. The markup is equal to $\eta/(\eta-1)$.

Unlike Romer's model, each firm's production function is $Y_i = K_i^{\alpha} L_i^{1-\alpha}$
where each firm's capital stock K_i is *fixed* and equal to one unit.

a) Write an equation that gives a firm's profit-maximizing price P_i^* as a function of the market wage W and the firm's labor input L_i.

b) In the absence of menu costs (or any other barriers to wage and price adjustment), what will be the value of $L_i = L_j$ in the economy?