

PART I

Each question consists of several parts. Complete each part to the best of your ability. Be sure to manage your time efficiently. Show all your work. Good luck!

1. Tax and consumption

1. Suppose the federal government decides to increase spending by \$800 billion in 2013. Consider the following two economic policy scenarios:
 - To finance the spending, the government raises taxes by \$800 billion in 2013. For simplicity, assume the government will collect a lump-sum tax from each household.
 - Instead of raising taxes, the government decides to run a deficit of \$800 billion. All the required funds will be financed by issuing 10 year Treasury bonds.

Questions:

- (a) According to the infinite horizon, rational expectations version of consumption theory you learned in class, do these two policy measures affect a rational consumer's consumption path differently? If yes, explain how the consumers' behavior would be different in each scenario. If not, explain why you do not think the economic outcomes would be different.
- (b) After providing a detailed explanation in words, try to write down an economic model to support your answers.

2. A growth model

Time is continuous and the horizon is infinite. Let $k(t)$ denote the economy's capital stock at time t . There is a continuum of technologies denoted by z . Lower values of z represent cleaner technologies that generate less pollution and higher values of z represent dirtier technologies that generate more pollution. At time t , the economy produces $Ak(t)z(t)$ units of output but also produces $p(t) = Ak(t)z(t)^\beta$ of pollution. $A > 0$ is a parameter (not a variable) governing the overall level of productivity. Also, $\beta > 1$. People dislike pollution, but dirtier technologies can be used to produce output at a lower cost.

The social planner chooses how to optimally divide output into consumption and investment, and an optimal level of pollution technology $z(t)$. The stock of capital evolves according to

$$\dot{k} = Ak(t)z(t) - \delta k(t) - c(t),$$

where $0 < \delta < 1$ is the depreciation rate of capital. The utility function of consumers is

$$\int_0^\infty e^{-\rho t} \left\{ \frac{c(t)^{1-\sigma} - 1}{1-\sigma} - v[p(t)] \right\} dt,$$

where $\sigma > 0$, $0 < \rho < 1$ and the disutility from pollution is defined as

$$v[p(t)] = \left(\frac{B}{\gamma}\right)p(t)^\gamma = \left(\frac{B}{\gamma}\right)[Ak(t)z(t)^\beta]^\gamma.$$

1. Write down the current value Hamiltonian for the social planner's problem.

2. Write down the first order conditions that characterize the optimal solutions.
3. Eliminate unnecessary variables, and reduced the solutions into a system of three equations.
4. The small system you obtained in part 3 can be used to derive a set of equations that we use to solve for the economy's long run steady state values of c , k and z . Write down this set of three equations. You do not need to actually solve for them. Just list the equations.

3. Business cycle models

1. According to the Real Business Cycle (RBC) theory, if there are no external disturbances, what is the growth rate of the economy? What is the external disturbance that causes the economy to fluctuate?
2. AFTER an external disturbance hits, how does it propagate through the economy? Use an example, carefully describe two ways that the disturbance can propagate through the economy.
3. According to the RBC theory, should the government take actions to stabilize these business cycles? Why or why not?
4. Do you agree with the RBC school's explanation of business cycles and policy recommendations? Why or why not? Explain.

PART II

Hanes' comp questions January 2013

- 4.) As of about 1800, India and sub-saharan Africa had similar levels of real output per person, but population density (number of people per acre) was much higher in India. Does this tell you anything about relative technological development in India *versus* sub-saharan Africa? Explain.
- 5.) Using equations, prove the “dynamic inconsistency of optimal monetary policy” proposition. That is, show that a central bank that aims to keep inflation at an optimal level, *and* keep output above the natural rate, will end up achieving neither. For simplicity, assume the optimal level of inflation is zero and the central bank directly sets the level of *inflation*.
- 6.) Suppose China and the U.S. are the only two countries in the world. Between China and the U.S., there is trade in goods and services and financial assets. The exchange rate is floating. In each country, the level of net exports depends only on the exchange rate. Each country has a central bank that sets the country's real interest rate attempting to keep output at the natural rate. For simplicity, assume “static exchange rate expectations,” and that in both countries expected inflation is always equal to zero. In your answers and graphs, make sure you make clear whether a variable is Chinese or American (e.g., whether “the interest rate” is the Chinese interest rate or the American interest rate).
- a) For many years, China's government authorities have been buying up foreign assets. What does this tell you about the nature of international capital mobility? That is, based on the fact that China's authorities have been buying up foreign assets, would you guess that international capital mobility is *perfect* or *imperfect*? Explain.
- b) Suppose the United States is in a “liquidity trap.” That is, the central bank has cut the interest rate as low as it possibly can, but output is still less than the natural rate of output. Suddenly, China's authorities stop buying foreign assets. Does this have any effect on output in the U.S., assuming your answer to a) is correct? Explain.
- 7.) Unless the price level can “jump,” output cannot always remain at the natural rate. Demonstrate with an example.
- 8.) Suppose an economy can be described by three equations, where y is the “output gap”:
- A Friedman-Phelps Phillips curve: $\pi_t = \pi_t^e + y_t$ note the coefficient on y here is 1
 - An AD curve: $y_t = a(m_t - p_t)$
 - Money supply: $m_t = \bar{m} + \epsilon_t$ where ϵ is a mean-zero i.i.d. random variable $\pi_{t-1}^e = 0$
- Assume the economy is in “rational expectations equilibrium.”
- a) Derive expressions for y_t and π_t .
- b) Consider the correlation between output and inflation in this economy. Is the correlation positive, negative or zero?
- 9.) One of the models we learned explains both of the following facts.
- According to many studies, the marginal product of capital in small business is always higher than the real interest rate on government bonds.
 - People who take out loans to start small businesses are, on average, wealthier than otherwise-similar people who do not start small businesses.
- Explain.

10. Consider a model having two periods and S states. The consumer maximizes expected lifetime utility, U , where there is a single tradable good and a single non-tradable good. Use the following notation:

$$U = u(c_{T,1}, c_{N,1}) + \beta \sum_{s=1}^S \pi(s) u(c_{T,2}(s), c_{N,2}(s))$$

where N denotes non-tradables and T denotes tradables. For each state, there is a security that pays out 1 unit of the tradable good if that state occurs and zero units otherwise. The price of one share of such a security is $p(s)/(1+r)$ for each state s .

$p_{N,1}$ is the price of non-tradables in period 1 in terms of tradables and $p_{N,2}(s)$ is the corresponding price in period 2 in state s . We now add a new type of security. For each state, there is a security that pays out enough units of the tradable good to purchase 1 unit of the non-tradable good if that state occurs and zero units otherwise.

All of these securities are traded internationally.

a) What should be the price of a security that pays out enough units of the tradable good to purchase 1 unit of the non-tradable good in state s and zero otherwise?

b) Using the result from part a, prove that the lifetime budget constraint of the consumer looks like this:

$$c_{T,1} + p_{N,1}c_{N,1} + \frac{1}{1+r} \sum_{s=1}^S p(s) (c_{T,2}(s) + p_{N,2}(s)c_{N,2}(s)) =$$

$$Y_{T,1} + p_{N,1}Y_{N,1} + \frac{1}{1+r} \sum_{s=1}^S p(s) (Y_{T,2}(s) + p_{N,2}(s)Y_{N,2}(s))$$

and explain the result clearly.

c) Find *all* optimality conditions and provide convincing economic interpretations of them.

d) While all securities are *traded* internationally, non-tradables must be *consumed* domestically. What does this imply in equilibrium?

e) In simpler model, we proved that $\pi(s)u'(c_2(s))/u'(c_1)$ will be equalized across countries. Can you prove something analogous here using the result from part d?

f) Explain what the result in part e would imply under CRRA preferences.

// . A firm faces a production function: $Y_t = A_t F(K_t)$. Labor is treated as being fixed for this problem as is the real interest rate r . The firm chooses investment to maximize the discounted present value of profits. The firm must pay $\frac{\chi}{2} I^2$ in adjustment costs for investment of I in each period. Capital is assumed not to depreciate.

a) Prove:

$$K_{t+1} - K_t = \frac{q_t - 1}{\chi}$$

$$q_{t+1} - q_t = r q_t - A_{t+1} F' \left(K_t + \frac{q_t - 1}{\chi} \right)$$

b) Assume that A is constant. Find the steady state. Does it depend on χ ?

c) Draw the phase diagram for the system and show (using arrows) the direction the variables move in each part.

d) What kind of stability does the model exhibit? If saddle path stability is indicated, what type of additional assumptions would you want to make and why?

e) Analyze what happens if A increases to A' permanently starting from a steady state.

f) Does marginal q equal average q in this model? Explain.