Micro Comp **(Second attempt)** **June 29, 2015**

1 Part A: Short answer or explanation. 4 questions.

1. (Jones) Why are the axioms of completeness and transitivity necessary for the existence of a utility function?

2. (Jones) The substitution matrix of a utility-maximizing consumer's demand system at prices $(p, 5)$ is

$$\begin{bmatrix} a & 1 \\ b & -3 \end{bmatrix}$$

   Find a, b and p.

3. (Pape) Find all Nash Equilibria of the following game, as a function of $x \in \mathbb{R}$.

	L	R
T	$0, 3$	$4, 0$
B	$2, 0$	$x, 1$

4. (Pape) Suppose you and your neighbor, both upstate NY dairy farmers, share a river which runs between your property. Each of you can dump cow manure into the river, but if there is too much manure in the river it will back up and flood both your and your neighbor's property. Suppose no one else is effected by manure in the river. What would Coase say about whether you and your neighbor will dump the socially optimal amount of manure in the river?

2 Part B: Mathematical or advanced questions. 6 questions.

5. (Jones) A risk averse consumer has initial wealth w_0 and VNM utility function $u(\cdot)$. The probability is $p \in (0, 1)$ that he will have an accident and incur a loss of L. Prove that he will buy full coverage if insurance is actuarially fair. What is the intuition for this result?

6. (Jones) Fix $x \in R^n_+$. Define the Slutsky-compensated demand function at x, $x^s(\mathbf{p}, x)$, by $x^s(p, x) = x(\mathbf{p}, \mathbf{p} \cdot x)$.
   (a) In a graph, illustrate Slutsky-compensated demand for the case of $n = 2$. Explain what it means.
   (b) Let $x^0 = x(\mathbf{p}^0, y^0)$. Prove that

$$\frac{\partial x_i^s(p^0, x^0)}{\partial p_j} = \frac{\partial x_i^h(p^0, u^0)}{\partial p_j}, \qquad i, j = 1, \ldots, n$$

   where $u^0 = u(x^0)$. Interpret.

7. (Jones) Let $p \geq 0$ and $\mathbf{w} \geq 0$ be a competitive firm's output price and input prices, respectively. Prove the following properties of the profit function $\pi(p, \mathbf{w})$:
   (a) Convex in $(p, \mathbf{w})$,
   (b) $\frac{\partial \pi(p,\mathbf{w})}{\partial p} = y(p, \mathbf{w})$, and $\frac{-\partial \pi(p,\mathbf{w})}{\partial w_i} = x_i(p, \mathbf{w})$, where $y(p, \mathbf{w})$ is the firm's output supply function and $x_i(p, \mathbf{w})$ the input demand function for input i.

8. (Pape) Consider the following game. There are I individuals, numbered $i = 1, \ldots, I$, who each have one dollar. (Assume utility is linear in money.) When the game begins, each person i simultaneously chooses a fraction $\alpha_i \in [0, 1]$ of their wealth to throw into a *magic well*. After each agent has selected an α, all αs are revealed, and the magic well produces a payoff for each player.

   Then a winner is selected, who wins an award equal to the amount of the money in the well: $A = \sum_{j=1}^{I} \alpha_j$. The winner is selected according to the following rule: The agent with the largest donation α wins the award. If two or more agents tie with a maximum donation, the winner is selected uniformly from those with the maximum donation.

   Find all Nash Equilibria of this game. Explain carefully.

9. (Pape) Consider a two-period bargaining game. As usual, the pie is of size 1; both players discount future payoffs at $\delta = \frac{1}{2}$. In period one, A offers, i.e. proposes a split $(x, (1 - x))$ of the pie for (A, B); B then says yes or no. In period two, a fair coin is flipped. If it comes up heads, A gets to offer again, and B says yes or no; if it comes up tails, then B gets to offer and A says yes or no. A rejection in period two means that both payoffs are zero.

   Find a subgame perfect Nash equilibrium of this game.

10. (Pape) Suppose that there are three agents in the economy, A, B, and C. There are two types of goods: left shoes and right shoes, and these shoes each have a color: red, blue or yellow. A is endowed with a yellow left shoe and blue right shoe. B is endowed with a blue left shoe and a red right shoe. C is endowed with a red left shoe and a yellow right shoe. All agents strictly prefer a right/left pair of shoes with matching colors over a right/left pair without matching colors. Unmatched shoes are worthless, as are shoes beyond the best matching pair.
    (a) Find a Walrasian Equilbrium of this economy.
    (b) Suppose A is endowed with an additional yellow left shoe. How does your answer in the previous part change?

3 Part C: Longer questions. 4 questions.

11. (Jones) Consider the following utility function defined on the positive orthant of R_+^2:

$$u(x_1, x_2) = x_1x_2 \text{ if } x_1x_2 < 4 \text{ or } x_1x_2 > 8, \text{ and } u(x_1, x_2) = 4 \text{ if } 4 \leq x_1x_2 \leq 8$$

   (a) Use a graph with indifference sets to show that the corresponding preference relation is convex.

   (b) Show that these preferences can *not* be represented by a concave utility function.

12. (Jones) Consider the insurance screening game with 2 insurance companies and 1 consumer. The consumer is low-risk with probability α and high-risk with probability $1 - \alpha$.

   (a) Explain the concept of "cream skimming" in this game.

   (b) Prove: there are no pure strategy pooling equilibria in the screening game. What are the intuitions?

13. (Pape) General Blotto and General Otto are getting ready to do battle. Blotto will attack Otto at one of *two possible battle sites S and T.* Blotto has two indivisible groups of soldiers, and Otto has one. The generals play a *zero sum game.* If Otto has $k \in \{0, 1\}$ groups of soldiers at a site, and Otto has one, then Blotto receives a payoff of $k + 1$. If Blotto has no groups of soldiers at a site, and Otto has one, then Blottow receives a payoff of -1. If Blotto and Otto have the same number of groups of soldiers, both recieve a payoff of 0. Both are proud soldiers: retreat is never an option for them, and they must use all their soldiers.

   (a) Represent this game as a normal (matrix) form.

   (b) Find *all* Nash equilibria. (Can Blotto play a pure strategy?)

14. (Pape) Consider a Walrasian equilibrium under risk setting, with the following characteristics: There are two states $s = 1, 2$. π_s denotes the probability of state s, where $\pi_1 + \pi_2 = 1$.

There are two goods, x and y.

There are I agents indexed $i = 1, \ldots, I$. Each agent has a type $\theta \in \{\theta_A, \theta_B\}$; suppose that α denotes the fraction of the I agents who are of type θ_A (so therefore $(1-\alpha) I$ are of type θ_B). All agents, regardless of type, have the following utilty function: $u(x, y) = \ln(x) + \ln(y)$. The agents' allocation differs by type, as given by this table:

	State 1		State 2	
	x	y	x	y
θ_A	0	1	1	0
θ_B	1	0	0	1

(a) Find the Arrow-Debreu equilibrium in this economy as a function of parameters.

For the next three parts of this question, consider an increase in α, which represents the conversion of some agents from type θ_A to type θ_B. For simplicity, suppose that α is continuous and that this is a marginal increase in α.

(b) Consider an agent who was of type θ_A before and after this marginal increase. **Claim:** This agent is made better off by the increase in α. Argue/show/explain whether this claim is true, false, or uncertain.

(c) Considering the previous part, is the opposite true for an agent who is type θ_B both before and after the change? Answer and explain.

(d) Now consider the agent, on the margin, who is converted from type θ_A to type θ_B. **Claim:** This agent is made better off by the increase in α. Argue/show/explain whether this claim is true, false, or uncertain.