#47a P.S.

SPRING 2004

MACROECONOMIC THEORY COMPREHENSIVE EXAMINATION, SECOND CHANCE

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1) Suppose that a consumer maximizes the lifetime utility function

$$\sum_{t=1}^{\infty} \beta^t \ln(c_t)$$

subject to constraints in each period of the form:

$$a_{t+1} + c_t = ra_t,$$

given initial wealth. In each period, a represents wealth and r represents a constant real interest rate.

a) Guess the functional form for the value function.

b) Use the method of "guess and verify" (a.k.a. the method of undetermined coefficients) to solve for the parameters of the value function.

c) What is the "marginal propensity to consume" in the model?

d) What is the marginal lifetime utility from an additional unit of initial wealth *in mathematical terms*? Interpret the expression you derive in *economic terms*. Does it make sense (explain)?

e) Explain *in words* the alternatives to the method you used to solve for the value function.

2) An economy can produce consumption goods C only by depleting a non-renewable resource A. This is described by the equation

$$\dot{A} = -C$$

A central planner maximizes the function

$$\int_0^{\infty} e^{-\rho t} \ln(C_t) dt$$

subject to the depletion condition, the initial amount of the resource, and the constraint that $A_t \geq 0$ for all t.

a) Write down the Hamiltonian and the necessary conditions for optimality (include the usual transversality condition as a constraint)

b) What do these conditions imply about the costate in *mathematical terms*?

c) What do these conditions imply about optimal consumption in *economic terms*?

d) Solve mathematically for optimal consumption in all periods.

e) How could you introduce productivity growth into this economy? What would the growth rate of productivity need to be to completely offset the effects of resource depletion?

f) Can you generalize your ideas in e) to talk about "decreasing returns to scale"? If so, what economic factors could lead to DRS?

g) How does this parallel (to some extent) the idea of "increasing returns to scale"? What economic factors could lead to IRS?

3) Consider the Ramsey (Ramsey-Cass-Koopmans) model, assuming that:

$$U = \int_{t=0}^{\infty} e^{-\rho t}\, u(c_t)\, dt \quad \text{where} \quad u(c) = \frac{c^{1-\theta}}{1-\theta}$$

The rate of growth in TFP (total factor productivity) is g.

Recall the graph depicting the steady-state loci of the model (points where $\dot{c} = 0$ and $\dot{k} = 0$), the stable arm (saddle path), and the long-run steady state. Using such a graph, show what happens to the economy over time in response to each of the events described below. Assume that the economy is initially in a long-run steady state. Use the following symbols to label points:

(1) to label the point that is the initial LRSS before the event
(2) to label the point that is the combination of c and k immediately after the event
(3) to label a combination of c and k some time after the event, but before the new LRSS
(4) to label the point that is the new LRSS after the event.

a) A decrease in the rate of time-discount ρ

b) A decrease in θ

c) A terrorist attack that fortunately kills no one, but destroys a great deal of capital.

4) Consider an economy that is in a long-run steady state of the Solow growth model, with an exogenous savings rate that is NOT necessarily consistent with dynamic optimization of a utility function as in question 2) above. For each case below, state whether the capital stock is smaller than, equal to, or greater than the "golden rule" capital stock.

a) The real interest rate (return to capital) is 5 percent. The rate of depreciation is 2 percent. The rate of population growth is 2 percent. There is no growth in total factor productivity.

b) The real interest rate (return to capital) is 5 percent. The rate of depreciation is 2 percent. The rate of population growth is 1 percent. The rate of total factor productivity growth is 3 percent.

5) Suppose that the current inflation rate is 2 percent, output is equal to the natural rate of output, the natural rate of unemployment is 4 percent, and it is known that Alan Greenspan (the chairman of the American central bank) believes the optimal inflation rate is 2 percent. Today, it is announced that Greenspan will retire next year (one year from today), and his successor will be Ralph Nader, who hates unemployment and believes that the central bank should try to drive the unemployment rate down to one percent. Assuming that the public has rational expectations, what will happen to the rate of inflation over the next six months if:

a) Aggregate supply follows a Friedman-Phelps expectations-augmented Phillips curve

b) Aggregate supply follows a Rotemberg/Calvo "new Keynesian" Phillips curve.

c) Aggregate supply follows a Taylor model "new Keynesian" Phillips curve.

6) Consider an economy where

y is the output gap and $r_t = i_t - {}_t\pi^e_{t+1}$ is the real interest rate

$$y_t = -\beta_t(r_t - \bar{r}) + \varepsilon_t$$

$$\pi_t = {}_{t-1}\pi^e_t + \alpha y_t$$

The central bank sets the interest rate to minimize an "as if" loss function (that is, the central bank acts as if the desired output level were the natural rate of output):

$L = \frac{1}{2} E[y_t^2] + \frac{1}{2} E[(\pi_t - \pi^*)^2]$ where y is the output gap.

When the central bank sets i_t, it knows ${}_t\pi^e_{t+1}$.

Note that I have *not* made assumptions about the processes determining ε_t or β_t, or the central bank's information about those variables. Also, I have *not* specified that the economy is always in "rational expectations equilibrium."

a) Describe circumstances under which it would be true that the output gap is always zero, while the interest rate i varies from period to period. That is to say, list and describe in words a set of assumptions, *in addition to* those listed above, under which i would vary but y would always equal zero.

b) Describe circumstances under which it would be true that i_t is *negatively* correlated with y_t in a long time-series of data from the economy. That is to say, describe a set of assumptions, *in addition to* those listed above, under which i_t would appear to be negatively correlated with y_t across many years of data.

7) Suppose a country is subject to the problem of "dynamic inconsistency" in monetary policy. To mitigate the problem, the country appoints central bank governors for terms two periods long. The public does not know whether any newly appointed central bank governor is "normal," with preferences like those of the public with respect to unemployment and inflation, or a weird anti-inflation maniac, who doesn't care about unemployment at all.

You collect data from this economy over many years, and many central bank governors. You calculate average inflation rates and average output levels in the first periods of central bank governors' terms, and average inflation rates and average output levels in the *second* periods of central bank governors' terms.

a) Will average inflation rates for the first periods of governors' terms be higher than, lower than, or the same as average inflation rates in the second periods of central bank governors' terms?

b) Will average output levels for the first periods of central bank governors' terms be higher than, lower than, or the same as average output levels in the second periods of central bank governors' terms?

8) In the Caplin-Spulber model, the rate of growth of aggregate demand m varies over time, aggregate output is always equal to the natural rate, and the rate of price inflation is always equal to the rate of growth in m .

What assumptions about the stochastic process determining m - that is, assumptions about the behavior of the rate of growth of m - are crucial for this result?

9) Suppose you observe that there is a great deal of serial correlation in output gaps: that is, if the economy is in a recession this year, it is more likely to be in a recession next year; if it is booming this year, it is more likely to be booming next year.

a) Is this observation consistent with the "Lucas Supply Function"? Explain briefly.

b) Is this observation consistent with the "Taylor model" of predetermined prices with staggering? Explain briefly.

10) Consider a "new Keynesian" model where each firm has a monopoly in the production of a good g, and each household has a utility function:

$$U_i = C_i^{\alpha} (M/P)^{1-\alpha} - \frac{1}{\gamma} L_i^{\gamma} \quad where\ \alpha < 1\ ,\ \gamma > 1$$

and

$$C_i = \left(\sum_{g=1}^{T} C_{gi}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \quad where \quad \eta > 1$$

a) Does an increase in the parameter η tend to increase, decrease, or have no effect on the *natural rate* of output? Explain briefly.

b) Does an ~~increase~~ increase in the parameter γ tend to increase, decrease, or have no effect on the *natural rate* of output? Explain briefly.

#47 PS

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MACROECONOMIC THEORY COMPREHENSIVE EXAMINATION

C. Hanes/R. Britto/B. Jones

1)

a) A firm maximizes

$$\int_0^{\infty} e^{-Rs}\left(PF(K(s)) - q_1 I(s) - \frac{q_2}{2} I^2(s) \right) ds$$

subject to the constraint:

$$\dot{K} = I - \delta K,$$

where $F(K) = b_1 K - \frac{b_2}{2} K^2$ and $b_1, b_2, q_1, q_2 > 0$. The control set for I is the set of positive real numbers.

i) Provide an economic interpretation of the optimization. Specifically, you must address the interpretation of the integrand of the objective function.
ii) Write down the Hamiltonian and state the necessary conditions for optimality. Are these conditions also sufficient? Intrepret the necessary conditions in economic terms *as best you can.*
iii) Define $\mu(t) = \lambda(t)e^{Rt}$, where $\lambda(t)$ is the costate variable. Derive a two-dimensional (linear) differential equation system for (μ, K).
iv) Derive steady state values of μ^*, K^*, I^* and interpret them.

b) Now suppose that $q_2 = 0$ and let the control set for I be $[0, \bar{I}]$.

i) Write down the Hamiltonian and state the necessary conditions for optimality.
ii) If K is in steady state, what does this imply about I?
iii) If I behaves as you specified in *ii*), what does it imply about the costate variable?
iv) If the costate variable behaves as you specified in *iii*), what does it imply about $F'(K)$ in steady state? Interpret your result.
v) Compare and contrast the steady state results in Part a) and Part b).

2) Consider the Ramsey (Ramsey-Cass-Koopmans) model, assuming that:

$$U = \int_{t=0}^{\infty} e^{-\rho t}\, u(c_t)\, dt \quad \text{where} \quad u(c) = \frac{c^{1-\theta}}{1-\theta}$$

The rate of growth in TFP (total factor productivity) is g.

Recall the graph depicting the steady-state loci of the model (points where $\dot{c} = 0$ and $\dot{k} = 0$), the stable arm (saddle path), and the long-run steady state. Using such a graph, show what happens to the economy over time in response to each of the events described below. Assume that the economy is initially in a long-run steady state. Use the following symbols to label points:

(1) to label the point that is the initial LRSS before the event
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(3) to label a combination of c and k some time after the event, but before the new LRSS
(4) to label the point that is the new LRSS after the event.

a) A one-time upward jump (increase) in TFP, with no change in the growth rate g

b) An increase in the rate of time-discount ρ

c) An increase in θ

3) Consider the "golden rule" savings rate and capital stock for an economy. Explain why it is possible for an economy to have a capital stock greater than the golden rule capital stock in the Solow model, but not in the Ramsey (RCK) model.

4) In all of the following models, the market economy is generally "inefficient." Pick ***just one*** of the following models, and explain why the market economy is inefficient in that model.

a) The Sidrauski model

b) A "spillover" model such as the one described by Romer

c) An R&D model such as the one described by Charles Jones (in his textbook)

5) How is a "New Keynesian" Phillips curve *different from* an Friedman-Phelps expectations-augmented Phillips curve?

6) How is a "New Keynesian" Phillips curve derived from the Taylor model different from a New Keynesian Phillips curve derived from the Rotemberg or Calvo model?

7) Consider an economy where

y is the output gap and $r_t = i_t - {}_t\pi^e_{t+1}$ is the real interest rate

$$y_t = -\beta_t(r_t - \bar{r}) + \varepsilon_t$$

$$\pi_t = {}_{t-1}\pi^e_t + \alpha y_t$$

The central bank sets the interest rate to minimize an "as if" loss function (that is, the central bank acts as if the desired output level were the natural rate of output):

$L = \frac{1}{2} E[y_t^2] + \frac{1}{2} E[(\pi_t - \pi^*)^2]$ where y is the output gap.

When the central bank sets i_t, it knows ${}_t\pi^e_{t+1}$.

The economy is always in "rational expectations equilibrium."

Note that I have *not* made assumptions about the processes determining ε_t or β_t, or the central bank's information about those variables.

a) Describe circumstances under which it would be true that the interest rate i varies from period to period, and the output gap y varies from period to period, but i_t is uncorrelated with y_t. That is to say, list and describe in words a set of assumptions, *in addition to* those listed above, under which i and y vary and i_t would be uncorrelated with y_t.

b) Describe circumstances under which it would be true that i_t is positively correlated with y_t. That is to say, describe a set of assumptions, *in addition to* those listed above, under which i_t would be positively correlated with y_t.

8) Assume that the desired inflation rate π^* is the same for all people in the world.

a) Suppose that all countries are equally subject to the problem of "dynamic inconsistency" in monetary policy - that is, all central bankers aim to minimize the same loss function, with a desired level of output that is greater than the natural rate of output. What economic parameter or parameters would determine a country's long-run average inflation rate?

b) Suppose that all countries in the world are subject to rulers who set money growth so as to maximize long-run steady state revenue from seigneurage. What economic parameter or parameters would determine a country's long-run average inflation rate?

9) Consider an economy where

y is the output gap and $r_t = i_t - \pi^e_{t+1}$ is the real interest rate

$$y_t = -\beta(r_t - \bar{r}) + \varepsilon_t$$

$$\pi_t = \pi^e_{t-1} + \alpha y_t$$

The central bank sets the interest rate to minimize an "as if" loss function (that is, the central bank acts as if the desired output level were the natural rate of output):

$L = \frac{1}{2} E[y_t^2] + \frac{1}{2} E[(\pi_t - \pi^*)^2]$ where y is the output gap.

The central bank does not know the value of ε_t when it sets i_t, but it *does* know the values of α and β. It has forecasts - rational expectations - for π_t and y_t.

a) Under what circumstances will the central bank's forecast for π_t be equal to π^* ?

b) Under what circumstances will the central bank's forecast for π_t be greater than π^*? Under these circumstances, will the central bank's forecast for the output gap y_t be greater than, less than, or equal to zero?

10) Consider two economies, A and B, described by the Lucas supply function model.

In economy A, there is no central bank. Aggregate demand m evolves as $m_t - m_{t-1} = \bar{x} + \varepsilon_t$, where $\bar{x}$ is a constant and ε is a serially-correlated random variable:
$\varepsilon_t = \rho\, \varepsilon_{t-1} + e_t$ *where* $0<\rho<1$ and e is mean-zero, i.i.d.

At time *t-1*, when the public forms its expectation of period *t*'s aggregate demand m_t, the public has observed ε_{t-1} and m_{t-1}, but not the upcoming value of ε_t or e_t.

Economy B is identical to A except that in B there is a central bank. Aggregate demand is

$m_t - m_{t-1} = x_t^{CB} + \bar{x} + \varepsilon_t$ where the central bank sets x_t^{CB} at time *t-1*. At time *t-1*, the central bank has observed ε_{t-1} and m_{t-1}, but not the upcoming value of ε_t or e_t.

The central bank in economy B acts to minimize an "as if" loss function (that is, the central bank acts as if the desired output level were the natural rate of output):

$L = \frac{1}{2} E[y_t^2] + \frac{1}{2} E[(\pi_t - \bar{x})^2]$ where y is the output gap and $\pi_t = p_t - p_{t-1}$.

a) What is the value of x_t^{CB} that minimizes the loss function of the central bank in economy B?

b) Is the variance of output greater in A, greater in B, or the same in both economies?

c) Is the variance of inflation π greater in A, greater in B, or the same in both economies?

11) Consider this household utility function:

$$U_i = C_i^{\alpha} (M/P)^{1-\alpha} - \frac{1}{\gamma} L_i^{\gamma} \quad \textit{where } \alpha < 1 \, , \, \gamma > 1$$

and

$C_i = \left(\sum_{g=1}^{T} Z_g^{\frac{1}{\eta}} C_{gi}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}$ is a CES aggregator of the household's consumption of individual goods.

Note that there are three parameters in this utility function: α , γ , η . This household utility function can be part of a "Lucas Supply Function" model. It can also be part of a "New Keynesian" model, in which case one can assume $Z = 1$ for all g.

a) Suppose this utility function is part of a Lucas Supply Function model. Does an *increase* in the value of the parameter γ tend to increase, decrease, or have no effect on the variance of aggregate real output y ?

b) If this utility function is part of a New Keynesian model, it is necessary to assume that $\eta > 1$. If the utility function is part of a Lucas Supply Function model, we need only assume that $\eta > 0$. Explain.

c) Suppose this utility function is part of a New Keynesian model where the output of a firm j is equal to the labor input to the firm: $q_j = l_j$, and the labor market is perfectly competitive. We want to know whether a small "menu cost" of price adjustment can keep the economy in a "fixed price equilibrium." Which parameter (or parameters) of the household utility function are relevant for this question? Explain briefly.

d) Again suppose this utility function is part of a New Keynesian model where the output of a firm j is equal to the labor input to the firm: $q_j = l_j$, and the labor market is perfectly competitive. Which parameter (or parameters) of the household utility function determine the long-run equilibrium real wage $\frac{W}{P}$ (in logs, $w - p$) ? Explain briefly.