Hanes' comp questions, June 2015

**4)** Consider the following model:

$$y_t = {}_t y^e_{t+1} - sr_t + u_t \quad where \; u_t = \rho u_{t-1} + \epsilon_t$$

$$\pi_t = {}_t \pi^e_{t+1} + y_t$$

$$r_t = \pi_t$$

where $\pi$ is the inflation rate, $y$ is the output gap, $r$ is the gap between the real interest rate and the natural rate of interest, and $\epsilon$ is a mean-zero i.i.d. random variable.

Assume agents in the economy have rational expectations. They know the economy must eventually converge to a long-run steady state with $y = 0$, $\pi = \pi^e = 0$ .

**a)** Solve for $\pi_t$ and $y_t$, in terms of $u_t$ (not $\epsilon_t$). I want to see the steps in the derivation.
*5 pts. See notes.*

**b)** Suppose that the first two equations were derived from a standard New Keynesian representative-agent DSGE model. What might $u_t$ represent? Be specific. *5 pts. See notes, "NKIS curve (following King)". It is:*

$$\frac{G_t}{Y_t} - E_t\left[\frac{G_{t+1}}{Y_{t+1}}\right]$$

*that is, expected value of decrease in government purchases share of output, from current period to upcoming period. It cannot be about (lump-sum) taxes.*

**5)** Consider a model in which:

$$y_t = {}_t y^e_{t+1} - sr_t + \epsilon^{IS}_t$$

$$\pi_t = {}_t \pi^e_{t+1} + y_t + \epsilon^{AS}_t$$

where $\pi$ is the inflation rate, $y$ is the output gap, $r$ is the gap between the real interest rate and the natural rate of interest. $\epsilon^{IS}$ and $\epsilon^{AS}$ are mean-zero i.i.d. random variables, uncorrelated with each other. Their values cannot be predicted in the previous period. Expectations are rational. "Society" has a loss function:

$$L_t = E_t \sum_{\tau=0}^{\infty} \left[\beta^\tau \left(\frac{1}{2}\pi^2 + \frac{1}{2}y^2\right)\right]_{t+\tau} \qquad where \quad 0 < \beta < 1$$

Consider two economies, *"Rulia"* and *"Discretia",* both described by this model.

In Discretia, the central bank policy committee freely chooses $r_t$ in period *t,* with no rules or preset commitments, aiming to minimize Society's loss function. When the central bank sets $r_t$ it knows the value of $\epsilon^{IS}_t$, but it does not know $\epsilon^{AS}_t$, $y_t$ or $\pi_t$.

In Rulia, there is a mechanism that automatically links $r_t$ to $y_t$ and $\pi_t$, such that $r_t = a\pi_t + by_t$

Answer the following questions and explain. Your answer might be "it depends." If your answer is "it depends," be sure to explain what it depends *on*.

*Rulia follows an interest-rate rule; its economy is described by the IS/MP model. Draw it. See: IS shocks affect output and inflation, creating loss, and cause positive co-movements between inflation and output. AS shocks also affect output and inflation, creating loss, and cause negative co-movements between inflation and output.*

*Discretia is the CGG model under discretion, except with no serial correlation in shocks and the addition of unobservable AS shocks. Recall that the central bank adjusts r to completely counteract observable IS shocks, so IS shocks affect nothing and create no loss. Also recall that in response to an observable AS shock, the central bank would minimize loss by adjusting r to push y away from zero and partly counteract the effect of the AS shock on inflation. But here the AS shocks are unobservable. In response to unobservable AS shocks, r remains fixed, output unaffected, but inflation is affected, creating loss.*

*Note that an AS shock of given magnitude will create greater loss in Discretia than in Rulia, because Rulia's interest-rate rule will reproduce (somewhat) what Disccretia's central bank would do if it could observe the AS shock.*

**a)** In a sample of data from many periods, what is the sign - positive, negative or zero - of the correlation between $y$ and $\pi$ in Discretia? In Rulia? *6 points, 3 for each. In Discretia, correlation is zero (or undefined, because y doesn't move). In Rulia, depends; becomes more positive (negative) as variance of IS (AS) shocks is relatively larger. Note it is incorrect to say that the correation between output and inflation can possibly be negative in Discretia.*

**b)** Is the value of Society's loss bigger in Rulia, or bigger in Discretia? *5 points. Depends, If variance of IS shocks is big enough, variance of AS shocks small enough, loss is bigger in Rulia. If variance of IS shocks is small enough, variance of AS shocks big enough, loss can be bigger in Discretia.*

**6)** Recall the elaborate New Keynesian model of Christiano, Eichenbaum and Evans (2005). What is the contraint on price adjustment in this model, and how does it differ from the constraint on price adjustment in the simple Calvo model? What is the resulting Phillips curve? *6 pts. See notes.*

**7)** Recall Romer's static imperfect-competition model, where the representative household's utility function is:

$$U = \left[ \int_{i=0}^{1} C_i^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} - \frac{1}{\gamma} L^{\gamma} \qquad \text{where } \eta > 1 \ , \lambda > 1$$

each firm is a monopolist, and the production function is such that one unit of labor input creates one unit of output: $Y = L$. Recall $\eta$ is the elasticity of demand for an individual good.

Consider two economics, *A* and *B,* both described by this model. The fixed "menu cost" of changing a price is the same in both economies, but the values of the utility-function parameters $\eta$ and $\lambda$ are not necessarily the same. Answer the questions below and *explain* your answers using graphs and/or equations as appropriate.

*Sorry! $\lambda$ should be $\gamma$ ! But I think you all knew that. Recall the elasticity of labor supply is $1/(\gamma - 1)$, that is, $l_i^S = [1/(\gamma - 1)] \ (w - p)$*

**a)** Suppose that the value of $\lambda$ is the same in both economies, but $\eta_A > \eta_B$ .
i) Is the long-run equilibrium level of output bigger in economy A, bigger in economy B, or the same in the two economies? *3 points. As stated in the question, $\eta$ is the elasticity of demand for an individual good. The economy with greater elasticity of demand has a smaller markup and larger output. See notes. So output is bigger in A.*

ii) For an aggregate demand (*M*) shock of a given size, is a "fixed-price equilibrium" more likely to prevail in economy A, economy B, or equally likely to prevail in both economies? *3 points. See notes. Recall a fixed-price equilibrium is more likely to prevail if an economy has more "real rigidity," which means $\phi$ is small in the "real rigidity equation" $p_i^* - p = c + \phi y$. In Romer's static imperfect-competition model, $\phi = \gamma - 1$. So here the real-rigidity parameter is the same in both economies. Answer: equally likely to prevail in both economies.*

*Note that for any value of $\eta$, the markup is constant (neither procyclical nor countercyclical) in this model.*

**b)** Suppose that the value of $\eta$ is the same in both economies, but $\lambda_A > \lambda_B$ .
i) Is the long-run equilibrium level of output bigger in economy A, bigger in economy B, or the same in the two economies? *3 points. See notes, where you'll find:*

$\bar{Y} = \left(\frac{\eta-1}{\eta}\right)^{\frac{1}{\gamma-1}}$

$\left(\frac{\eta-1}{\eta}\right) < 1$. *If* $\gamma$ *is bigger, then* $\frac{1}{\gamma-1}$ *is smaller, and Y is bigger. So output is bigger in A, smaller in B.*

ii) For an aggregate demand (*M*) shock of a given size, is a "fixed-price equilibrium" more likely to prevail in economy A, economy B, or equally likely to prevail in both economies? *3 pts. More real rigidity in B (* $\phi$ *is smaller in B). Answer: more likely to prevail in B.*

**8)** Consider an economy described by the following model:

$$y_t = {}_t y_{t+1}^e - sr_t$$

$$\pi_t = {}_t\pi_{t+1}^e + y_t$$

$$r_t = a\pi_t + by_t + u_t \qquad \text{where } u_t = \rho u_{t-1} + \epsilon_t$$

where $\pi$ is the inflation rate, $y$ is the output gap, $r$ is the gap between the real interest rate and the natural rate of interest, and $\epsilon$ is a mean-zero i.i.d. random variable. Assume agents in the economy have rational expectations. They know the economy must eventually converge to a long-run steady state with $y = 0$, $\pi = \pi^e = 0$ .

Ordinarily, the people in this economy have no advance information about future realizations of $\epsilon$: that is $E_t[\epsilon_{t+\tau} = 0]$ for all $\tau$. Suppose, however, that in one period *t*, for which $u_t = 0$ , the people receive a copy of the *Wall Street Journal* from a future period $t+1000$. (The newspaper fell into a black hole in that future period and popped out of a toaster today, having travelled backward through time.) From this newspaper, people deduce that $u_{t+100} < 0$. What, if any effect does this news have on output and inflation in period *t*? Explain your answer, using equations as appropriate.

*6 pts. I* think *the answer is that the news must affect output and inflation in period t, but we can't be sure how.*

*Going forward from t+100, this will be the economy described in our notes and problem set about the simple New Keynesian economy with an interest-rate rule and serially correlated shocks. From that you know that if* $u_{t+100} < 0$, *then it will be true that* $y_{t+100} > 0$, $\pi_{t+100} > 0$.

*How do we know this must affect output and inflation in period t? Iterating back from period t+100,*

$$y_t = {}_t y_{100}^e - sE_t\left[\sum_{\tau=1}^{99} r_{t+\tau}\right] \qquad \text{and} \qquad \pi_t = {}_t\pi_{100}^e + E_t\left[\sum_{\tau=1}^{99} y_{t+\tau}\right]$$

*So for any given expected path for the interest rate in the future to period t+100,* $y_t$ *is bigger.*

*And for any given expected path for output in the future to period t+100,* $\pi_t$ *is bigger.*

*But why do I say the effects are ambiguous? Think about period t+99. The fact that* ${}_t y_{t+100}^e > 0$ *means that for any given* $r_{t+99}$, *the value of* $y_{t+99}$ *is bigger - this is like a positive IS shock. And the fact that* ${}_t\pi_{t+100}^e > 0$ *means that for any given* $y_{t+99}$, *the value of* $\pi_{t+99}$ *is bigger - this is like a positive AS (cost-push) shock. Given the interest-rate rule, the result of a positive IS shock combined with a positive AS shock must be an increase in* $\pi_{t+99}$. *The effect on* $y_{t+99}$ *is ambiguous (depends on values of coefficients in interest-rate rule).*

*Now think about period t+98. The fact that* $\pi_{t+99}^e > 0$ *is like a positive AS (cost-push) shock, as in t+99. If* $y_{t+99}^e > 0$,

*we've again got a positive IS shock as in t+99, and results are similar to what I said for t+100. But if* $y^{e}_{t+99}<0$*, we've got a negative IS shock. The results of a positive AS shock and a negative IS shock for output and inflation in period t+98 are ambiguous.*

*Going back to period t, continues to be ambiguous.*

**9)** Recall Bernanke, Gertler and Gilchrist's (1999) model of a New Keynesian economy with imperfect capital markets. *Based on that model*, some people argued for a program whereby the Federal Government would force creditors to "forgive" large amounts of mortgage debt. They argued such a program would have tended to boost economic recovery from the recession of 2008-09. Explain. *6 points. Recall in BGG, a transfer of wealth to debtors (entrepreneurs) from creditors can boost output. See notes for how. So, if forgiveness of mortgage debt would increase the wealth of potential entrepreneurs, then it might boost output.*