COMP QUESTIONS, MACRO RETAKE, SPRING 2005
Answer all questions.

1. A useful utility function in microeconomic theory is the constant relative risk aversion utility function. It is, for example, commonly used in models of insurance and gambling. The same utility function is also widely used in macroeconomics, but for a different purpose. Explain carefully how the function is used in macroeconomics and explain in detail the macroeconomic interpretation given to the coefficient of relative risk aversion.

2. New growth theory has been concerned with the development of improvements in productivity as the result of research and development (R&D). First, explain how Romer modeled productivity in his groundbreaking paper on R&D models. Second, explain the structure of his model in terms of different "sectors" of the economy. Third, explain the efficiency implications of the model. Finally, explain how R&D models improved/modified upon the earlier external economies of scale approach.

3. A commonly used alternative to the money-in-the-utility function approach is the so-called cash-in-advance approach. We can represent this approach as a standard Ramsey model, but with an added constraint. Some constant percentage of consumption, B, must be financed directly with currency, m. Thus, the consumer maximizes

$$\int_0^\infty e^{-\rho t} U\left(c(t)\right) dt$$

subject to

$$\dot{k} + \dot{m} = f(k) - c - \delta k + \tau - \pi m \quad [n = 0 \text{ by assumption}]$$

and

$$c = Bm$$

where π is the rate of inflation and τ is a money transfer from the central bank. The agent takes π and τ to be exogenous, but not constant. All variables are in real, per-capita terms, just like in class.

A) Do a substitution to get rid of the equality constraint. Do another one to get a single state variable.
B) Write down the Hamiltonian.
C) Write down the necessary conditions for optimality using the maximum principle.
D) Interpret (in economic terms) the intra-temporal optimality conditions.
E) Assuming that $\tau = \sigma m$ (our standard assumption from class), derive the steady state equations for the model.
F) Explain the super-neutrality properties of the model (if any).
G) What "real world" factors do you think should affect B.

ANSWER ALL QUESTIONS

1) Consider the Ramsey (Ramsey-Cass-Koopmans) model, assuming that:

$$U = \int_{t=0}^{\infty} e^{-\rho t}\, u(c_t)\, dt \quad \text{where} \quad u(c) = \frac{c^{1-\theta}}{1-\theta}$$

The rate of growth in TFP (total factor productivity) is g.

Recall the graph depicting the steady-state loci of the model (points where $\dot{c} = 0$ and $\dot{k} = 0$), the stable arm (saddle path), and the long-run steady state. Using such a graph, show what happens to the economy over time in response to each of the events described below. Assume that the economy is initially in a long-run steady state. Use the following symbols to label points:

(1) to label the point that is the initial LRSS before the event
(2) to label the point that is the combination of c and k immediately after the event
(3) to label a combination of c and k some time after the event, but before the new LRSS
(4) to label the point that is the new LRSS after the event.

a) An increase in θ.

b) A sudden in-migration of workers who arrive with no capital, but have exactly the same preferences and behavior as the natives.

2) Consider a closed-economy IS-LM model with a fixed price level and a fixed money supply M, described by the following expressions:

$$\frac{M}{P} = L(i,Y,H) \quad \textit{where} \quad L_i<0,\ L_Y>0, L_H>0$$

$$Y = E(Y,r,G,T,H) \quad \textit{where}\ 0 < E_Y < 1\ ,\ E_r < 0\ ,\ E_H > 0 \quad \textit{and} \quad r = i - \pi^e$$

Note the new variable H: it denotes the average annual temperature (heat).

a) Derive expressions showing the effect of a change in H on output Y and the nominal interest rate i, assuming expected inflation remains fixed.

b) Does an increase in H tend to increase output Y, reduce Y, have no effect on Y, or are you unable to say? Explain your answer.

c) Does an increase in H tend to increase the interest rate i, reduce i, have no effect on i, or are you unable to say? Explain your answer.

3) Consider an open economy with *imperfect* capital mobility (*not* the Mundell-Fleming model), "static exchange rate expectations" ($\dot{\varepsilon}^e/\varepsilon = 0$, always), a fixed domestic price level P, a fixed foreign price level P^*, a fixed domestic money supply M, and a *floating* exchange rate.

a) Using a graph, illustrate the effect on output and the nominal interest rate i of an increase in the foreign interest rate i^*. Also, state clearly what happens to the exchange rate e - does the country's currency appreciate or depreciate?

b) Using a graph, illustrate the effect on output and the nominal interest rate i of a positive "IS shock," that is, an increase in the level of domestic real output that would be purchased at any given real interest rate and real exchange rate. Also, state clearly what happens to the exchange rate e - does the country's currency appreciate or depreciate?

4) Consider an economy where

y is the output gap and $r_t = i_t - {}_t\pi^e_{t+1}$ is the real interest rate

$$y_t = -\beta_t(r_t - \bar{r}) + \varepsilon_t$$

$$\pi_t = {}_{t-1}\pi^e_t + \alpha y_t$$

The central bank sets the interest rate to minimize an "as if" loss function (that is, the central bank acts as if the desired output level were the natural rate of output):

$L = \frac{1}{2} E[y_t^2] + \frac{1}{2} E[(\pi_t - \pi^*)^2]$ where y is the output gap.

When the central bank sets i_t, it knows ${}_t\pi^e_{t+1}$.

The economy is always in "rational expectations equilibrium."

Note that I have *not* made assumptions about the processes determining ε_t or β_t, or the central bank's *information* about those variables.

a) Describe circumstances under which it would be true that the interest rate i varies from period to period, and the output gap y varies from period to period, but i_t is uncorrelated with y_t. That is to say, list and describe in words a set of assumptions, *in addition to* those listed above, under which i and y vary and i_t would be uncorrelated with y_t.

b) Describe circumstances under which it would be true that i_t is positively correlated with y_t. That is to say, describe a set of assumptions, *in addition to* those listed above, under which i_t would be positively correlated with y_t.

5) Suppose that the current inflation rate is 2 percent and has been 2 percent for a while. Output is equal to the natural rate of output, and it is known that Alan Greenspan (the chairman of the American central bank) believes the optimal inflation rate is 2 percent. Today, it is suddenly and surprisingly announced that Greenspan has retired and his successor will aim for a *higher* rate of inflation, 4 per cent. Explain what will happen over the next few years to:

- inflation
- output
- the nominal interest rate

assuming that:

a) Aggregate supply follows a Friedman-Phelps expectations-augmented Phillips curve, and the public's expectations are *not* rational, but rather backward-looking: expected inflation is always equal to recently experienced inflation.

b) Aggregate supply follows a Rotemberg/Calvo "new Keynesian" Phillips curve, and the public's expectations are rational.

6) In the Caplin-Spulber model, the rate of growth of aggregate demand m varies over time, aggregate output is always equal to the natural rate, and the rate of price inflation is always equal to the rate of growth in m.

What assumptions about the stochastic process determining m - that is, assumptions about the behavior of the rate of growth of m - are crucial for this result?

7) Consider an economy with perfectly competitive labor markets and an upward-sloping aggregate labor supply curve (as a function of the real wage). Aggregate demand is determined by $y = m - p$, where y is the log of aggregate output, p is the log price level and m is a variable like the log "money supply." Output is produced by identical price-setting monopoly firms. Each firm produces a differentiated product, using labor as an input, according to a production function $Y_i = AL_i$. The elasticity of demand for an individual firm's good is denoted η.

Consider the size of the "menu cost" needed to maintain a "fixed-price equilibrium" in the face of an unexpected variation in m. Is the required menu cost bigger if:

- each firm's demand elasticity η is a fixed parameter, unaffected by the level of output, *or*

- each firm's demand elasticity is an endogenous variable that increases with the level of output.

Explain your answer.

8) Suppose you are on the central bank's policy committee that sets interest rates. Your economic staff advises you that they forecast an upcoming decrease in spending (an "IS shock") of about 600 billion real-GDP dollars. According to their best estimate of the slope of the IS curve, you can counteract this IS shock and keep output at the natural rate if you cut the interest rate by one percent.

You have one more hour before you must make your interest-rate decision. That is enough time to hear one more report from the staff. Would you rather hear:

- a report describing the degree of uncertainty in the forecast of the 600-billion dollar IS shock *or*

- a report describing the degree of uncertainty in the estimate of the slope of the IS curve?

Explain your answer.

#48 PS

SPRING 2005

MACROECONOMIC THEORY COMPREHENSIVE EXAMINATION

C. Hanes/B. Jones

ANSWER BOTH QUESTIONS 1 AND 2

1. Consider the problem to maximize

$$\sum_{t=0}^{\infty} \beta^t U(c_t)$$

subject to the following constraints:

$$c_t + x_t + g_t = F(k_{t+1}, g_t)$$
$$k_{t+1} = (1-\delta)k_t + x_t$$

where c, x, g, and x are always non-negative. c, x, and g are the controls (one is redundant, however) for the problem.

a) Setup Bellman's equation (use a substitution to eliminate the redundant control).

b) Derive the Euler equation(s).

c) Derive conditions for the existence of a steady state and characterize the steady state.

d) What does the steady state condition for g mean in economic terms? (This requires you to interpret what g is!)

2. An infinitely-lived representative agent solves the following problem:

$$\text{Max} \int_0^{\infty} e^{-\rho t} U\left(\left(1-s(t)\right) A k_1^{\alpha}(t) k_2^{\beta}(t)\right) dt$$

subject to the following constraints:

$$\dot{k}_1 = s\gamma A k_1^{\alpha} k_2^{\beta} - \delta k_1$$
$$\dot{k}_2 = s(1-\gamma) A k_1^{\alpha} k_2^{\beta} - \delta k_2$$

and $1 \geq s(t) \geq 0, 1 \geq \gamma(t) \geq 0$ for all t.

Throughout the question, assume $\alpha + \beta < 1$.

a) Interpret the model in economic terms. How does it (does it?) relate to models from your core macro courses? How does it (if it does) differ from such models? Hint: maybe you would like to answer all the other questions first.

b) Write down the Hamiltonian. List the states and controls clearly.

c) Under what condition(s) would you expect to have an interior solution for s? Interpret in economic terms.

d) Under what condition(s) would you expect to have an interior solution for γ? Interpret in economic terms.

e) State all of the necessary conditions for optimality using the maximum principle (assuming an interior solution for s, but not necessarily for γ).

f) What does the necessary condition for s mean in economic terms? Be specific.

g) Derive all of the conditions needed to characterize the steady state.

ANSWER ALL QUESTIONS 3 THROUGH 9

3) Consider the Malthusian model of population and economic growth. The aggregate production function is $Y = AF(L)$ where Y is output and L is labor input, equivalent to population. The "subsistence" level of consumption or real wage is denoted σ. Assume the economy is initially in a state of long-run equilibrium with a stable population L.

Suppose the economy experiences an increase in real wages, accompanied by a falling population. Eventually the population stabilizes at a lower level, while the real wage remains higher than it was initially. How could you explain this using the model - for example, as a result of an exogenous shock to population L or the parameter A or the parameter λ? Use a graph to illustrate your answer.

4) Consider the Ramsey (Ramsey-Cass-Koopmans) model, assuming that:

$$U = \int_{t=0}^{\infty} e^{-\rho t}\, u(c_t)\, dt \quad \text{where} \quad u(c) = \frac{c^{1-\theta}}{1-\theta}$$

The rate of growth in TFP (total factor productivity) is g.

Recall the graph depicting the steady-state loci of the model (points where $\dot{c} = 0$ and $\dot{k} = 0$), the stable arm (saddle path), and the long-run steady state. Using such a graph, show what happens to the economy over time in response to each of the events described below. Assume that the economy is initially in a long-run steady state. Use the following symbols to label points:

(1) to label the point that is the initial LRSS before the event
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(3) to label a combination of c and k some time after the event, but before the new LRSS
(4) to label the point that is the new LRSS after the event.

a) An increase in the rate of time-discount ρ

b) An increase in the rate of growth of TFP g.

5) Consider a closed-economy IS-LM model with a fixed price level and a fixed money supply M, described by the following expressions:

$$\frac{M}{P} = L(i,Y) \quad \textit{where} \quad L_i<0,\ L_Y>0$$

$$Y = E(Y,r,G,T) \quad \textit{where}\ 0 < E_Y < 1\ ,\ E_r < 0 \quad \textit{and} \quad r = i - \pi^e$$

Consider the effect of an exogenous change in expected inflation π^e.
Derive an expression showing the effect of a change in π^e on output Y, and an expression showing the effect of a change in π^e on the nominal interest rate i.

6) Consider an open economy with perfect capital mobility (Mundell-Fleming model), "static exchange rate expectations" ($\dot{\varepsilon}^e/\varepsilon = 0$, always), a fixed domestic price level P, a fixed foreign price level P^*, a fixed domestic money supply M, and a *floating* exchange rate.

a) Using a graph, illustrate the effect on output and the exchange rate of an increase in the foreign interest rate i^*.

b) Using a graph, illustrate the effect on output and the exchange rate of a positive "IS shock," that is, an increase in the level of domestic real output that would be purchased at any given real interest rate and real exchange rate.

7) Consider an economy where y is the output gap (the difference between log output and log potential output) and $r_t = i_t - {}_t\pi^e_{t+1}$ is the real interest rate. Inflation follows a Friedman-Phelps Phillips curve:

$$\pi_t = {}_{t-1}\pi^e_t + \alpha y_t$$

The economy has a central bank. The central bank observes ${}_t\pi^e_{t+1}$ and ${}_{t-1}\pi^e_t$, and sets the nominal interest rate i_t to minimize a loss function.

a) Suppose the central bank's loss function is:

$L = \frac{1}{2} E[(y_t - \varphi)^2] + \frac{1}{2} E[(\pi_t - \pi^*)^2]$ where φ is some positive number.

The output gap is determined by $y_t = -\beta(r_t - \bar{r})$. The central bank knows the value of β.

What will be the inflation rate π, the output gap y, and the nominal interest rate i in "rational expectations equilibrium," that is assuming the public is rational and knows the true structure of the economy and preferences of the central bank?

b) Suppose again that output in the economy is given by $y_t = -\beta(r_t - \bar{r})$ and the central bank knows the value of β. But the central bank's loss function is instead:

$L = \frac{1}{2} E[y_t^2] + \frac{1}{2} E[(\pi_t - \pi^*)^2]$

and the public's expectations are *not* rational. Instead, in every period,

${}_{t-1}\pi^e_t = {}_t\pi^e_{t+1} = \pi^* + \varepsilon_t$ where ε is an i.i.d. (no serial correlation), mean-zero random variable.

i) What will be π, y, and i in a period t?

ii) Consider correlations between these variables.

Will the period-by-period correlation between i_t and y_t be positive, negative, or zero?

Will the period-by-period correlation between π_t and y_t be positive, negative, or zero?

c) Suppose again that the central bank's loss function is

$$L = \frac{1}{2} E[y_t^2] + \frac{1}{2} E[(\pi_t - \pi^*)^2]$$

But now the output gap in the economy is given by:

$$y_t = -(\beta + g_t)(r_t - \bar{r}) + h_t$$

g and h are two uncorrelated, i.i.d. mean-zero random variables. The public observes neither g nor h. The central bank observes the realized value of h_t before it sets i_t, but it does not observe g: all it knows is that the variance of g is equal to σ_g^2. Finally, assume that the value of α in the Phillips curve is one (1).

i) What will be π , y, and i in a period t, in "rational expectations equilibrium," that is assuming the public is rational and knows the true structure of the economy and preferences of the central bank?

ii) Consider correlations between these variables.

Will the period-by-period correlation between i_t and y_t be positive, negative, or zero?

Will the period-by-period correlation between π_t and y_t be positive, negative, or zero?

8) Suppose one regressed the change in inflation $(\pi_{t+1} - \pi_t)$, on the output gap in period t, with a regression equation like $(\pi_{t+1} - \pi_t) = \beta y_t + \varepsilon_t$ where ε is the residual in the regression.

a) What sign would you expect to find for β - positive, negative, or zero - assuming that inflation is determined by a Calvo-style New-Keynesian Phillips curve and the public's inflation expectations are rational? Explain.

b) What sign would you expect to find for β - positive, negative, or zero - assuming that inflation is determined by the Lucas supply function, and the public's expected value for inflation is always equal to the central bank's inflation target π^*? Explain.

9) Consider an economy with perfectly competitive labor markets and an upward-sloping aggregate labor supply curve (as a function of the real wage). Aggregate demand is determined by $y = m - p$, where y is the log of aggregate output, p is the log price level and m is a variable like the log "money supply." Output is produced by identical price-setting monopoly firms. Each firm produces a differentiated product, using labor as an input. Consider the size of the "menu cost" needed to maintain a "fixed-price equilibrium" in the face of an unexpected variation in m. Is the required menu cost bigger if each firm's production function looks like: $Y_i = AL_i$ (production function one)
or if each firm's production function looks like: $Y_i = AL_i^{\alpha}$ *where* $0<\alpha<1$ (production function two)?

Note that these possible production functions are *not* in log terms. Explain your answer.