Micro Comp Second Round June 30, 2014

Part A: Short Questions

1. (Jones) True or false? "If the preference relation is convex, the optimal consumption bundle will be unique."

2. (Jones) Calculate the elasticity of substitution for the production function $f(x_1, x_2) = k(1 + x_1^{-0.5} x_2^{-0.5})^{-1}$, where $k > 0$.

3. (Pape) Find a simultaneous game with an even number of Nash Equilibria, or argue that one cannot exist.

4. (Pape) Suppose that two consumers have different utility functions for two goods, x and y. In equilibrium, they will agree about how much x they are willing to give up in order to secure one more unit of y. True/False/Uncertain and justify.

Part B: Medium-Length Questions

5. (Jones) State and prove the Slutsky equation.

6. (Jones) Let $u(w)$ be an individual's utility-over-wealth function with $u'(w) > 0$ and $u''(w) < 0$. The individual has initial wealth w_0. He can take part in a betting game. If he bets an amount x, he will gain x dollars with probability p and lose x dollars with probability $1 - p$.
   a. Show that if $p = 0.5$, the individual will bet zero; and if $p > 0.5$, he will bet a positive amount. What is the intuition behind this result?
   b. Suppose $p > 0.5$. Show that if $u(w)$ exhibits absolute risk aversion, the individual's optimal bet will be independent of w_0. What is the intuition behind this result?

7. (Pape) Consider a 2 player game (players A and B) that works in the following fashion: First, player A chooses U or D. If player A chooses U, the game is over and the players receive payoffs (5, 5). If player A chooses D, then player 2 must choose R or L. The payoffs are : (10, 0) if R, (0, 10) if L. Find all pure strategy Nash Equilibria of this game. Then, find all pure strategy Subgame-perfect Nash Equilibria of this game. Compare the two sets: do they differ? If so, why?

8. (Pape) Consider an exchange economy with two individuals, A and B. Suppose that there are two goods, x and y. Suppose that their utility functions are:

   $u_A(x,y) = \ln(x) + \ln(y)$
   $u_B(x,y) = \ln(x) + y$

   Suppose that the total amount of x and y available is (4,2). Find the set of all Pareto Optimal allocations and illustrate them in the Edgeworth box. Precisely communicate the set of PO allocations.

Part C: Long Questions

9. (Jones) A price-taking firm produces output y from inputs x_1 and x_2 according to a production function $f(x_1, x_2)$. The price of its output is $p > 0$ and the prices of the inputs are $(w_1, w_2) \gg 0$. However, there are two unusual things about this firm. First, rather than maximizing profit, the firm maximizes revenue (the manager wants her firm to have the biggest dollar sales possible). Second, the firm is cash constrained. In particular, it has only C dollars on hand before production and, as a result, its total expenditures on inputs cannot exceed C.

   Suppose an econometric model has determined that the firm's revenue R can be expressed as the following function of p, w_1, w_2 and C:

   $$R(p, w_1, w_2, C) = p(\beta + \ln C - \alpha \ln w_1 - (1 - \alpha) \ln w_2)$$

   Where β and α are constants.

   a. What is the firm's use of inputs x_1 and x_2 when prices are (p, w_1, w_2) and it has C dollars of cash on hand?
   b. Can you determine what kind of production technology this firm has?

10. (Jones) Consider the insurance signaling game with 2 types of consumers (high-risk and low-risk). Show that there exists no pooling equilibrium satisfying the *intuitive criterion*. Be sure to draw a graph.

11. (Pape) Find at least one SPNE of the following game:

Two players play the following card game with a deck consisting of (A,2,3). (A = Ace.)

A dollar is placed in the pot by some third party, and player 1 is dealt a card. If it is an A, he has a winning card, otherwise he has a losing card.

Player 1 can decide whether to fold (conceding the dollar to player 2) or bet, by placing an additional dollar in the pot.

If Player 1 bets, player 2 can decide to fold (conceding the pot to player 1) or call, by placing an additional dollar in the pot.

If player 2 calls, player 1 reveals his card - if it is an A, he takes the pot, otherwise player 2 takes the pot.

12. (Pape) Consider the following economy:

Two goods, x and y.

200 consumers with utility functions:

$$u(x,y) = x\,y$$

150 consumers are endowed with 1 unit of x. 50 consumers are endowed with one unit of y.

Suppose initially there is no production in this economy. Then, technology is developed in which 100 firms which produce y from x with the production function $f(x) = a\,x$, where $a<0$. Assume that the first firm is jointly and equally owned by the first two consumers and so on.

Consider the competitive equilibria before and after this change. Which consumers, if any, are made better off, and under what conditions? Which consumers are made worse off, if any, and under what conditions?