Micro Comp **(Second attempt)** **July 1, 2013**

1 Part A: Short answer or explanation. 4 questions.

1. (Jones) Suppose the preference relation is locally non-satiated. Let $\mathbf{p} \gg 0$ be the price vector, $y \geq 0$ income, and $x^\star$ the most preferred bundle in the budget set. Prove: $\mathbf{p} \cdot x^\star = y$ (i.e., at the optimum budget is exhausted).

2. (Jones) Let $\pi(p, \mathbf{w})$ be a competitive firms profit function given the output price p and input price vector $\mathbf{w}$. Prove: $\pi(p, \mathbf{w})$ is convex in $(p, \mathbf{w})$.

3. (Pape) Find all Nash Equilibria of the following game:

	b_1	b_2	b_3
a_1	3, 3	5, 1	1, 2
a_2	3, 1	4, 1	0, 2
a_3	2, 2	1, 9	0, 1

4. (Pape) If an allocation is Pareto Optimal, then prices can be found such that this allocation is a Walrasian Equilibrium. True/False/Uncertain and Justify.

2 Part B: Mathematical or advanced questions. 6 questions.

5. (Jones) Suppose two goods, $x_1(p, y)$ and $x_2(p, y)$, have equal income elasticity at (p^0, y^0). Prove that $\frac{\partial x_1}{\partial p_2} = \frac{\partial x_2}{\partial p_1}$ at (p^0, y^0).

6. (Jones) A consumer of two goods faces positive prices and has a positive income. His utility function is $u(x_1, x_2) = \max\left[\frac{1}{3}x_1, \frac{1}{3}x_2\right] + \min\left[x_1, x_2\right]$. Derive his Marshallian demand functions, indirect utility function, Hicksian demand functions and expenditure function.

7. (Jones) Suppose a consumer's utility function is additively separable, i.e.,

$$u(x) = \sum_{i=1}^{n} f_i(x_i) \tag{1}$$

Suppose u is strictly quasiconcave with $f_i'(x_i) > 0$ for all i. Let $\mathbf{p} \gg 0$ be the price vector and $y > 0$ be income. Assume $\mathbf{x}(\mathbf{p}, y) \gg 0$.

(a) Show that if one good displays increasing marginal utility at $\mathbf{x}(\mathbf{p}, y)$, all other goods must display diminishing marginal utility there.

(b) Prove that if one good displays increasing marginal utility and all others diminishing marginal utility at $\mathbf{x}(\mathbf{p}, y)$, then one good is normal and all other goods are inferior.

(c) Show that if all goods display diminishing marginal utility at $\mathbf{x}(\mathbf{p}, y)$, then all goods are normal.

8. (Pape) Consider the market for professional basketball players. Suppose that there are teams $t = 1, \ldots, T$, and each team has a fixed set of fans F_t. The utility of each fan for team t is an increasing function of the number of wins w_t of their team. Suppose there are a fixed number of games per season, and each team plays each other team exactly once per season. For simplicity, suppose that fans pay their team for wins, which they enjoy, again for simplicity, suppose that each team earns $\pi_t > 0$ of profit for each win.

   (a) Suppose the market for professional basketball players is competitive, so they are paid their marginal value to the team. Claim: That means that basketball player x will be paid π_t times $\big(\text{Prob}\,(\text{team } t \text{wins with player } x) - \text{Prob}\,(\text{team } t \text{ wins without player } x)\big)$. True or False? Justify your answer.

   (b) Claim: In this setting, basketball players are overpaid relative to their marginal social value. True or False? Justify your answer.

9. (Pape) Consider this description of the game **The Stag Hunt**. In the stag hunt, two hunters must each independently decide whether to hunt the stag together or hunt rabbits alone. Half a stag is better than the rabbits one would get by hunting alone, but the stag will only be brought down with a combined effort: hunting a stag alone yields nothing. Rabbits, on the other hand, can be hunted by an individual without any trouble.

   Write a normal form representation of this game and find all Nash Equilibria. Compare the game to the Prisoner's Dilemma.

10. (Pape) An airline loses two suitcases belonging to two different travelers. Both suitcases happen to be identical and contain identical items. An airline manager tasked to settle the claims of both travelers explains that the airline is liable for a maximum of \$100 per suitcase (he is unable to find out directly the price of the items), and in order to determine an honest appraised value of the antiques the manager separates both travelers so they can't confer, and asks them to write down the amount of their value at no less than \$2 and no larger than \$100. He also tells them that if both write down the same number, he will treat that number as the true dollar value of both suitcases and reimburse both travelers that amount. However, if one writes down a smaller number than the other, this smaller number will be taken as the true dollar value, and both travelers will receive that amount along with a bonus/malus: \$2 extra will be paid to the traveler who wrote down the lower value and a \$2 deduction will be taken from the person who wrote down the higher amount. What are the Nash Equilibria of this game? (Assume that these travelers can only write down whole numbers of dollars.)

3 Part C: Longer questions. 4 questions.

11. (Jones) Analyze the insurance signaling game with 2 types of consumers when benefit B is restricted to being equal to L (the loss). Assume that the low-risk consumer strictly prefers full insurance at the high-risk competitive price to no insurance.

    (a) Show that there is a unique sequential equilibrium when attention is restricted to those in which the insurance company earns zero profits.

    (b) Show that among all sequential equilibria, there are no separating equilibria. Why is this intuitive?

    (c) Show that there are pooling equilibria in which the insurance company earns positive profits.

12. (Jones) An agent lives two periods, $t = 0, 1$. Let x_t denote his consumption in period t, y_t his income in period t, and $r > 0$ the interest rate at which the agent can freely borrow or lend. His intertemporal utility function is

$$u(x_0, x_1) = \sum_{t=0}^{1} \beta^t \Big(-\frac{1}{2} (x_t - 2)^2 \Big) \qquad \text{where } 0 < \beta < 1. \tag{2}$$

The intertemporal budget constraint requires that the present value of expenditures not exceed the present value of income:

$$\sum_{t=0}^{1} \left(\frac{1}{1+r} \right)^t x_t \leq \sum_{t=0}^{1} \left(\frac{1}{1+r} \right)^t y_t \tag{3}$$

(a) If $y_0 = 1$, $y_1 = 1$, and $\beta = \frac{1}{1+r}$, solve for optimal consumption in each period and calculate the level of lifetime utility the agent achieves.

(b) Suppose, now, that the agent knows that income in the initial period will be $y_0 = 1$. However, there is uncertainty about what next period's income will be. It could be high, $y_1^H = \frac{3}{2}$; or it could be low, $y_1^L = \frac{1}{2}$. He knows it will be high with probability $\frac{1}{2}$. His problem now is to choose the initial period consumption, x_0; the future consumption if income is high, x_1^H; and the future consumption if income is low, x_1^L, to maximize intertemporal *expected utility*. Again, assuming that $\beta = \frac{1}{1+r}$, formulate the agents optimization problem and solve for the optimal consumption plan and the level of lifetime expected utility.

(c) How do you account for any difference or similarity in your answers to parts (a) and (b)?

13. (Pape) Consider the game Rock, Paper, Scissors. It works as follows: the two players simultaneously choose (R)ock, (P)aper, or (S)cissors by making appropriate shapes with their hands ("Rock" is a fist, "Paper" is a flat hand, and "Scissors" is a 'v' shape with the index and middle finger.) The winner is determined as follows: R beats S, S beats P, and P beats R, and any move ties with itself. The payoffs are: beating your opponent earns you 1, and gives your opponent −1. Tying yields 0 for both players.

(a) Represent the game in an appropriate form. What are the Nash equilibria of this game? What is the expected payoff for each player?

(b) Often, Rock, Paper, Scissors is played 'best two out of three.' Does this change the Nash equilibria of the subgames? Explain.

(c) In the show "Bob's Burgers," Bob is unable to play Scissors because of surgery on his hand, and his opponent knows this. Represent the transformed game in an appropriate form. What are the Nash Equilibria of this transformed game? How do the expected payoffs in NE change? Explain.

(d) Now suppose that winning with Scissors (i.e. when your opponent plays Paper) brings a value of 2 and gives your opponent −2; and the reverse is also true. (Ignore the part (c) when considering this part.) Represent this transformed game in an appropriate form. In NE, does one play Scissors more or less often? Explain.

14. (Pape) Consider a production economy with two goods x and y. There are I identical consumers: they each have an endowment $\omega = (0, 10)$ and have a utility function $u(x, y) = \ln(x) + \ln(y)$. There are also J identical firms. These firms can produce x and use y as an input. The production function of each firm is $f(z) = \beta z$, where $\beta \in (0, 1)$, and z is the input of y used by the firm in question. Each consumer owns an equal share of each firm.

(a) Find the Walrasian equilibrium for this economy.

(b) Suppose that you are a government entity, who has money to invest in infrastructure. Suppose you can build more firms at the cost of p_{firm} each, or invest in technological research, which increases β at the cost of p_β for each unit that β increases. For what values of p_{firm}, p_β is building firms a better investment? For what values of p_{firm}, p_β is investing in technological research a better investment? Explain.

(c) Consider immigration of $\hat{I}$ new agents who have the same utility function and an endowment of $\hat{\omega} = (0, \hat{y})$. For what values of $\hat{y}$ does this immigration make the original agents better off? Worse off? Explain.