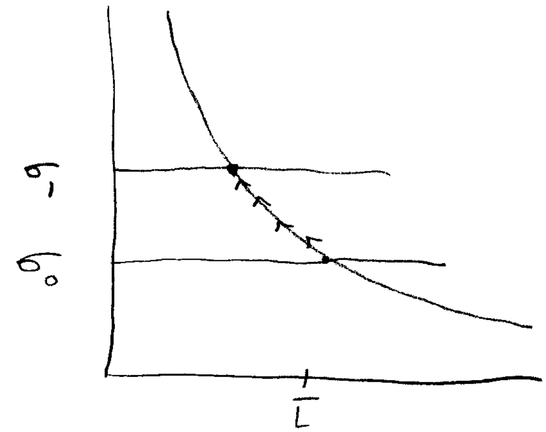
ANSWER ALL QUESTIONS 3 THROUGH 9

3) Consider the Malthusian model of population and economic growth. The aggregate production function is $Y = AF(L)$ where Y is output and L is labor input, equivalent to population. The "subsistence" level of consumption or real wage is denoted σ. Assume the economy is initially in a state of long-run equilibrium with a stable population $\bar{L}$.

Suppose the economy experiences an increase in real wages, accompanied by a falling population. Eventually the population stabilizes at a lower level, while the real wage remains higher than it was initially. How could you explain this using the model - for example, as a result of an exogenous shock to population L or the parameter A or the parameter σ? Use a graph to illustrate your answer. *Highest grade: 5 pts. This event can only be explained as a result of an increase in the subsistence level of consumption of real wage σ. A shock to any other parameter, or to L, is irrelevant.*



4) Consider the Ramsey (Ramsey-Cass-Koopmans) model, assuming that:

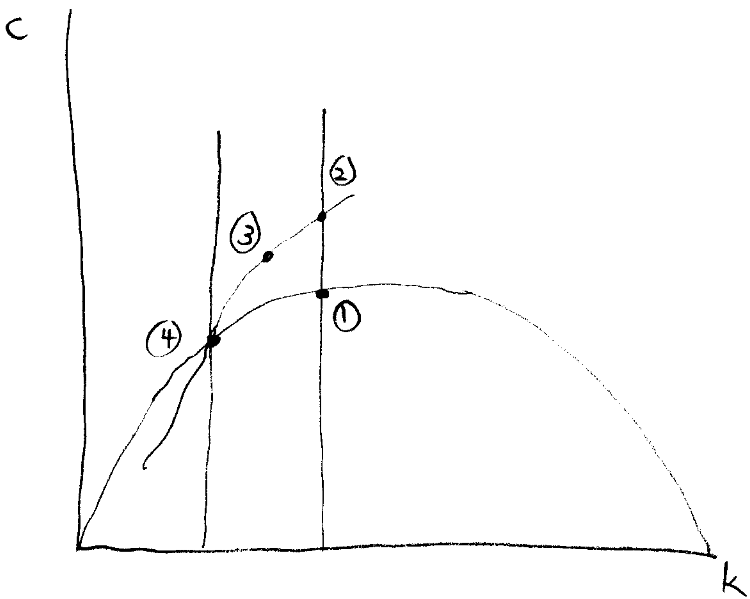
$$U = \int_{t=0}^{\infty} e^{-\rho t} u(c_t)\, dt \quad \text{where} \quad u(c) = \frac{c^{1-\theta}}{1-\theta}$$

The rate of growth in TFP (total factor productivity) is g.

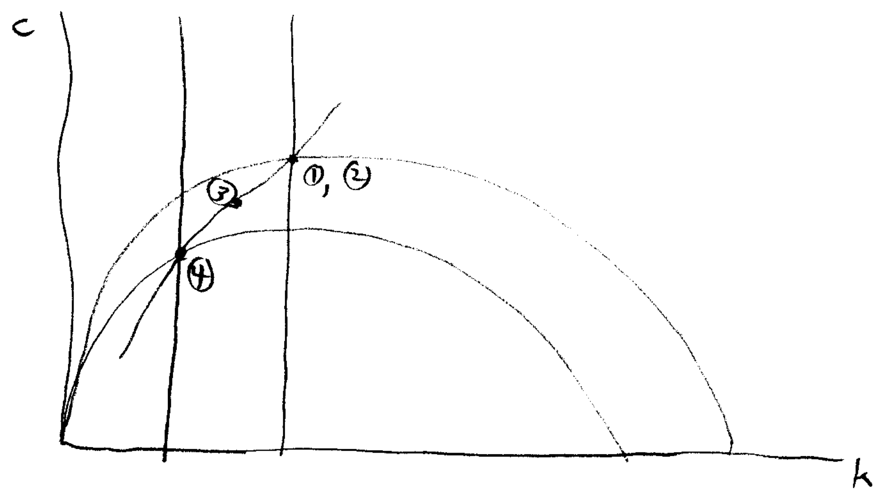
Recall the graph depicting the steady-state loci of the model (points where $\dot{c} = 0$ and $\dot{k} = 0$), the stable arm (saddle path), and the long-run steady state. Using such a graph, show what happens to the economy over time in response to each of the events described below. Assume that the economy is initially in a long-run steady state. Use the following symbols to label points:

(1) to label the point that is the initial LRSS before the event
(2) to label the point that is the combination of c and k immediately after the event
(3) to label a combination of c and k some time after the event, but before the new LRSS
(4) to label the point that is the new LRSS after the event.

a) An increase in the rate of time-discount ρ. *3 pts. The economy jumps up to the new saddle path, then crawls to the new LRSS. A key point is that point 2) CANNOT be to the left (or right) of point 1): the capital stock can't jump.*



b) An increase in the rate of growth of TFP *g*. *3 pts. Again, the economy jumps to the new saddlepath. The new saddlepath may be above, below, or on the old LRSS point. The particular way I have drawn it, the new saddle path is on the old LRSS point.*



5) Consider a closed-economy IS-LM model with a fixed price level and a fixed money supply *M*, described by the following expressions:

$\frac{M}{P} = L(i,Y)$ *where* $L_i<0,\ L_Y>0$

$Y = E(Y,r,G,T)$ *where* $0 < E_Y < 1\ ,\ E_r < 0$ *and* $r = i - \pi^e$

Consider the effect of an exogenous change in expected inflation π^e.

Derive an expression showing the effect of a change in π^e on output *Y*, and an expression showing the effect of a change in π^e on the nominal interest rate *i*.

5 pts. A key point here is that $\frac{\partial r}{\partial \pi^e} = \frac{\partial i}{\partial \pi^e} + \frac{\partial r}{\partial \pi^e} = \frac{\partial i}{\partial \pi^e} - 1$

$$\frac{M}{P} = L(i, Y)$$

$$Y = E(Y, r, G, T)$$

$$0 = L_i \frac{\partial i}{\partial \pi^e} + L_Y \frac{\partial Y}{\partial \pi^e}$$

$$\frac{\partial Y}{\partial \pi^e} = E_Y \frac{\partial Y}{\partial \pi^e} - E_r + E_r \frac{\partial i}{\partial \pi^e}$$

$$\frac{\partial i}{\partial \pi^e} = -\frac{L_Y}{L_i} \frac{\partial Y}{\partial \pi^e}$$

$$(1 - E_Y)\frac{\partial Y}{\partial \pi^e} = -E_r + E_r \frac{\partial i}{\partial \pi^e}$$

$$\frac{\partial Y}{\partial \pi^e} = \frac{-E_r}{1 - E_Y} + \frac{E_r}{1 - E_Y} \frac{\partial i}{\partial \pi^e}$$

$$\frac{\partial Y}{\partial \pi^e} = -\frac{E_r}{1 - E_Y} + \frac{E_r}{1 - E_Y}\left(-\frac{L_Y}{L_i}\right) \frac{\partial Y}{\partial \pi^e}$$

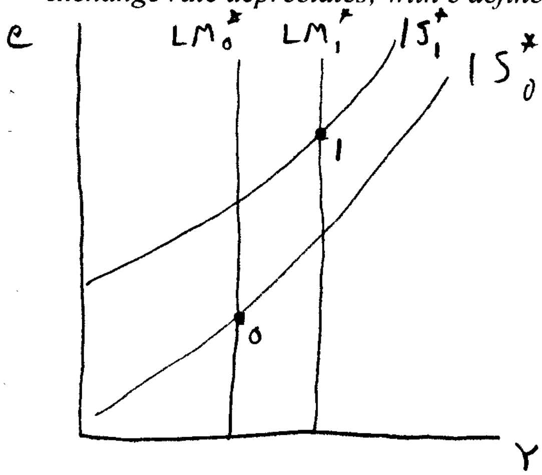
$$\vdots$$

$$\frac{\partial Y}{\partial \pi^e} = \frac{-E_r}{1 - E_Y + E_r \frac{L_Y}{L_i}} = -\frac{1}{\frac{1 - E_Y}{E_r} + \frac{L_Y}{L_i}} > 0$$

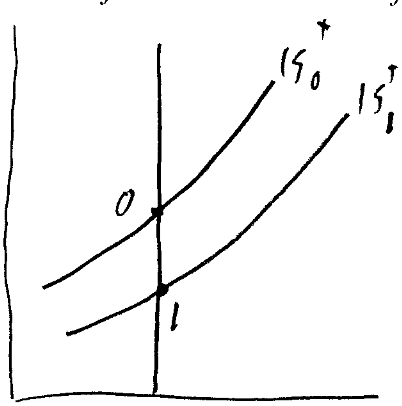
$$\frac{\partial i}{\partial \pi^e} = -\frac{L_Y}{L_i}\left(-\frac{1}{\frac{1 - E_r}{E_r} + \frac{L_Y}{L_i}}\right) = \frac{\frac{L_Y}{L_i}}{\frac{1 - E_r}{E_r} + \frac{L_Y}{L_i}} = \frac{1}{1 + \frac{L_i}{L_Y}\frac{1 - E_r}{E_r}} > 0$$

6) Consider an open economy with perfect capital mobility (Mundell-Fleming model), "static exchange rate expectations" ($\dot{\varepsilon}^e/\varepsilon = 0$, always), a fixed domestic price level P, a fixed foreign price level P^*, a fixed domestic money supply M, and a *floating* exchange rate.

a) Using a graph, illustrate the effect on output and the exchange rate of an increase in the foreign interest rate i^*. *3 pts. Recall that (under the specified conditions) the domestic interest rate equals the foreign interest rate, so the hike in i* means a hike in i. With the supply of real balances fixed, the hike in i must be accompanied by an increase in Y (money demand depends on i and Y; if i is higher Y must be bigger in order to keep money demand equal to the fixed supply). So the LM* curve shifts out. At the same time, the hike in i means a higher real interest rate, which shifts the IS* curve in (output depends on the real interest rate and the real exchange rate; at any given real exchange rate, output would be lower). Hence Y increases while the exchange rate depreciates; with e defined as domestic currency units/foreign, e rises.*



b) Using a graph, illustrate the effect on output and the exchange rate of a positive "IS shock," that is, an increase in the level of domestic real output that would be purchased at any given real interest rate and real exchange rate. *3 pts. With the interest rate and real balances fixed, the LM* curve remains fixed. The IS* curve shifts out. Hence e falls (currency appreciates).*



7) Consider an economy where y is the output gap (the difference between log output and log potential output) and $r_t = i_t - {}_t\pi^e_{t+1}$ is the real interest rate. Inflation follows a Friedman-Phelps Phillips curve:

$$\pi_t = {}_{t-1}\pi^e_t + \alpha y_t$$

The economy has a central bank. The central bank observes $_t\pi^e_{t+1}$ and $_{t-1}\pi^e_t$, and sets the nominal interest rate i_t to minimize a loss function.

a) Suppose the central bank's loss function is:

$$L = \frac{1}{2} E[(y_t - \varphi)^2] + \frac{1}{2} E[(\pi_t - \pi^*)^2] \quad \text{where } \varphi \text{ is some positive number.}$$

The output gap is determined by $y_t = -\beta(r_t - \bar{r})$. The central bank knows the value of β.

What will be the inflation rate π, the output gap y, and the nominal interest rate i in "rational expectations equilibrium," that is assuming the public is rational and knows the true structure of the economy and preferences of the central bank? *4 pts. This is Kydland-Prescott dynamic inconsistency. The easiest way to do this one is to describe the central bank as choosing π directly, as we did in discussing that model (one can do this because the central bank has no uncertainty). First, take π^e as given and find the value of y or π that minimizes the central bank's loss function. Then, to describe rational expectations equlibrium, set $\pi^e = \pi$, which implies $y = 0$ and $r = \bar{r}$. And $i = \pi^e + \bar{r}$.*

$$L = \frac{1}{2}\left(\frac{1}{\alpha}(\pi - \pi^e) - \phi\right)^2 + \frac{1}{2}(\pi - \pi^*)^2$$

$$\frac{\partial L}{\partial \pi} = 0 = \left(\frac{1}{\alpha}(\pi - \pi^e) - \phi\right)\frac{1}{\alpha} + (\pi - \pi^*)$$

gives

$$\pi = \frac{1}{1+\alpha^2}\pi^e + \frac{\alpha}{1+\alpha^2}\phi + \frac{\alpha^2}{1+\alpha^2}\pi^* = \frac{1}{1+\alpha^2}(\pi^e - \pi^*) + \frac{\alpha}{1+\alpha^2}\phi + \pi^*$$

Setting $\pi = \pi^e$,

$$\pi = \pi^* + \frac{1}{\alpha}\phi, \quad y = 0, \quad r = \bar{r}$$

$$i = \bar{r} + \pi^e = \bar{r} + \pi = \bar{r} + \pi^* + \frac{1}{\alpha}\phi$$

b) Suppose again that output in the economy is given by $y_t = -\beta(r_t - \bar{r})$ and the central bank knows the value of β. But the central bank's loss function is instead:

$$L = \frac{1}{2} E[y_t^2] + \frac{1}{2} E[(\pi_t - \pi^*)^2]$$

and the public's expectations are *not* rational. Instead, in every period,

$_{t-1}\pi_t^e = {}_t\pi_{t+1}^e = \pi^* + \varepsilon_t$ where ε is an i.i.d. (no serial correlation), mean-zero random variable.

i) What will be π , y, and i in a period t?

ii) Consider correlations between these variables.
Will the period-by-period correlation between i_t and y_t be positive, negative, or zero?
Will the period-by-period correlation between π_t and y_t be positive, negative, or zero?

4 pts. *The easiest way to do this one is to describe the central bank as choosing π or y directly (again the central bank has no uncertainty). Take $\pi^e = \pi^* + \varepsilon$ as given and find the value of π or y that minimizes the central bank's loss function. Then find the value of y that corresponds to that value of π, or vice-versa. Then find the value of r that corresponds to that value of y. Then* $i = r + \pi^* + \varepsilon$.

$$L = \frac{1}{2}\left(\frac{1}{\alpha}(\pi - \pi^* - \varepsilon_t)\right)^2 + \frac{1}{2}(\pi - \pi^*)^2$$

$$\frac{\partial L}{\partial \pi} = 0 = \frac{1}{\alpha}(\pi - \pi^* - \varepsilon_t)\frac{1}{\alpha} + (\pi - \pi^*)$$

gives

$$\pi_t = \pi^* + \frac{1}{1+\alpha^2}\varepsilon_t$$

$$y_t = \frac{1}{\alpha}(\pi - \pi^e) = \frac{1}{\alpha}\left(\pi^* + \frac{1}{1+\alpha^2}\varepsilon_t - \pi^* - \varepsilon_t\right)$$

$$= -\frac{\alpha}{1+\alpha^2}\varepsilon_t$$

$$r_t - \bar{r} = \frac{-1}{\beta} y_t = \frac{1}{\beta}\frac{\alpha}{1+\alpha^2}\varepsilon_t$$

$$i_t = \bar{r} + \frac{1}{\beta}\frac{\alpha}{1+\alpha^2}\varepsilon_t + \pi^* + \varepsilon_t$$

$$= \bar{r} + \pi^* + \left(1 + \frac{\alpha}{\beta(1+\alpha^2)}\right)\varepsilon_t$$

c) Suppose again that the central bank's loss function is

$$L = \frac{1}{2} E[y_t^2] + \frac{1}{2} E[(\pi_t - \pi^*)^2]$$

But now the output gap in the economy is given by:

$$y_t = -(\beta + g_t)(r_t - \bar{r}) + h_t$$

g and h are two uncorrelated, i.i.d. mean-zero random variables. The public observes neither g nor h. The central bank observes the realized value of h_t before it sets i_t, but it does not observe g: all it knows is that the variance of g is equal to σ_g^2. Finally, assume that the value of α in the Phillips curve is one (1).

i) What will be π , y, and i in a period t, in "rational expectations equilibrium," that is assuming the public is rational and knows the true structure of the economy and preferences of the central bank?

ii) Consider correlations between these variables.

Will the period-by-period correlation between i_t and y_t be positive, negative, or zero?

Will the period-by-period correlation between π_t and y_t be positive, negative, or zero?

4 pts. This is about "multiplier" or "Brainard" uncertainty. It can be solved easily using a little intuition, or grimly cranked out. Intuition: the central bank is aiming for a zero output gap, expectations are rational, and there is no lag in the effect of the interest rate on output, so you know (from what we did in class) that in the absence of Brainard uncertainty the central bank would entirely counteract h in every period, keeping the output gap at zero and inflation at the target inflation rate; expected inflation would always be equal to π^*. *You should guess that with Brainard uncertainty, the central bank will under-respond to h (the shock it observes), so that a high value of h will be associated with high output and inflation greater than expected inflation. It is still true, however, that expected inflation always equals* π^*, *since the public can't observe h. Based on this intuition, without solving for precise values of anything, you can say that the correlation between* $\square_t$ *and* y_t *is positive, and the correlation between* π_t *and* y_t *is positive.*

To solve: conjecture that $\pi^e = \pi^*$. *Solve for the optimal value of r as a function of h. Applying that r, get y and* π_t. *Finally, check the original conjecture that* $\pi^e = \pi^*$

Assuming $\pi^e = \pi^*$, $\pi = \pi^* + \alpha y$

$$L = \frac{1}{2} E[y_t^2] + \frac{1}{2} E[(\pi^e + \alpha y - \pi^*)^2]$$

$$= \frac{1}{2}(1+\alpha^2) E[y^2] = \frac{1}{2}(1+\alpha^2)\left((E[y])^2 + Var(y)\right)$$

$$= \frac{1}{2}(1+\alpha^2)\left((-\beta(r-\bar{r})+h)^2 + \sigma_g^2 (r-\bar{r})^2\right)$$

$$\frac{\partial L}{\partial (r-\bar{r})} = 0 = \frac{1}{2}(1+\alpha^2)\left((-\beta(r-\bar{r})+h)(-\beta) + \sigma_g^2(r-\bar{r})\right)$$

hence

$$r - \bar{r} = \frac{\beta h}{\beta^2 + \sigma_g^2}$$

$$y_t = -(\beta+g)\frac{\beta h}{\beta^2+\sigma_g^2} + h = \left(1 - \frac{\beta^2}{\beta^2+\sigma_g^2} - \frac{\beta}{\beta^2+\sigma_g^2} g\right) h$$

$$\pi_t = \pi^* + \alpha\left(1 - \frac{\beta^2}{\beta^2-\sigma_g^2} - \frac{\beta}{\beta^2+\sigma_g^2}\right) h \Leftarrow \left(\text{Since } g \,\&\, h \text{ uncorrelated, } E[\pi_t] = 0!\right)$$

8) Suppose one regressed the change in inflation $(\pi_{t+1} - \pi_t)$, on the output gap in period *t*, with a regression equation like $(\pi_{t+1} - \pi_t) = \beta y_t + \varepsilon_t$ where ε is the residual in the regression.

a) What sign would you expect to find for β - positive, negative, or zero - assuming that inflation is determined by a Calvo-style New-Keynesian Phillips curve and the public's inflation expectations are rational? Explain. 3 pts. $\hat{\beta}$ *must be negative (when output is high, inflation is falling.) See your notes from class.*

b) What sign would you expect to find for β - positive, negative, or zero - assuming that inflation is determined by the Lucas supply function, and the public's expected value for inflation is always equal to the central bank's inflation target π^*? Explain. *3 pts. Again,* $\hat{\beta}$ *must be negative. Under the Lucas supply function and rational expectations, there is no serial correlation in y, that is* y_{t+1} *is independent of* y_t, *or* $y_{t+1} - y_t = e_{t+1} - e_t$ *where e is i.i.d. If* $\pi^e = \pi^*$, *then the LSF is* $\square_t = \varphi(\pi_t - \pi^*)$, *that is, there is no serial correlation in inflation either. Thus, if output and inflation are high this period, they are likely to be lower next period. . Or,* $\pi_{t+1} - \pi_t = \frac{1}{\varphi}(y_{t+1} - y_t) = \frac{1}{\varphi}(e_{t+1} - e_t)$. *The regression of the change in inflation on output is thus a regression of* $\frac{1}{\varphi}(e_{t+1} - e_t)$ *on* e_t. *The coefficient on* e_t *will be* $\hat{\beta} = -\frac{1}{\varphi} < 0$.

9) Consider an economy with perfectly competitive labor markets and an upward-sloping aggregate labor supply curve (as a function of the real wage). Aggregate demand is determined by $y = m - p$, where *y* is the log of aggregate output, *p* is the log price level and *m* is a variable like the log "money supply." Output is produced by identical price-setting monopoly firms. Each firm produces a differentiated product, using labor as an input. Consider the size of the "menu cost" needed to maintain a "fixed-price equilibrium" in the face of an unexpected variation in *m*. Is the required menu cost bigger if each firm's production function looks like: $Y_i = AL_i$ (production function one)
or if each firm's production function looks like: $Y_i = AL_i^\alpha$ *where* $0 < \alpha < 1$
(production function two)?

Note that these possible production functions are *not* in log terms. Explain your answer.
5 pts. Recall Romer's discussion of real rigidity and the equation $p_i^* = p + c + \varphi y$ *(Romer's 6.45),*

where p_i is an individual firm's optimal price apart from menu costs, and p is the price level, that is the average of prices charged by all other firms. The bigger is φ, the larger is the menu cost required to support a fixed-price equilibrium. Recall also that, in general, the value of φ can reflect:

- the relation between output and the profit-maximizing markup over marginal cost
- the relation between output and marginal cost at a given the cost of labor w, that is the slope of the short-run marginal cost curve
- the relation between output and the real wage w - p that clears the labor market.

This question is about the second of these three factors. Production function one implies the marginal product of labor is constant. At a given wage w, a firm's marginal cost is constant. In the positive value of φ would then reflect the following: higher output means higher employment; higher employment requires a higher real wage because of the competitive labor market and upward-sloping labor supply; a higher real wage means higher w at given p. In production function two, there is diminishing marginal product of labor, which implies:
- at given w, marginal cost is inrecasing with output.
- as y increases, labor demand increases more than one-to-one, so (given upward-sloping labor supply) the real wage must increase faster with y.

For both of these reasons, φ must be bigger with production function two. Hence the required menu cost is bigger for production function number two.
a firm's optimal price p rises with output*