#46 PS

SPRING 2003

MACROECONOMIC THEORY COMPREHENSIVE EXAMINATION

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REQUIRED QUESTION

The attachment is a selection from "Macroeconomics and Reality" by Christopher Sims, which appeared in *Econometrica* in 1980. Sims discusses the "Lucas critique" in detail. Read the section.

Write an essay that addresses the following issues:

1) State the Lucas critique and interpret it. Illustrate it using one or more examples from the original article by Lucas.

2) Write an essay that addresses the following: what areas of agreement do you find between Lucas and Sims; how do their positions differ; how does Sims view the relationship between policy and reduced forms and how does this view differ from Lucas' view; how do they differ in their definitions of policy rules and how does this effect their thinking; what is the difference between changing a policy rule and "effectively implementing an existing rule". Which view do you find more convincing and why?

CHOOSE ONE OF THE FOLLOWING TWO QUESTIONS:

1. Politicians are assumed to maximize the following:

$$\int_0^T e^{rt} V(U(t), \pi(t)) dt$$

U is unemployment, π is inflation, r is positive, V is a "vote function". High inflation and high unemployment lower the vote function.

The constraints are as follows:

1) $\dot{\pi}^e = b(\pi - \pi^e)$

and

2) $\pi = \phi(U) + a\pi^e$.

There are no terminal state restrictions.

a) What is the economic interpretation of 1)?

b) What is the economic interpretation of 2)?

c) What are the signs of a and b? Why?

d) What is the sign of ϕ'? Why?

e) Write down conditions that determine the optimal control solution.

f) Assume that $\phi(U) = \bar{U} - U$ and $V(U,\pi) = -U^2 - h\pi$. Get as far as you can in solving for the optimal controls.

g) If you solve it successfully, then provide interpretation. If you did not, then take a guess as to the properties of the optimal controls.

2. A representative agent maximizes

$$\sum_{t=0}^{\infty} \beta^t U(C_t / N_t, M_t / P_t N_t)$$

subject to $N_t f(K_{t-1} / N_t) + \tau_t N_t + (1-\delta)K_{t-1} + \frac{M_{t-1}}{P_t} = C_t + K_t + \frac{M_t}{P_t}$.

All variables have their usual meanings. τ is a lump-sum per-capita transfer. Let n be the population growth rate.

a) Convert the problem to per-capita terms.

b) Find optimality conditions using dynamic programming.

c) Interpret them.

d) Find steady state.

e) What is the social optimum? How can it be decentralized?

f) What is the connection between money, scarcity, and efficiency?

ANSWER ALL THE QUESTIONS BELOW.

A. (7%) Explain what is meant by Excess Smoothness of Consumption. (Don't derive any relations, but explain clearly).

B. (19%) Derive the Consumption-CAPM model. (The version in the book and the version in the handout Professor Sorensen posted for the class are both valid answers. You do not need to prove the Euler-equation.)

C. (24%) Assume that 2 agents live for 2 periods in an economy with perfect Arrow-Debreu markets and no storage. Assume that the endowment of the first agent is $y_0=1$, $y_1=2$ and that the endowment of the second agent in period 0 is $y_0^*=1$. In period 1 his or her endowment is $y_1^*=3$ in the "good state" g and in the "bad state" b the endowment of the second agent is $y_1^*=1$. Assume that the good state happens with probability 1/2. Assume until further notice that each agent maximizes a utility function $-1/C_0 - E_0\{1/C_1\}$.

i) Find the period 0 prices of the Arrow securities that pay out one unit in the good and the bad state, respectively. (I suggest that you follow Obstfeld-Rogoff and parameterize such that the period 0 price of 1 unit delivered in the good state is $p\_g/(1+r)$ and in the bad state it is $p\_b/(1+r)$ which implies that $p\_g+p\_b = 1$ when r is the safe rate of interest. The price of one unit of period-0 consumption is normalized to 1.)

ii) Find the safe rate of interest r.

iii) Find the value (in terms of period 0 output) of the second ("*") agent's output stream (i.e., the total value of period 0 and period 1

iv) Find the level of consumption of each of the agents in periods 0 and 1 and both states of the world. Which agent has a higher level of consumption? Interpret why using economic terms from class.

v) Argue, using words, whether the consumption of agent 1 would increase or decrease (compared to the model above) if the utility function were $-1/C_0^2 - E_0\{1/C_1^2\}$. (Spell this out in terms of economic concepts.)

vi) Demonstrate what would happen to the interest rate (i.e., would it go up or would it go down) if the output of the second agent in period 1 were constant rather than a random variable---assume that the mean value of period 1 output is the same (the first agent's output is unchanged and so is the output of the second agent in the first period).

independent variation between the time path of expected future *levels* of P and expected future *derivatives* of P. With ρ or δ nonzero, the operator applied to $\hat{P}_s$ on the right side of (12) differs from that applied to K on the left by more than a reflection. Even if we know ρ a priori (by reading the financial press), first-order Markov behavior for P implies that δ is not identified (assuming still that past P makes up the information set). In the first-order Markov case with $\dot{P} = -rP + e$, we have $(d/dt)\hat{P}_s(t) = -r\hat{P}_s(t)$ for all $t > s$. Thus (12) becomes, when we replace $\hat{P}_s$ by its observable counterpart,

$$(14) \qquad (D + M_1)K(s) = -(\delta + \rho + r)P(s)/2\Theta(M_2 + r) + (\alpha/2\Theta M_2).$$

The separate coefficients on $\hat{P}_s$ and its derivative in (12) have merged into one, leaving the structural parameters unidentifiable from the relation of the observable variables. In particular, one can see by examining (14) and (13) that one could vary δ, Θ, α, and λ in such a way as to leave coefficients in (14) unchanged even for fixed ρ and r, so that knowing r from the data and ρ a priori will not suffice to identify the model.

D. *Concrete Implications*

Were any one of the categories of criticism of large-model identification outlined in the preceding three sections the only serious criticism, it would make sense to consider existing standard methodology as a base from which to make improvements. There is much good work in progress on estimating and specifying systems of demand relations. Some builders of large models are moving in the direction of specifying sectoral behavior equations as systems.[10] There is much good work in progress on estimating dynamic systems of equations without getting fouled up by treating knowledge of lag lengths and orders of serial correlation as exact. There is much good work treating expectations as rational and using the implied constraints in small systems of equations. Rethinking structural macromodel specification from any one of these points of view would be a challenging research program. Doing all of these things at once would be a program which is so challenging as to be impossible in the short run.

On the other hand, there is no immediate prospect that large-scale macromodels will disappear from the scene, and for good reason: they are useful tools in forecasting and policy analysis.

How can the assertion that macroeconomic models are identified using false assumptions be reconciled with the claim that they are useful tools? The answer is that for forecasting and policy analysis, structural identification is not ordinarily needed and that false restrictions may not hurt, may even help a model to function in these capacities.

Textbook discussions sometimes suggest that structural identification is necessary in order for a model to be used to analyze policy. This is true if "structure"

[10] For example, Fair [6] takes this approach in principle, though his empirical equations are specified with a single-equation approach to forming lists of variables. Modigliani [20] reports that the MPS model (like the Fair model) has interest rates turning up in many household behavioral equations.

and "identification" are interpreted in a broad way. A structure is defined (by me, following Hurwicz [12] and Koopmans [13]) as something which remains fixed when we undertake a policy change, and the structure is identified if we can estimate it from the given data. But in this broad sense, when a policy variable is an exogenous variable in the system, the reduced form is itself a structure and is identified. In a supply and demand example, if we contemplate introducing an excise tax into a market where none has before existed, then we need to be able to estimate supply and demand curves separately. But if there has previously been an excise tax, and it has varied exogenously, reduced form estimation will allow us accurately to predict the effects of further changes in the tax. Policy analysis in macromodels is more often in the latter mode, projecting the effect of a change in a policy variable, than in the mode of projecting the effect of changing the parameters of a model equation.

Of course, if macroeconomic policy-makers have a clear idea of what they are supposed to do and set about it systematically, macroeconomic policy variables will not be at all exogenous. This is a big if, however, and in fact some policy variables are close enough to exogenous that reduced forms treating them or their proximate determinants as exogenous may be close to structural in the required sense.[11] Furthermore, we may sometimes be able to separate endogenous and exogenous components of variance in policy variables by careful historical analysis, in effect using a type of instrumental variables procedure for estimating a structural relation between policy variables and the rest of the economy.

Lucas' [16] critique of macroeconomic policy making goes further and argues that, since a policy is not really just one change in a policy variable, but rather a rule for systematically changing that variable in response to conditions, and since changes in policy in this sense must be expected to change the reduced form of existing macroeconometric models, the reduced form of existing models is not structural even when policy variables have historically been exogenous—institution of a nontrivial policy would end that exogeneity and thereby change expectation formation rules and the reduced form.

There is no doubt that this position is correct, if one accepts this definition of policy formation. One cannot choose policy rules rationally with an econometric model in which the structure fails to include realistic expectation formation. However what practical men mean by policy formation is not entirely, probably not even mainly, choice of rules of this sort. Policy makers do spend considerable effort in comparing projected time paths for variables under their control. As Prescott and Kydland [23] have recently shown, making policy from such projections, while ignoring the effect of policy on expectation-formation rules, can lead to a very bad time path for the economy, under some assumptions. Or, as Sargent and Wallace [29] have shown, it can on other assumptions be merely a charade, with the economy's real variables following a stochastic process which cannot be affected by any such exercises in choice of time paths for policy variables.

[11] We shall see below, for example, that in Germany and the U.S. money supply, while not entirely exogenous, has an exogenous component which accounts for much of its variance.

I do not think, however, that practical exercises in conditional projection of effects of policy are either charades or (usually) Prescott and Kydlund's case of "Peter White" policy making.[12] Suppose it were true that the policy rule did make a difference to the economy. There are many ways to argue that this is true in the face of Sargent's [28] or Sargent and Wallace's [29] analysis, all being suggestions for forms of non-neutrality of money. To be concrete, suppose that the real variables of the economy do follow a stochastic process independent of the money supply rule but that for some reason the rate of inflation enters the social utility function.[13] Then the optimal form for macropolicy will be stabilization of the price level.[14] If we could agree on a stable model in which all forms of shock to the aggregate price level were specified a priori, then it would be easy in principle to specify an appropriate function mapping past values of observed macrovariables into current levels of policy variables in such a way as to minimize price variance. However, if disturbances in the economy can originate in a variety of different ways, the form of this policy reaction function may be quite complicated. It is much easier simply to state that policy rule is to minimize the variance of the price level. Furthermore, if there is uncertainty about the structure of the economy, then even with a fixed policy objective function, widely understood, the form of the dependence of policy on observed history will shift over time as more is learned about (or as opinions shift about) the structure of the economy. One could continually re-estimate the structure and, each period, re-announce an explicit relation of policy variables to history. However it is simpler to announce the stable objective function once and then each period solve only for this period's policy variable values instead of computing a complete policy reaction function. This is done by making conditional projections from the best existing reduced form model, and picking the best-looking projected future time path. Policy choice is then most easily and reliably carried out by comparing the projected effects of alternative policies and picking the policy which most nearly holds the price level constant. Accurate projections can be made from reduced form models fit to history because it is not proposed to change the policy rule, only to implement effectively the existing rule.

[12] Peter White will ne'er go right/ Would you know the reason why?/ He follows his nose where'er he goes/ And that stands all awry.—Nursery Rhyme.

[13] It is a little hard to imagine why the rate of inflation should matter if it affects no real variables. A more realistic and complicated scenario would suppose that there are costs to writing contingencies into contracts, and enforcing contracts with complicated provisions, so that a macropolicy which stabilizes certain macroeconomic aggregates—prices, wages, unemployment rates, etc.—may simplify contract-writing and thus save resources. This has been made the basis of an argument against inflation by Arthur Okun [22].

[14] Discussion of such a policy seems particularly appropriate in the Fisher-Schultz lecture, as Irving Fisher supported such a policy: "The more the evidence in the case is studied, the deeper will grow the public conviction that our shifting dollar is responsible for colossal social wrongs and is all the more at fault because those wrongs are usually attributed to other causes. When these who can apply the remedy realize that our dollar is the great pickpocket, robbing first one set of people, then another, to the tune of billions of dollars a year, confounding business calculations and convulsing politics, and, all the time, keeping out of sight and unsuspected, action will follow and we shall secure a boon for all future generations, a true standard for contracts, a stabilized dollar" [7].

In fact, it appears to be a mistake to assume that the economy's real variables follow a process even approximately unrelated to nominal aggregates. Thus stabilization of the price level alone is not likely to be the best policy. However, it is not clear that the existing pattern of policy in most countries, in which there is weight given to stabilization of inflation, unemployment, and income distribution, is very far from an optimal policy. Simply implementing policy according to these objectives in the way the public expects is a highly nontrivial task, and one in which reduced-form modeling may be quite useful.

To summarize the argument, it is admitted that the task of choosing among policy regimes requires models in which explicit account is taken of the effect of policy regime on expectations. On the other hand, it is argued that the choice of policy regime probably does have important consequences, and that an optimal regime and the present regime in most countries are both most naturally specified in terms of the effects of policy on the evolution of the economy, rather than in terms of the nature of the dependence of policy on the economy's history. Effectively implementing a stable optimal or existing policy regime therefore is likely appropriately to involve reduced-form modeling and policy projection.

But I have argued earlier that most of the restrictions on existing models are false, and the models are nominally over-identified. Even if we admit that a model whose claimed behavioral interpretation is spurious may have a useful reduced form, isn't it true that when the spurious identification results in restrictions on the reduced form, the reduced form is distorted by the false identifying restrictions? The answer is yes and no. Yes, the reduced form will be infected by false restrictions and may thereby become useless as a framework within which to do formal statistical tests of competing macroeconomic theories. But no, the resulting infection need not distort the results of forecasting and policy analysis with the reduced form. Much recent theoretical work gives rigorous foundation for a rule of thumb that in high dimensional models restricted estimators can easily produce smaller forecast or projection errors than unrestricted estimators even when the restrictions are false. Of course very false restrictions will make forecasts worse, but in large macromodels restrictions very false in the sense of producing very bad reduced-form fits are probably usually detected and eliminated. Thus models whose self-proclaimed behavioral interpretation is widely disbelieved may nonetheless find satisfied users as tools of forecasting and policy projection.

Because existing large models contain too many incredible restrictions, empirical research aimed at testing competing macroeconomic theories too often proceeds in a single- or few-equation framework.[15] For this reason alone it appears worthwhile to investigate the possibility of building large models in a style which does not tend to accumulate restrictions so haphazardly. In addition, though, one might suspect that a more systematic approach to imposing restrictions could lead to capture of empirical regularities which remain hidden to the standard procedures and hence lead to improved forecasts and policy projections.[16]

[15] Modigliani [**20**] has used the MPS model as an arena within which to let macroeconomic theories confront each other, however.

[16] The work of Nelson [**21**] and Cooper and Nelson [**5**] provides empirical support for this idea.

Empirical macroeconomists sometimes express frustration at the limited amount of information in economic time series, and it does not infrequently turn out that models reflecting rather different behavioral hypotheses fit the data about equally well. This attitude may account for the lack of previous research on the possibility of using much less parsimoniously parameterized multiple-equation models. It might be expected that in such a model one would find nothing new except a relatively larger number of "insignificant" t statistics. Forecasts might be expected to be worse, and the accurate picture of the relation of data to theory one would obtain might be expected to be simply the conclusion that the data cannot discriminate between competing theories.

In the next section of this paper we discuss a general strategy for estimating profligately (as opposed to parsimoniously) parameterized macromodels, and present results for a particular relatively small-scale application.

2. AN ALTERNATIVE STRATEGY FOR EMPIRICAL MACROECONOMICS

It should be feasible to estimate large-scale macromodels as unrestricted reduced forms, treating all variables as endogenous. Of course, some restrictions, if only on lag length, are essential, so by "unrestricted" here I mean "without restrictions based on supposed a priori knowledge." The style I am suggesting we emulate is that of frequency-domain time series theory (though it will be clear I am not suggesting we use frequency-domain methods themselves), in which what is being estimated (e.g., the spectral density) is implicitly part of an infinite-dimensional parameter space, and the finite-parameter methods we actually use are justified as part of a procedure in which the number of parameters is explicitly a function of sample size or the data. After the arbitrary "smoothness" or "rate-of-damping" restrictions have been used to formulate a model which serves to summarize the data, hypotheses with economic content are formulated *and tested* at a second stage, with some perhaps looking attractive enough after a test to be used to further constrain the model. Besides frequency-domain work, such methods are implicit or explicit in much distributed lag model estimation in econometrics, and Amemiya [**1**] has proposed handling serial correlation in time domain regression models in this style.

The first step in developing such an approach is evidently to develop a class of multivariate time series models which will serve as the unstructured first-stage models. In the six-variable system discussed below, the data are accepting of a relatively stringent limit on lag length (four quarters), so that it proves feasible to use an otherwise unconstrained (144 parameter) vector autoregression as the basic model. In the larger systems one will eventually want to study this way, some additional form of constraint, beyond lag length or damping rate constraints, will be necessary. Finding the best way to do this is very much an open problem. Sargent and I [**28**] have published work using a class of restricted vector time series models we call index models in macroeconomic work, and I am currently working on applying those methods to systems larger than that explored below. Priestly, Rao, and Tong [**24**] in the engineering literature and Brillinger [**4**] have suggested related classes of models. All of these methods in one way or another