Micro Comp **(Second attempt)** **February 1, 2013**

Part A: Short questions. Be thorough in your answers or explanations.

1. (Khanna) A perfectly competitive firm faces a price of $10 and is currently producing a level of output where marginal cost is equal to $10 on a rising portion of its short run marginal cost curve. Other information available about this firm:
   - Long run marginal cost at the current quantity = $12.
   - Short run average variable cost at the current quantity =$ 8.
   - The minimum point on its long run average cost curve = $10.

   Is this firm earning an economic profit in the short run? Should it change its output in the short run? What should this firm do in the long run?

2. (Jones) Calculate the elasticity of substitution for the production function $f(x_1, x_2) = (x_1^\alpha + x_2^\alpha)^{1/\alpha}$, where $0 \neq \alpha < 1$.

3. (Jones) Let $\pi(p, \boldsymbol{w})$ be a competitive firm's profit function given output price p and input price vector **w**. Prove: $\pi(p, \boldsymbol{w})$ is convex in (p, **w**).

4. (Pape) "Positive externalities can be thought of as increasing the social marginal benefit above the private marginal benefit, but not decreasing the social marginal cost below the private marginal cost." True or false? Explain.

5. (Pape) "In perfectly competitive games, there is no incentive for players to share information before the game begins." True, false, or uncertain? Justify your answer.

Part B: Mathematical / advanced questions

6. (Khanna) Huimei has the option of renting a car for $40 per day or $200 per week. Her total travel budget is $360 which she divides between the car rental and a composite good representing all other travel expenses which cost $1 per unit.

   (a) Draw Huimei's budget constraint. Put rental car days on the horizontal axis and the composite good on the vertical axis.

   (b) Find Huimei's best affordable bundle if her travel preferences are such that she requires exactly $140 worth of all other goods per day of rental car consumption.

   (c) Now suppose that Huimei views a day of car rental consumption as a perfect substitute for $35 worth of other goods. What is her best affordable bundle in this case?

7. (Khanna) Consider a firm that produces output, *Q*, using two inputs capital, *K*, and labor, *L*. Its production function *Q = f(K,L)* satisfies all the usual properties of a well-behaved production function so that you need to consider only interior solutions. Assume perfectly competitive input and output markets so that the firm takes all prices as given.

   (a) Write out the firm's cost minimization problem and the first order condition that determines the firm's optimal capital-labor ratio. Label this capital-labor ratio $\left(K/L\right)_1$.

   Now suppose that the firm's emissions, *e*, are determined by its input use via an emission production technology, $e = e_L L + e_K K$, where $e_L > 0$ and $e_K > 0$ are the emission coefficients of capital and labor, respectively. Further suppose that capital is the relatively more emission intensive input so that $e_K > e_L > 0$.

   (b) Write out and solve the firm's cost minimization problem in the presence of a binding emission constraint, $\bar{e} = e_L L + e_K K > 0$. Label the capital-labor ratio obtained under the emission constraint $\left(K/L\right)_2$.

   (c) Provide an intuitive interpretation for the first order condition that you obtain in part (b) above. What do you expect is the sign of the Lagrange multiplier?

   (d) Compare $\left(K/L\right)_1$ with $\left(K/L\right)_2$. Is $\left(K/L\right)_2$ larger, smaller, or equal to $\left(K/L\right)_1$? Support your answer with an intuitive or graphical exposition.

8. (Jones) Let $u(x)$ be a consumer's utility function where x denotes the vector of goods. Prove: $u(x)$ is strictly quasiconcave if and only if the consumer's preference relation is strictly convex.

9. (Jones) A consumer's utility function is $u(x_1, x_2, x_3) = \min\{x_1, x_2\} + \ln x_3$.
   (a) State the conditions under which his utility-maximization problem has a corner solution.

   (b) Find his Marshallian demand function assuming interior solution.

   (c) Are all three goods normal?

10. (Pape) Consider the following game:

	L	R
U	4,1	0,3
D	0,4	4,2

Suppose that the row player is offered the following choice:

(a) play the simultaneous game above, or
(b) pay an amount *x>0* and have the privilege of moving first
(c) pay an amount *x>0* and have the privilege of moving second

Question: what is the largest amount *x* that will result in the row player choosing *b,* or is there no such *x?* What is the largest amount *x* that will result in the row player choosing *c,* or is there no such *x?*

11. (Pape) Consider a small, open economy that has two output goods *x* and *y* and two inputs, labor *l* and capital *k.* Suppose the output goods are traded on international markets with international prices p_x and p_y, but labor and capital are not traded on international markets (i.e. they are not mobile) and have local prices *w* and *r*.

    Suppose that *x* is produced by a firm with production function $f_x(l,k) = l^a k^{1-a}$ for some *a* between 0 and 1, and suppose that *y* is produced a different firm with a production function $f_y(l,k) = l^b k^{1-b}$ for some *b* between 0 and 1, and where $b<a$. Suppose there are *K* units of capital and *L* units of labor in this country.

    Suppose that there is a technological shift in the production of *x,* which causes *a* to increase to $a' > a$. How does this change the Walrasian equilibrium allocations of inputs between the firms and the input and output prices?

    Assume that we only need to consider internal equilibria.

Part C: Longer questions

12. (Jones) Analyze the insurance signaling game when benefit is restricted to being equal to L (the loss). Assume that the low-risk consumer strictly prefers full insurance at the high-risk competitive price to no insurance.

    (a) Show that there is a unique sequential equilibrium when attention is restricted to those in which the insurance company earns zero profits.

    (b) Show that among all sequential equlibria, there are no separating equilibria. Is this intuitive?

    (c) Show that there are pooling equilibria in which the insurance company earns positive profits.

13. (Jones) Consider this modified version of the consumer's utility-maximization problem: Suppose that, instead of a fixed income, the consumer only has an initial *endowment* of goods $w = (w_1, \ldots, w_n) \gg 0$, where w_i is the quantity of good i that he is endowed with. (For example, my initial endowment could be 5 apples, 3 bananas and 7 coffees.) His "income" is the market value of his endowment, i.e., $y = p \cdot w$.

(a) Use the Slutsky equation in the conventional setting (with fixed income and no endowments) as a basis, derive the Slutsky equation for this new setting, i.e., an expression for $\frac{\partial x_i}{\partial p_j}$ which shows the substitution and income effects.

(b) Explain what it means for the consumer to be a *net seller* of a good and a *net buyer* of a good.

(c) Assuming only 2 goods, draw a graph to illustrate the income and substitution effects in this setting.

14. (Pape) Consider the following game of Chicken:

	T	C
T	-4,-4	8,0
C	0,8	1,1

Part 1) Find all Nash Equilibria of this game.

Part 2) Suppose there is a coin flip, that both players know will come up heads with probability of .5 and tails with probability of .5. Suppose that the players observe the coin flip before they choose their actions. Is there a Subgame Perfect Nash Equilibrium of this augmented game (coin flip followed by chicken) which (a) provides equal expected payoffs to both player, and (b) Pareto dominates all mixed strategy Nash Equilibria found in Part 1? If such an SPNE exists, find it. If it does not exist, explain why.

15. (Pape) Consider an economy with two consumers: *A*be (consumer A) and *B*en (consumer B), who consume a single good *x.* Their vNM utility functions are u(x)=ln(x) and there are two possible states of the world, s=1 or 2. Let the probability of state 1 be $\pi < {}^{1}/_{2}$. Their endowments are:

Consumer A: (2, 1)

Consumer B: (1, 3)

Part 1) Is there aggregate risk in this economy? Explain.

Part 2) Suppose that Consumer A can pay an amount *x* to increase the probability of state 1 to 2π. What is the largest value of *x* that Consumer A will pay for this privilege, or is there no such *x*?