Part A: Short Questions

1. (Jones) Let $\mathbf{p}$ denote the price vector a consumer faces and y his income. Suppose $x_1(\mathbf{p}, y)$ and $x_2(\mathbf{p}, y)$ have equal income elasticity at $(\mathbf{p}^0, y^0)$. Show that $\frac{\partial x_1}{\partial p_2} = \frac{\partial x_2}{\partial p_1}$ at $(\mathbf{p}^0, y^0)$.

2. (Jones) Consumer 1 has expenditure function $e^1(\mathbf{p}, u)$. Consumer 2 has expenditure function $e^2(\mathbf{p}, u) = \frac{1}{2} e^1(2\mathbf{p}, u)$. Will the Marshallian demand of these two consumers be identical?

3. (Pape) In 2001, the rock band Wilco released their most famous album, ``Yankee Hotel Foxtrot,'' which was successful in part because of leaked songs on the Internet, according to the lead singer Jeff Tweedy. About the economics of music, Tweedy said: ``Music is not a loaf of bread. When someone steals a loaf of bread from the store, that's it. The loaf of bread is gone.'' But music is not like that in the digital age. ``People who look at music as commerce don't understand that. They are talking about pieces of plastic they want to sell (CDs and DVDs).'' Explain what Tweedy means from an economics perspective. Do you agree? Explain.

4. (Pape) Can Iterated Elimination of Dominated Strategies eliminate a Nash Equilibrium? Explain your answer.

Part B: Medium-Length Questions

5. (Jones) An investor must decide how much of his initial wealth w to put into a risky asset. The risky asset can have any of the positive or negative rates of return r_i with probabilities p_i, $i = 1, \cdots, n$. If β is the amount of wealth put into the risky asset, the investor's final wealth under outcome i will be $(w - \beta) + (1 + r_i)\beta = w + r_i\beta$. The investor's utility-of-wealth function is $u(\cdot)$, with $u' > 0$ and $u'' < 0$. Under what conditions will the investor decide to put *no* wealth into the risky asset? Under what conditions will he put a positive sum into it? What are the intuitions behind these results?

6. (Jones) Suppose the utility function $u(x)$ is continuous and strictly increasing. Prove: if u is strictly quasiconcave,
   a. the expenditure-minimizing bundle, $x^h(p, u)$, is unique;
   b. the expenditure function, $e(\mathbf{p}, u)$, is strictly concave in $\mathbf{p}$.

7. (Jones) A competitive firm uses 2 inputs to produce an output. The firm has profit function $\pi(w_1, w_2) = s_1(w_1) + s_2(w_2)$, where w_i is the price of input i, $i = 1,2$. The price of the output is normalized to be 1.
   a. What do you know about the first and second derivatives of the functions $s_i(w_i)$?
   b. If $x_i(w_1, w_2)$ is the factor demand function for factor i, what is the sign of $\frac{\partial x_i(w_1,w_2)}{\partial w_j}$ for $i \neq j$?
   c. Let $f(x_1, x_2)$ be the production function that generated the profit function of this form. What can you say about the form of this production function?

8. (Pape) Consider an economy with two consumers, 1 and 2, and two firms, A and B. The economy has two products: a consumption good *x* and time *t*. If time is consumed by a consumer, it is leisure; if it used in production, it is labor.

   The consumer have the same utility function $u(x,t) = x\,t$, and they are each endowed with 1 unit of time and zero units of *x*. Consumer 1 owns firm A and consumer 2 owns firm B.

   The firm A has the production function $f(t) = t^a$ and firm B has the production function $g(t)=t^b$, where $0 < a < b < 1$. Both firm A and firm B produce *x*.

   a) Find the Walrasian Equilibrium of this economy.
   b) Is it better to own firm A or firm B, or does it not matter?

9. (Pape) Consider an exchange economy under uncertainty with two goods, *x* and *y*. There are two consumers, A and B, and two states of the world, 1 and 2. State 1 happens with probability π, where $0 < \pi < 1$. The endowments in the two states are:

	State 1	State 2
Mr. A	(3,1)	(1,1)
Mr. B	(1,1)	(3,1)

   Their utility functions identical: u(x,y) = ln(x) + ln(y) .

Compare the Arrow-Debreu equilibrium to the alternative, with no trade at time 0. Show that both Mr. A and Mr. B are better off through trade at time 0 than the alternative.

10. (Pape) Three oligopolists operate in a market with inverse demand given by $p(Q) = a - Q$, where $Q=q_1 + q_2 + q_3$ and q_i is the quantity produced by firm i. Each firm has a constant marginal cost of production, c, and no fixed cost. The first choose their quantities as follows: (1) firm 1 chooses q_1 ; (2) firms 2 and 3 observe q_1 and then simultaneously choose q_2 and q_3 respectively. What is/are the subgame-perfect outcome(s)?

Part C: Long Questions

11. (Jones) Consider the insurance signaling game with two types of consumers, high-risk and low-risk. Let $\alpha \in (0,1)$ be the fraction of low-risk consumers. With the help of a graph, show that when α is large enough, there are pooling equilibria that make both consumer types better off than they would be in every separating equilibrium. What is the intuitive reason behind this result?

12. (Jones) Consider an English (*open-cry, ascending-price*) auction in which there are only 2 bidders. The bidders' values are drawn independently from a common distribution on [0, 1] with density $f(\cdot)$ and distribution function $F(\cdot)$. The seller sets a reserve price $c \in [0,1]$. I.e., bidding starts at price c. If no one bids, the object will not be sold. Otherwise it is sold at the price equal to the last (highest) bid.
    a. Describe a bidder's optimal bidding strategy in this auction.
    b. Let v_i be bidder i's value and order the bidders so that $v_1 > v_2$. Compute the seller's expected revenue.
    c. Suppose v_i is uniformly distributed on [0, 1]. Compute the seller's expected revenue. What value of c maximize the seller's expected revenue?

13. (Pape) Consider this version of the Prisoner's Dilemma:

	C	D
C	3,3	0,5
D	5,0	2,2

Suppose that there are three players, a, b, and c. Play proceeds as follows: a plays PD with b, then b plays with c, then c plays with a, and then the cycle repeats, forever.

Find an SPNE in which (C,C) is played in every turn. Find the set of discount rates which support this SPNE. Compare it to the set of discount rates required to support this SPNE if there were only two players; how do they compare? Explain.

14. (Pape) Consider the following economy:

Two goods, x and y, and also pollution Z, where Z is the total amount of pollution in the economy.

100 consumers, with endowments of 1 unit of x each, and utility functions:

$$u(x,y,Z) = x\,y - .0001\,Z$$

100 firms with production functions $f(x) = c\,x$, $c > 0$ and the output is y. Firm 1 is owned by consumer 1 and so on. Suppose that the production of y causes pollution: each unit of y produced also produces 1 unit of pollution.

a. Find the WE of this economy, assuming that all agents are price-takers and there is no market for Z. Is this WE Pareto-optimal? Explain.
b. Propose a tax or subsidy on the production of y that maximize social surplus. Note that you must find the level of this tax or subsidy and show that it is surplus-maximizing.