Spring 2002

MACROECONOMIC THEORY COMPREHENSIVE EXAMINATION

B. Jones/Sorensen

Section I: Answer the following 6 short answer/definition questions

a) Assume the Consumption CAPM is true. Now suppose that asset A pays $300 next period for sure, while asset B pays $800 when consumption is high and $1000 when consumption is low. Will asset A or asset B have the highest rate of return?

b) State a relation that holds between individual and aggregate consumption when perfect risk sharing holds. Mention several (2 at least) assumptions necessary for this relation to hold.

c) Explain Campbell and Mankiw's "rule-of-thumb" consumer model. (Write down the model and explain the meaning of all variables and parameters; you do not need to explain how it can be estimated empirically.)

d) "Increasing returns to scale are both necessary and sufficient for endogenous growth." Discuss this statement.

e) State the "certainty equivalence (CE) principle". What assumptions would be sufficient for CE to hold? How is CE used?

f) Assuming rational expectations, when is it desirable to target a fixed nominal interest rate? When is it desirable to target a fixed nominal stock of money? What happens if you target a fixed nominal interest rate under adaptive expectations? Your answer should emphasize economic intuition rather than math.

Section II: Answer all parts (a-g) of this section

Suppose a worker maximizes lifetime utility given by the following equations:

$$\sum_{t=0}^{\infty} \beta^t u_t$$

$$u_t = \begin{cases} w - e_t, & if\ x_t = 1 \\ -e_t, & if\ x_t = 0 \end{cases}$$

At each time t, workers choose e_t to be either 0 or $\overline{e}$. The state variable is a stochastic process described as follows:

$$x_{t+1} = \begin{cases} 1 \quad if\ x_t = 1\ and\ e_t = \overline{e} \\ 1\ with\ probability (1-q) \quad if\ x_t = 1\ and\ e_t = 0 \\ 0\ with\ probability\ q \quad if\ x_t = 1\ and\ e_t = 0 \\ 1\ with\ probability\ a \quad if\ x_t = 0 \end{cases}$$

Answer the following questions:

a) What does it mean in economic terms if $x = 0$?
What does it mean in economic terms if $x = 1, e = 0$?
What does it mean in economic terms if $x = 1, e = \overline{e}$?
What is the economic meaning of q?
What is the economic meaning of a?

b) State the stochastic version of Bellman's equation for the state $x = 0$. What is the optimal value of e if $x = 0$? Solve the Bellman equation for $V(0)$, the value function in state $x = 0$. [Hint: $V(0)$ is a function of $V(1)$ the value function in state $x = 1$]

c) State Bellman's equation for the state $x = 1$.

d) Suppose that the worker in state $x = 1$ is indifferent between $e = 0$ and $e = \overline{e}$. Using results from parts b and c, show that $V(1) - V(0) = \frac{\overline{e}}{q}$. Interpret this equation.

e) Show how to derive the value of w that insures the worker is indifferent between $e = 0$ and $e = \overline{e}$. [Hint: assume that $e = \overline{e}$ whenever $x = 1$ and solve for $V(1)$. Then use the expressions derived in parts b and d.]

f) This model is a simplified version of the Shapiro-Stiglitz Model (AER, 1984). Romer states "Theories in which there is both a cost as well as a benefit to lowering wages are known as *efficiency-wage* theories." Explain how the Shapiro-Stiglitz environment is related to the existence of efficiency wages.

g) Is the decentralized equilibrium efficient "in the sense that the MPL equals the marginal cost of labor in equilibrium" in the Shapiro-Stiglitz model? Explain carefully.

Section III: Answer all parts (a-e) of this section
Assume the income follows the stationary ARMA(1,1) process

$y(t) = 0.7*y(t-1) + u(t) + 0.2*u(t-1)$.

a) What is the variance of y(t)?

b) What is the first order autocorrelation of the process?

c) Now consider Hall's version of the Permanent Income Hypothesis. Explain carefully in intuitive terms (preferably without mathematical symbols) why consumption would react less strongly to the innovation u(t) if the coefficient to y(t-1) in the ARMA-process was 0.1 instead of 0.7.

d) Explain what is meant by "excess sensitivity."

e) Assume that the rate of interest is 5 percent. What will be the impact on period t consumption of a $100 innovation u(t) in period t, according to the Permanent Income Hypothesis?

Section IV
Answer 1 of the following 2 questions

IV - 1. Suppose a central planner solves the following problem:

$$\max \int_0^\infty e^{-\rho s} \ln(C(s)/N(s))ds$$

subject to the following constraints:

i) $\dot{K} = Y - C - \delta K$

ii) $\dot{R} = -E$

iii) $\dot{B} = gB$

iv) $\dot{N} = nN$

v) $Y = BK^{\alpha}E^{\beta}N^{1-\alpha-\beta}$

given $R(0) = R_0, K(0) = K_0, B(0) = 1$

B is TFP, K is capital stock, N is labor force, and E is energy flow. The flow of energy is derived by reducing the stock of natural resources R.

a) Derive the necessary conditions for optimality using the maximum principle.

b) What is the main implication of transversality for *R*?

c) Suppose the central planner chooses a steady state solution. What is the steady state value of $\dot{R}/R$? Interpret.

d) Using what you know about the relationship between returns to scale and steady state growth, form a guess as to the steady state behavior of this economy if there is no TFP growth.

e) Suppose that production is decentralized. Private agents derive energy flows for use in private production by depleting the stock of resources. Private agents do not pay for the "right" to deplete the resource stock. What are the efficiency implications of decentralized production? What would you recommend to policy makers? Be specific.

IV - 2. Suppose a central planner maximizes the lifetime utility of an infinitely lived representative agent:

$$\sum_{t=0}^{\infty} \beta^t u(c_t)$$

subject to the following constraints:

$$c_t + x_t + g_t = zf(k_t, g_t)$$
$$k_{t+1} = (1-\delta)k_t + x_t$$

a) Use Bellman's method to formulate necessary conditions for optimality, assuming the existence of an interior solution.

b) State conditions under which a steady state solution will exist. Describe the steady state as thoroughly as you can.

c) Provide the economic intuition for the condition that determines g.

d) What happens to each variable if β increases? Explain the economic intuition behind this result.

e) Suppose that production is decentralized. The government finances g using taxes levied on production. The variable factor of production, k, is owned by households. The tax levied on each agent is proportional to income. The tax rate is τ, so that private agents face constraints:

$$c_t + x_t = (1-\tau)zf(k_t, g_t)$$
$$k_{t+1} = (1-\delta)k_t + x_t$$

The value of g is assumed to be exogenous to the agent. How does this type of taxation affect the steady state of the model? What are the efficiency implications for this type of taxation in steady state? What would you recommend to policy makers? Be specific.

#45a re-take PS

Spring 2002-2

MACROECONOMIC THEORY COMPREHENSIVE EXAMINATION

B. Jones/Sorensen

Section I. Answer the following 7 short answer questions.

1. Suppose that prices are generated by the following equations:

$$P_t^i = P_t + z_t^i$$

$$P_t = P_0 + v_t$$

where z^i and v are mean zero, uncorrelated, random shocks. Firm i observes P_t^i, but not P_t .

State the Lucas supply curve for firm *i*. Explain (in economic not mathematical terms) how the variances of the two random shocks affect the firm's behavior.

2. State the Lucas (econometric policy evaluation) critique. Explain how the model in question 1 relates to the Lucas critique?

3. The neoclassical investment model has a strong implication for the path of investment over time, explain. There are alternatives to the neoclassical investment model, discuss two of them.

4. Explain how to compare productivity across countries, quantitatively, using a Malmquist index.

5. A consumer lives for 3 periods and expects to earn 100\$, 200\$, and 300\$ in periods 1, 2, and 3 respectively. The consumer has a quadratic utility function and is---in period 1---allowed to freely borrow and lend at an interest rate that equals his or her rate of time preference. The consumer is not allowed to borrow or lend in period 2. Let C_i be the consumption of the representative consumer in period i. Is $C_1 = E(C_2)$ and is $C_2 = E(C_3)$?

6. Asset A and asset B exist for one period and their returns have identical covariance with the market return. The rate return of asset B has a variance that is twice as large as the variance of the rate of return of asset A. Which asset will---if the CAPM holds---have the highest expected rate of return?

7. Explain (without deriving any formulas) what is meant by the equity premium puzzle? Explain the empirical observation that the puzzle refers to, and under which assumptions it is or isn't a puzzle.

Section II. Answer 1 of the following 2 questions.

Question II.1.

Suppose that the aggregate production function is as follows:

$$Y = \frac{AN}{\left(1+\left((K/N)-\bar{k}\right)^2\right)^{1/2}} \text{ if } K/N \leq \bar{k}$$

and $$Y = AN \text{ if } K/N > \bar{k}$$

where K is capital stock, N is units of labor, and A is TFP.

a) Is this production function neoclassical (please be thorough)?

b) What model from ECON 613 is closest to this production function?

c) Suppose that the economy consumes a constant proportion of income each period, so that $C(t) = (1-s)Y(t), 0 < s < 1$

Discuss the steady state properties of the model. Compare and contrast them with the model you identified in part b.

d) Now suppose that a central planner maximizes lifetime utility of an average consumer:

$$\int_0^\infty e^{-\rho s} U(c(s))ds$$

subject to $\dot{k} = y - c - (n+\delta)k$, where y is production per unit of labor.

Write out necessary conditions for optimality using the maximum principle.

e) Discuss the steady state properties of the model.

f) Qualitatively, how would per-capita consumption and the capital labor ratio respond (in steady state) to a positive TFP shock?

g) Compare and contrast the results from parts c and e (in <u>economic not mathematical</u> terms).

Question II.2.

You are writing a new undergraduate textbook on economic growth theory. You have data, which shows that the growth rate of technology is constant over time, but the rival factors of production used in the research and development (R&D) sector have grown significantly over time. You want to explain how this is possible to undergraduate students who know basic calculus.

In this section of your textbook, you will assume that there is a single rival factor of production (call it N), which grows at a constant rate n, but which cannot be accumulated. Output cannot be stored. Call the non-rival factor of production A

a) Explain to your <u>undergraduate readers</u> the difference between rival and non-rival factors of production. Give some examples that will help your readers understand the difference.

b) Output is produced using rival and non-rival factors of production. Write down a production function for output that is CRS in rival factors. What condition(s) are necessary for steady state growth in output/N?

c) A is produced by the R&D sector. A is produced using rival and non-rival factors of production, write down a production function for $\dot{A}$. [Hint: you want to show that rival factors of production used in the R&D sector grow over time, even though the rate of technical progress does not!]

d) Derive the steady state growth rate of output/N based on a) and b). Explain your equation to your <u>undergraduate readers</u>.

e) Show that the rate of technical progress is constant in steady state, but that the quantity of the rival factors of production devoted to R&D are growing.

f) What are the returns to scale in the R&D sector? Why is this significant? Explain to your <u>undergraduate readers</u> why this assumption makes sense.

g) The "fishing out effect" is that over time it becomes harder to discover technical innovations, because the easiest technical innovations are discovered first. Incorporate this into your model [or, if it already is incorporated, then explain it] and discuss the implications of the fishing out effect for the steady state.

Section III. Answer 1 of the following 2 questions.

Question III.1.
Consider an "endowment economy" with identical (representative) consumers. Assume there is no capital and output is "fruit from trees," i.e., the production of output requires no effort and output is non-storable and cannot be invested. Agents can freely trade in financial assets. Assume that the (representative) agent has a quadratic utility function $U(C_1) + EU(C_2) + EU(C_3) + \dots$ where $U(C) = C - \frac{C^2}{200}$.

a) Under which condition on the interest rate will the level of consumption of the representative consumer be a martingale (a "random walk")? (Note that this condition is not necessarily true in the questions below.)

From now on assume that output in period 1 is 100 and in period 2 it is 120 if "sun" and 100 if "rain."

b) What is the safe rate of interest from period 1 to period 2?

c) Write down the period 1 Euler Equation for the representative consumer and an asset with stochastic rate of return R_2. (Please be precise).

d) Consider an Arrow security that pays one unit of consumption in period 2 if it is "sun." What is the period 1 price of the Arrow security (the price of period 1 consumption is normalized to 1)?

Question III.2.
Consider a 1-period economy with many agents. Consumption and labor supply of individual i is C_i and L_i, respectively. Individual i has a utility-function $U_i(C_i,L_i) = C_i - \frac{(L_i)^\gamma}{\gamma}$. Output is given as $Q_i = L_i$ and the demand for the output of individual i is $Q_i = Y\,(P_i/P)^{-\eta}$, where $\eta>1$, Y is aggregate output and P is the average price-level. Consumption of individual i is equal to real income (income divided by P). $\text{Income}_i = (P_i - W)*Q_i + W*L_i$, where W is the wage level.

a) Derive (and state) the indirect utility function that expresses the utility of individual i as a function of P_i and L_i (the choice variables of individual i).
b) Derive the first order condition for optimality with respect to P_i.
c) Derive the first order condition for optimality with respect to L_i.
d) Find the level of output in equilibrium.
e) Explain if this level of output is efficient. (Use derivations or words, in the latter case only *precise* explanations are valid.

Section IV. Answer one additional question from either section II or section III.