

MACROECONOMICS COMPREHENSIVE EXAM RE-TAKE

FEBRUARY 15, 2013

Each question consists of several parts. Complete each part to the best of your ability. Be sure to manage your time efficiently. Show all your work. Good luck!

1 A growth model

During each period $t \in [0, \infty)$, a representative consumer divides up a total stock of physical capital $k(t)$ into a fraction $u(t)$ used to produce goods for consumption and investment in physical capital and a fraction $1-u(t)$ used to produce additional human capital. Likewise, the consumer “divides up” a total stock of human capital $h(t)$ by spending a fraction $v(t)$ of his or her time producing goods for consumption and investment in physical capital and a fraction $1-v(t)$ of his or her time accumulating additional human capital. Hence, $u(t)k(t)$ and $v(t)h(t)$ measure the total amounts of physical and human capital used to produce goods and $[1-u(t)]k(t)$ and $[1-v(t)]h(t)$ measure the total amounts of physical and human capital used to accumulate more human capital.

Suppose that goods and new human capital both get produced according to the same Cobb-Douglas production function in which the parameter α , satisfying $0 < \alpha < 1$, measures the share of physical versus human capital in production. Suppose also that both capital stocks depreciate at the same rate, measured by the parameter δ satisfying $0 < \delta < 1$. Then in this economy, physical capital gets accumulated according to the constraint

$$[u(t)k(t)]^\alpha [v(t)h(t)]^{1-\alpha} - \delta k(t) - c(t) = \dot{k}(t), \quad (1)$$

for all $t \in [0, \infty)$. In (1), $c(t)$ denotes the amount of goods consumed during period t .

Meanwhile, human capital gets accumulated according to the constraint

$$\{[1-u(t)]k(t)\}^\alpha \{[1-v(t)]h(t)\}^{1-\alpha} - \delta h(t) = \dot{h}(t), \quad (2)$$

for all $t \in [0, \infty)$.

Comparing (1) and (2) also reveals that even under the simplifying assumptions made above that the production functions and depreciation rates are the same for physical and human capital, there is a key economic distinction between goods and human capital, because goods can be eaten but human capital cannot.

Suppose, finally, that the representative consumer’s utility from consuming $c(t)$ units of the good at each date $t \in [0, \infty)$ is given by

$$\int_0^\infty e^{-\rho t} \ln(c(t)) dt, \quad (3)$$

where the single-period utility function takes the natural log form and where the discount rate ρ satisfies $\rho > 0$.

1. Provide an explanation of the economic meaning of equations (1) and (2).
2. Write down the Hamiltonian for the consumer’s problem.
3. Now write down all first-order conditions solve the consumer’s problem.
4. The equations that you derived in part 3 above imply that this economy has a “balanced growth path,” along which the fractions $u(t)$ and $v(t)$ are constant, with $u(t) = u$ and $v(t) = v$ for all $t \in [0, \infty)$ and along which consumption $c(t)$ and the two capital stocks $k(t)$ and $h(t)$ all grow at the same constant rate γ . Use your results from above to obtain solutions for these three constants, u, v , and γ , in terms of the model’s parameters: α, δ , and ρ .

2 Consumption theory

Follow the steps below to answer questions about the random walk theory of consumption

1. What is the random walk theory of consumption? Use plain English, explain the general idea of the theory.
2. Suppose Congress passes a law on January 1, 2013 to increase income tax rates on all households. The law will take effect on January 1, 2014. According to the random walk theory, will consumers alter their existing consumption plan on January 1, 2013? Will consumers alter their existing consumption plan on January 1, 2014? Give an answer to each question and explain.
3. Set up a mathematical model to present the theory in a rigorous way. You need to a. specify the economic environment, including preferences, technology, and constraints; b. derive the optimizing conditions; c. derive the random walk equation.
4. What critical conditions must exist for the random walk theory to hold exactly?
5. Describe two empirical tests of the random walk theory. Make sure to a. write down the hypothesis of the test and the empirical equations used to conduct the tests; b. briefly discuss the results.

- 3) Consider an open economy where domestic inflation, domestic expected inflation and ROW expected inflation are all fixed (π , π^e and π^{e^*} are all fixed). Net exports depends *only* on the exchange rate according to a function $NX(\epsilon)$. Exchange-rate expectations have been "static." What happens to the economy if financial-market participants come to expect the economy's currency will depreciate in the future, so that $\dot{\epsilon}^e/\epsilon$ becomes positive? For each case below, *clearly state* what happens to:
- domestic output
 - the domestic interest rate
 - the exchange rate ϵ
 - capital flow CF

Illustrate with appropriate graphs!

- a) Perfect capital mobility, reserve gain zero, central bank fixes the money supply, exchange rate floats.
- b) Imperfect capital mobility, reserve gain zero, central bank follows an interest-rate rule $r(\pi, Y)$, exchange rate floats.

- 4) Consider an economy that can be described by the Diamond OLG model.

The aggregate production function is Cobb-Douglas: $Y = K^\alpha L^{1-\alpha}$ where $0 < \alpha < 1$

There is no depreciation. The rate of growth of population is n . Note that there is *no* improvement over time in technology. A person's lifetime utility function (lifetime utility as a function of first-period consumption C_1 and second-period consumption C_2) is:

$$U = \ln(C_1) + \frac{1}{1+\rho} \ln(C_2)$$

Let r denote the real interest rate and w denote the real wage.

- a) Derive an expression that gives C_{1t} as a function of w_t and any other relevant variables.
- b) Using a), write down an expression for s , that is the fraction of first-period labor income that a young person saves.

- 5) Suppose an economy has a Friedman-Phelps Phillips curve: $\pi_t = \rho_{t-1}\pi_t^e + \lambda y_t + \epsilon_t$ and an IS curve corresponding to: $y_t = -ar_t$ and the central bank follows an interest-rate rule: $r_t = by_t + c(\pi_t - \bar{\pi})$ where y denotes the log of the output gap (y is zero if output is equal to the natural rate of output), r denotes the difference between the real interest rate and the natural rate of interest, and $\bar{\pi}$ is a fixed value. Expectations are model-consistent rational expectations (what you get in rational expectations equilibrium). ϵ_t is a potential mean-zero, i.i.d. random shock to inflation (a "supply shock").
- a) Solve for $\rho_{t-1}\pi_t^e$.
 - b) Derive π_t , y_t , r_t , i_t in terms of ϵ_{t-1} and $\bar{\pi}$.
 - c) Illustrate this situation in an IS/MP graph and an AS/AD graph.

- 6) Why do central-bank policy committees nowadays choose interest rates and adjust the money supply from day to day to hit that chosen interest rate, rather than simply fix the money supply?

- 7) Can you use the IS/MP graph to describe an economy in which at least one economic sector is subject to the financial-market imperfection described by the Romer textbook's model of asymmetric information and costly state verification? How do such financial-market imperfections show up in the graph? Suppose, for example, that there is an increase in the cost of foreclosing on a defaulting debtor and seizing his remaining assets (an increase in the model's variable c). Would that shift the IS curve, the MP curve, neither or both?

Section 3: Jones

8. Consider a model in which there are two types of tradable goods, two factors of production (capital and labor) and one non-tradable good. Choose one of the tradable goods as the numeraire and let p be the price of the non-tradable good and p_T be the price of the other tradable good, both in terms of units of the first tradable good. Let p_T be determined by world markets. But, p , w and r are NOT determined by world markets. This differs from class, where we assumed that r was determined by world markets (here, the difference is that we are not assuming capital mobility). Assume a common rate of productivity growth for the two tradables (call it \hat{A}_T).

a) Write down the zero profit conditions.

b) Derive conditions that could be used to determine \hat{p} , \hat{w} and \hat{r} as functions of \hat{A}_T and \hat{A}_N .

c) Show that $\hat{w} = \hat{r} = \hat{A}_T$.

d) Assuming that $\hat{w} = \hat{r} = \hat{A}_T$, show that $\hat{p} = \hat{A}_T - \hat{A}_N$.

e) How does this result relate to the Harrod-Balassa-Samuelson result?

9. Suppose that there are two countries and that there are two distinct goods (call them X and Y). Home and foreign have identical preferences and Home maximizes

$$E_t \left[\sum_{s=t}^{\infty} \beta^{s-t} U(C_{X,s}, C_{Y,s}) \right]$$

subject to

$$C_{X,s} + p_s C_{Y,s} + \theta_{X,s+1} V_{X,s} + \theta_{Y,s+1} V_{Y,s} + B_{s+1} = (1+r)B_s + \theta_{X,s}(X_s + V_{X,s}) + \theta_{Y,s}(Y_s + V_{Y,s})$$

p refers to the price of good Y in terms of good X . Home receives a stochastic endowment of good X and Foreign receives a stochastic endowment of good Y each period. Home and Foreign trade in shares representing claims against future endowments of the two goods. θ_X refers to Home's share of such claims for good X and θ_Y refers to the share of such claims for good Y . B is a risk-free bond indexed to good X . V_X and V_Y are the asset prices.

- a) Find all relevant first-order conditions for Home and provide economic interpretations of them.
- b) Prove that if Home and Foreign start with perfectly pooled identical portfolios of risky claims, then they will keep these portfolios forever.
- c) Interpret this finding in economic terms.