Micro Comp **(First attempt)** **June 1, 2015**

1 Part A: Short answer or explanation. 4 questions.

1. (Jones) Calculate the elasticity of substitution for the production function

$$f(x_1, x_2) = k\left(1 + x_1^{-0.5} x_2^{-0.5}\right)^{-1}$$

where $k > 0$.

2. (Jones) A consumer has lexicographic preferences over $\mathbb{R}^2_+$. I.e., $x^1 \succsim x^2$ if and only if $x_1^1 > x_1^2$, or $x_1^1 = x_1^2$ and $x_2^1 \geq x_2^2$. Sketch and indifference map for these preferences. Do these preferences satisfy the continuity axiom? Explain.

3. (Pape) Find all Nash Equilibria of the following game:

	L	C	R
T	$0,5$	$6,7$	$6,3$
M	$0,8$	$2,6$	$9,4$
B	$2,2$	$4,6$	$1,1$

4. (Pape) What is the first fundamental theorem of welfare economics and why is it true? You need not provide a complete proof; a concise argument is sufficient.

2 Part B: Mathematical or advanced questions. 6 questions.

5. (Jones) A consumer of 2 goods has an indirect utility function given by $v(p_1, p_2, y) = \frac{y}{p_1+p_2}$, where p_1 and p_2 denote the price of good 1 and good 2 and y denotes income. Find his Marshallian demand function, expenditure function and Hicksian demand function.

6. (Jones) Let $x_i(p, y)$ be the Marshallian demand for good i. Prove: $x_i(p, y)$ is homogeneous of degree k in y if and only if the income elasticity of good i equals k.

7. (Jones) Suppose the production function is $f(x_1, x_2) = \ln(x_1 + 1) + \ln(x_2 + 1)$. Let p be output price, and w_1 and w_2 price of input 1 and 2, respectively.
   (a) Find the output supply function $y(p, w)$ by solving the profit maximization problem directly: $\max_{x_1,x_2} pf(x_1, x_2) - (w_1x_1 + w_2x_2)$.
   (b) Find the cost function $c(w, y)$.
   (c) Now find the output supply function indirectly by solving the problem: $\max_y py - c(w, y)$.

8. (Pape) Frank and Johnny play a game of Chicken on bicycles every Sunday on the Exchange Street bridge in downtown Binghamton. Frank and Johnny have two identical actions which yield the same payoffs. The actions are Tough and Chicken. The payoffs are: $u(C, C) = 1$, $u(C, T) = 0$, $u(T, C) = 3$, and $u(T, T) = -1$.

   (a) Write down the normal form of this game. Find all NE of this game.

   (b) The weather in Binghamton is Rainy with probability π and it is Sunny with probability $(1 - \pi)$. Find all π for which there is a Bayes-Nash Equiliubrium (or "Perfect Bayesian Equilibrium") in which (1) the payoffs are symmetric across players in expectation and (2) it Pareto dominates any symmetric equilibria found in part (a). **Hint:** Begin with the case where $\pi = .5$.

9. (Pape) Consider a two-by-two production model setting: a small, open economy with two goods, x and y, which are sold on the world market at exogenous prices p_x and p_y. These goods are produced locally with Cobb-Douglas production functions f_x and f_y, using local inputs labor L and capital K with associated local prices w and r. Suppose the exponent in the production of x is α and the exponent in the production of y is β. Suppose as usual that $\bar{L}$ and $\bar{K}$ represent the total quantities of labor and capital available locally. Suppose that the production of x is capital intensive.

   Let $(w_0, r_0, l_{x,0}, k_{x,0}, l_{y,0}, k_{y,0})$ be the original equilibrium values before a policy change.

   This is the policy: Suppose that the government implements a tax τ on capital, so if the firms pay r_d for each unit of capital, then the owners of capital receive only $r_s = \frac{r_d}{1+\tau}$ (because $r_d = r_s(1 + \tau)$). Let $(w_1, r_d, r_s, l_{x,1}, k_{x,1}, l_{y,1}, k_{y,1})$ be the equilibrium values after this policy change.

   (a) Without formally modeling (unless you wish) consider a comparison r_0 with r_d and r_s: That is, think about whether r_0 is strictly between the other two values, or equal to one or the other, or higher than both, or lower than both.

   (b) Using the argument from part (a), discuss the implications for the other variables in the model. You may wish to use the standard diagrams associated with the two-by-two production model.

10. (Pape) Suppose that there are I people in an economy, indexed $i = 1, \ldots, I$, with the following utility functions over a private good, x, and a public good, G:

$$u_i(x_i, G) = \ln(x_i) + \theta_i \ln(G) \qquad \text{where } \theta_i < \theta_{i+1} \text{ and } \theta_i > 0 \text{ for all } i$$

   Suppose that G is produced the constant cost of c units of x per unit of G and let the price of x be normalized to 1. Suppose that each agent is endowed with $\omega > 0$ units of x. Find the privately provided level of the public good $G^\star$ and the optimal level of the public good G^o. Then, devise a tax scheme which will provide the optimal level of the public good.

3 Part C: Longer questions. 4 questions.

11. (Jones) A consumer lives 2 periods, $t = 0, 1$. Let x_t denote consumption spending in period t, y_t denote income in period t, and $r > 0$ the market interest rate. The consumer's intertemporal utility function is given by

$$u^{\star}(x_0, x_1) = -\frac{1}{2}(x_0 - 2)^2 + \beta\left(-\frac{1}{2}(x_1 - 2)^2\right)$$

where $0 < \beta < 1$. The intertemporal budget constraint requires that the present value of expenditures not exceed the present value of income.

   (a) If $y_0 = 1$, $y_1 = 1$, and $\beta = \frac{1}{1+r}$, solve for optimal consumption in each period and calculate the consumer's lifetime utility.

   (b) Suppose, now, that the consumer knows that income in the initial period will be $y_0 = 1$. However, income in the future period will be $y_1^H = \frac{3}{2}$ with probability 0.5 and $y_1^L = \frac{1}{2}$ with probability 0.5. Again, assuming $\beta = \frac{1}{1+r}$, formulate the consumer's problem of maximizing intertemporal expected utility, and solve for the optimal consumption plan $\left(x_0, x_1^H, x_1^L\right)$ (i.e., consumption in the initial period, and consumption in the future period if income is high and if income is low).

   (c) How do you account for any difference or similarity in your answers to parts (a) and (b)?

12. (Jones) Consider a signaling game between a worker whose type is hidden and a firm. The worker is either of type 1 ("good") or type 2 ("bad"). The probability that he is type 1 is $\alpha \in (0, 1)$. Let w denote wage and e amount of education. A type-i worker's utility is given by $u^i(e, w) = f(w) - c_i e$, where $f' > 0$, $f'' < 0$, and c_i is a constant with $c_2 > c_1 > 0$. The value of a type-i worker to the firm is $v_i = a_i e$, where $a_1 > a_2 > 0$. The worker, knowing what type he is, sends a signal $\psi = (e, w)$ to the firm (i.e., he acquires e amount of education and asks for wage w). The firm receives the signal and either accepts or rejects to hire the worker at the proposed wage.

   Let $\psi_i = (e_i, w_i)$ be type-i worker's strategy, $\beta(e, w)$ the probability that the firm assigns to the event that the worker is type 1 given (e, w), and $\sigma(e, w) \in \{\text{accept}, \text{reject}\}$ the firm's strategy.

   (a) Show that the 2 types of workers' utility functions satisfy the single crossing property. In a diagram with e on the horizontal axis and w on the vertical, draw an indifference curve for each type of worker and indicate the direction in which utility increases.

   (b) Draw the firm's zero-profit line corresponding to each type of worker and indicate the direction in which profit increases.

   (c) Let $\psi_2^c = (e_2^c, w_2^c)$ be the solution to the problem: $\max_{e,w} u^2(e, w)$ s.t. $w \leq a_2 e$. Show that in every separating equilibrium, $\psi_2 = \psi_2^c$. Interpret this result.

   (d) Give a characterization of separating equilibrium (a proof is not required). Draw a graph and shade in the set of outcomes that can arise for the good worker in some separating equilibrium. Explain intuitively why the good worker's outcome must be in the shaded area.

13. (Pape) Consider the following game. There are I individuals, numbered $i = 1, \ldots, I$, who each have one dollar. (Assume utility is linear in money.) When the game begins, each person i simultaneously chooses a fraction $\alpha_i \in [0, 1]$ of their wealth to throw into a *magic well.* After each agent has selected an α, all αs are revealed, and the magic well produces a payoff for each player. The award is the same for each player, regardless of contribution. The award a is:

$$a = d \sum_{i=1}^{I} \alpha_i$$

where $d > 0$.

(a) Find all Nash Equilibria of this game, as a function of d.

(b) This is a game theory problem. However, the magic well in this problem is an example of a concept we encountered when we studied Walrasian equilibrium. What is that concept? Why do you think so? Explain!

For part (c), consider these two modifications to the game:

1. Let $d = .5$.
2. Assume that there is a second round of play which occurs just after the magic well gives its awards, in which each agent i simultaniously chooses a number $x_i = 0, 1, \ldots, I$. Then, the xs are revealed, and any agent whose number is revealed (i.e. any i such that $i = x_j$ for some j) *immediately loses a dollar.* (Note that no one has $i = 0$, which is an available x; therefore agents do have the option of hurting no one.)

(c) Is there an SPNE in which $\alpha_i > 0$ for all i in round 1? If so, describe it. If not, say why not. How does this relate to what you found in part (a)?

14. (Pape) Consider a Walrasian equilibrium under risk setting, with the following characteristics: There are two states $s = 1, 2$. There are two goods, x and y. And there are two agents, A and B. π_s denotes the probability of state s, where $\pi_1 + \pi_2 = 1$.

Suppose that A and B's allocations are given by the following table:

	State 1		State 2	
	x	y	x	y
A	1	1	1	0
B	0	0	0	1

(a) Consider an Arrow-Debreu equilibrium in this economy. **Claim:** A is made no better off by an increase in π_1. Argue/show/explain whether this claim is true, false, or uncertain.

(b) Consider a Radner equilibrium in which an asset which pays off in x is sold at time zero. **Claim:** Since Mr. A is perfectly insured in x in both states, he will not want to buy or sell these assets at time zero. Argue/show/explain whether this claim is true, false, or uncertain.