Macro Comps, Summer 2006, Answers to first part of exam questions 1-9

1) Consider the Ramsey (Ramsey-Cass-Koopmans) model, assuming that:

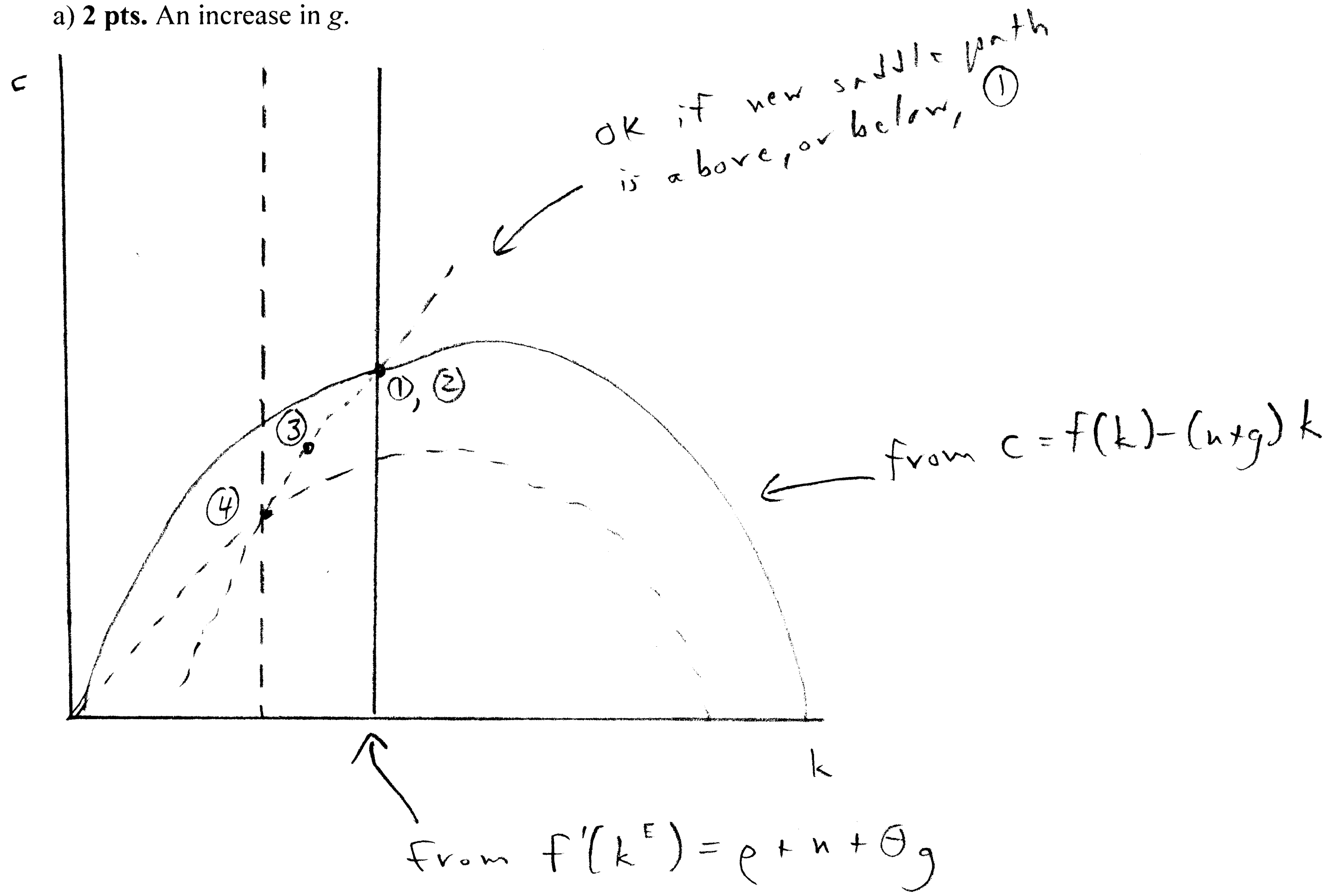
$$U = \int_{t=0}^{\infty} e^{-\rho t} u(c_t)\, dt \quad \text{where} \quad u(c) = \frac{c^{1-\theta}}{1-\theta}$$

The rate of growth in TFP (total factor productivity) is g.

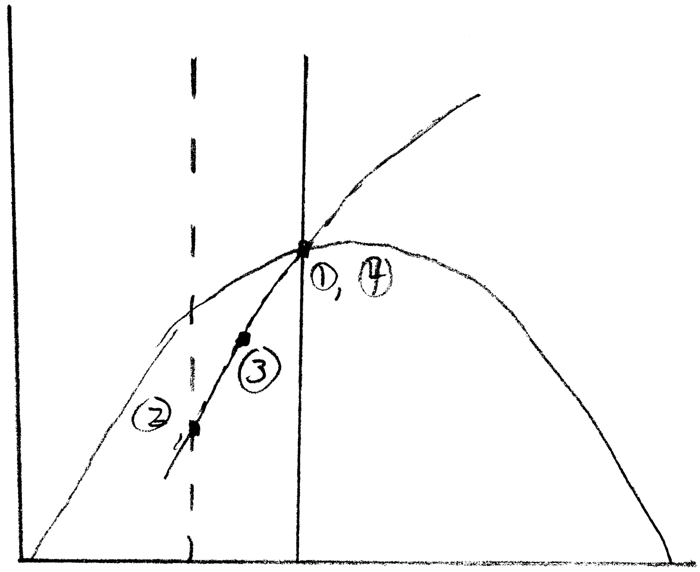
Recall the graph depicting the steady-state loci of the model (points where $\dot{c} = 0$ and $\dot{k} = 0$), the stable arm (saddle path), and the long-run steady state. Using such a graph, show what happens to the economy over time in response to each of the events described below. Assume that the economy is initially in a long-run steady state. Use the following symbols to label points:

(1) to label the point that is the initial LRSS before the event
(2) to label the point that is the combination of c and k immediately after the event
(3) to label a combination of c and k some time after the event, but before the new LRSS
(4) to label the point that is the new LRSS after the event.

a) **2 pts.** An increase in g.



b) **2 pts.** A sudden one-time in-migration of workers who arrive with no capital, but have exactly the same preferences and behavior as the natives.



c) **2 pts.** Consider how the event described in b) would affect an economy that can be described by the Solow model with a fixed savings rate. Would the response to the event of this economy over time be exactly the same as the response you described in b)? Carefully describe any differences.

The response would not be exactly the same. As in b), consumption, capital and output per efficiency-unit of labor (or per person) would fall, then gradually return to original LRSS. But the process would be faster in the RKC model. In the Solow model, savings rate remains the same before the event, on the transition to the new LRSS, and after the event. In RKC, the savings rate rises after the event and remains higher while the economy is returning to the LRSS. See Romer 2.6.

2) 5 pts. Consider a closed-economy IS-LM model with a fixed price level and a fixed money supply M, described by the following expressions:

$$\frac{M}{P} = L(i,Y) \quad where \quad L_i<0, \; L_Y>0$$

$$Y = E(Y,r,G,T) \quad where \; 0 < E_Y < 1 \; , \; E_r < 0 \; , \; and \quad r = i - \pi^e$$

Derive expressions showing the effect of a change in expected inflation on output Y and the nominal interest rate i, assuming the money supply M remains fixed.

(−1)

$$\frac{M}{P} = L(i, Y) \qquad\qquad Y = E(Y, r, G, T)$$

$$0 = L_i \frac{\partial i}{\partial \pi^e} + L_Y \frac{\partial Y}{\partial \pi^e} \qquad\qquad \frac{\partial Y}{\partial \pi^e} = E_Y \frac{\partial Y}{\partial \pi^e} + E_r \frac{\partial i}{\partial \pi^e} + E_r \frac{\partial r}{\partial \pi^e}$$

$$= \; \text{- - - - - - - - -} \; - E_r$$

then solve.

3) 5 pts. Consider two economies that can be described by the baseline RBC model, where the deviation from trend in the productivity parameter A is:

$$A_t = \rho_A A_{t-1} + \varepsilon_{A,t}$$

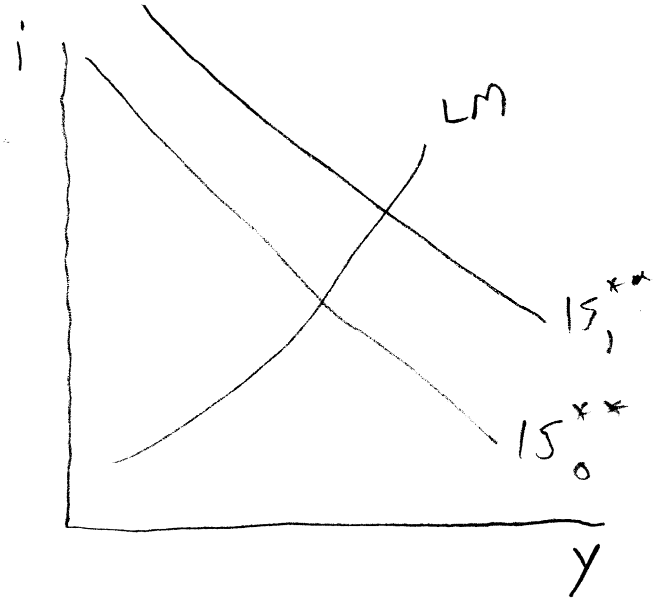
In economy I, $\rho_A = 0.1$.
In economy II, $\rho_A = 0.5$.

In which economy, I or II, would you expect to observe bigger variations in employment L, as measured by the magnitude of the initial response to productivity shocks? Explain.

Economy I. Your grade on question depended on explanation. This is about income (or wealth) effect of a productivity shock versus the substitution or real-wage effect. A good productivity shock raises the current period's real wage relative to the expected future path for the real wage. That tends to increase the quantity of labor supplied. At the same time, it makes the representative agent wealthier - raises his lifetime income. Since leisure is a normal good, that tends to reduce the quantity of labor supplied. The magnitude of the shock to lifetime income depends on the persistence of technology shocks. Less persistent technology shocks means smaller increase in lifetime income/wealth, so income/wealth effect is smaller relative to the substitution or current-real-wage effect, so labor response is bigger.

4) Consider an open economy with a fixed money supply M and a floating exchange rate. Let ε denote the nominal exchange rate (as in class, ε is dollars per euro, so ε increases if the dollar depreciates). Assume static exchange-rate expectations ($\dot{\varepsilon}^e/\varepsilon = 0$). In the "short run," the price level is fixed. Suppose there is a sudden, unexpected increase in the foreign interest rate i^*.

a) **3 pts.** Assuming imperfect capital mobility, what happens to output *Y*, the exchange rate ε , and the domestic interest rate *i* in the short run? Illustrate with a graph.

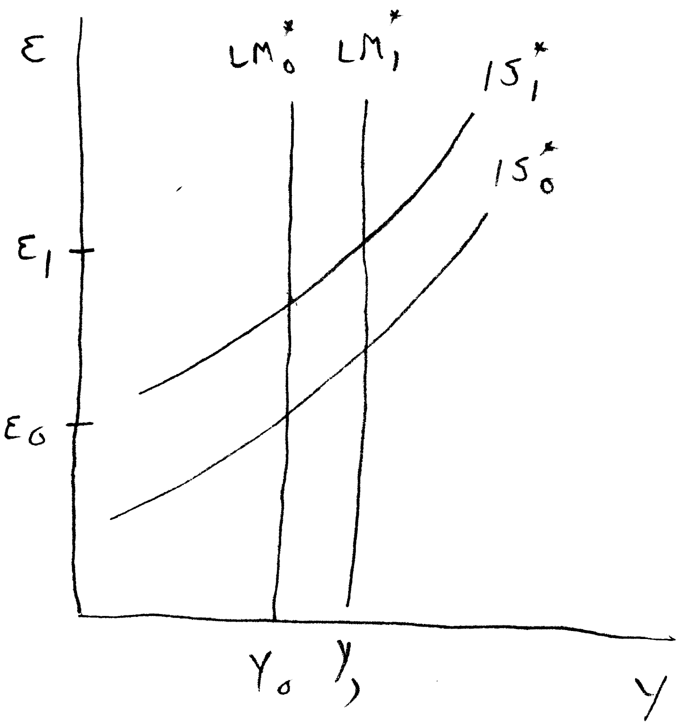


$CF(i - i^*) + NX\left(\frac{\varepsilon P}{P^*}, \ldots\right) = 0$

When $i^* \uparrow$, at given i, $CF \downarrow$, hence $NX \uparrow$, hence $\frac{\varepsilon P}{P^*} \uparrow$, at given P, $\varepsilon \uparrow$.

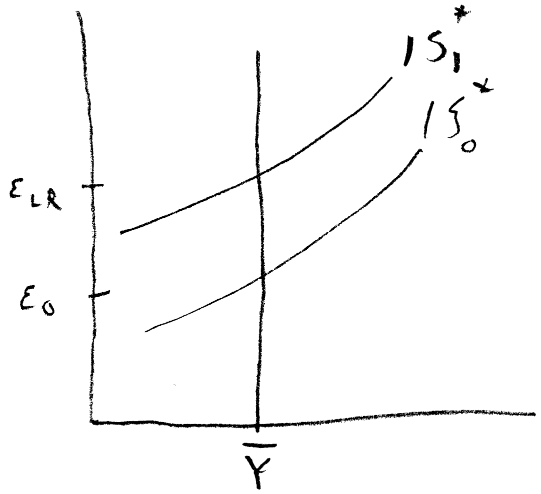
At given i, $\varepsilon \uparrow \rightarrow Y \uparrow$ hence IS^{**} shifts out.

b) **3 pts.** Assuming perfect capital mobility, what happens to output *Y*, the exchange rate ε , and the domestic interest rate *i* in the short run? Illustrate with a graph.



IS^* shifts back at given ε, higher $i = i^*$ reduces Y.

c) **4 pts.** Continuing to consider case b) (perfect capital mobility), suppose that the original level of output in the economy (before the increase in i^*) was the natural rate of output, and the natural rate of output does not change when i^* rises. In the "long run," the economy must return to the natural rate of output. Explain how this can happen. In the long run, is the (nominal) exchange rate ε the same as, greater than, or smaller than its original value (before the increase in i^*)? *In the long run, the price level adjusts - it rises - reducing M/P and hence shifting the LM* curve back to the original position. You can see from the graph that the resulting exchnage rate is higher than it was before the event - depreciation.*



5) Consider an economy where

y is the output gap and $r_t = i_t - {}_t\pi^e_{t+1}$ is the real interest rate

$y_t = -\beta_t(r_t - \bar{r})$ where $\bar{r}$ is the natural rate of interest

Each period, the central bank sets the interest rate to minimize an "as if" loss function (that is, the central bank acts as if the desired output level were the natural rate of output):

$L = \frac{1}{2} y_t^2 + \frac{1}{2} (\pi_t - \pi^*)^2$ where y is the output gap.

When the central bank sets i_t, it knows the public's expectations for future inflation.

a) **3 pts.** Assume that "aggregate supply" follows the "Lucas supply function." Can $\pi_t > \pi^*$ be a "rational expectations equilibrium"? Explain.

It can't be an REE. The keys here were to use the central bank's loss function, the definition of REE, and the LSF. The LSF means:
$\pi = {}_{t-1}\pi^e_t + \textit{something}\ y_t$ *or* $y_t = \textit{something}\ (\pi - {}_{t-1}\pi^e_t)$

Because there are no stochastic terms, REE means that the expected value of a time-t variable as of an earlier period t-i must be equal to the realized value of the variable. The loss function implies that the central bank will set y < 0 (we can describe the central bank as simply setting output because there are no stochastic terms in the model - each interest-rate setting corresponds to a certain y). If y < 0, realized inflation must be less than last period's expectation *of this period's inflation rate. That can't be an REE. Here's how the find the REE:*

Because no uncertainty in IS, and central bank knows π^e, might as well say c.b. controls y directly

$$L = \frac{1}{2} y_t^2 + \frac{1}{2}(\pi_t - \pi^*)^2 \qquad \pi_t = \pi_t^e + \alpha y_t$$

$$\Rightarrow L = \frac{1}{2} y_t^2 + \frac{1}{2}(\pi_t^e + \alpha y_t - \pi^*)^2$$

$$\frac{\partial L}{\partial y} = y + (\pi_t^e + \alpha y - \pi^*)\alpha = 0$$

$$\Rightarrow y_t = -\frac{\alpha}{1+\alpha^2}(\pi_t^e - \pi^*)$$

note this says that if $\pi_t^e > \pi_t^e$ then $y_t < 0$ which would mean $\pi_t < \pi^e$ which can't be REE

$$\pi_t = \pi_t^e - \frac{\alpha^2}{1+\alpha^2}(\pi_t^e - \pi^*)$$

$$= \left(1 - \frac{\alpha^2}{1+\alpha^2}\right)\pi_t^e + \frac{\alpha^2}{1+\alpha^2}\pi^*$$

In REE, $\pi_t = \pi_t^e$

$$\pi_t = (\qquad)\pi_t + \frac{\alpha^2}{1+\alpha^2}\pi^*$$

Subtract π_t from both sides

$$0 = \frac{\alpha^2}{1+\alpha^2}(\pi^* - \pi_t)$$

Only solution: $\pi_t = \pi^*$

b) **3 pts.** Assume that "aggregate supply" is derived from the Calvo or Rotemberg model of pricesetting. Can $\pi_t > \pi^*$ be a "rational expectations equilibrium"? Explain.
Yes, it can be an REE. Calvo or Rotemberg means

$\pi_t = \pi^e_{t+1}$ + *something* y_t *which means* $\pi^e_{t+1} - \pi_t = -$ *something* y_t

The central bank's loss function still means that the output gap must be negative if π_t *is greater than* π^*. *That's OK with the AS equation as long as* $\pi^e_{t+1} > \pi_t$. *And that's OK with rat. exp. as long as realized* π_{t+1} *is equal to* π^e_{t+1}. *Which is to say, the inflation rate is speeding up. Here's how to find this REE:* From loss fn. and $\pi_t = \pi^e_{t+1} + \alpha y_t$,

$$L = \tfrac{1}{2} y_t^2 + \tfrac{1}{2}\left(\pi^e_{t+1} + \alpha y_t - \pi^*\right)^2$$

$$\frac{\partial L}{\partial y_t} = 0 = \text{------}$$

$$\Rightarrow y_t = -\frac{\alpha}{1+\alpha^2}\left(\pi^e_{t+1} - \pi^*\right)$$

Note this means $y_t < 0$ if $\pi^e_{t+1} > \pi^*$. If $y_t < 0$, then $\pi_t < \pi^e_{t+1}$. That doesn't violate REE. It just means we expect accelerating inflation

$$\pi_t = \pi^e_{t+1} - \frac{\alpha^2}{1+\alpha^2}\left(\pi^e_{t+1} - \pi^*\right)$$

In REE, $\pi^e_{t+1} = \pi_{t+1}$

$$\pi_t = \pi_{t+1} - \frac{\alpha^2}{1+\alpha^2}\left(\pi_{t+1} - \pi^*\right)$$

Subtract π^* from both sides

$$\pi_t - \pi^* = \pi_{t+1} - \pi^* - \frac{\alpha^2}{1+\alpha^2}\left(\pi_{t+1} - \pi^*\right) = \left(1 - \frac{\alpha^2}{1+\alpha^2}\right)\left(\pi_{t+1} - \pi^*\right)$$

$$= \left(\frac{1+\alpha^2}{1+\alpha^2} - \frac{\alpha^2}{1+\alpha^2}\right)\left(\pi_{t+1} - \pi^*\right) = \frac{1}{1+\alpha^2}\left(\pi_{t+1} - \pi^*\right)$$

$$\Rightarrow \pi_{t+1} - \pi^* = \left(1+\alpha^2\right)\left(\pi_t - \pi^*\right)$$

If $\pi_t > \pi^*$, accelerating inflation.

6) Consider the basic model Romer uses to discuss "menu costs," with imperfectly competitive product markets and a perfectly competitive labor market. Suppose one changed the assumptions of this model to introduce "indivisible labor" plus "employment lotteries" plus "consumption insurance" to the model. For any given magnitude of menu costs, would this change make it harder or easier to maintain a "fixed-price equilibrium"? Explain.

5 pts. *Makes it easier to maintain FPE. Recall Romer says it is hard to maintain a fixed-price equilibrium in a model with a perfectly competitive labor market and reasonably inelastic labor supply. Consider an increase in M holding prices fixed. That means output and employment go up. But with inelastic labor supply, an increase in employment must be accompanied by a big increase in the real wage. That is, given P, W must rise. But the inrcease in W raises firms' marginal costs and gives them incentive to raise their prices, spoiling the FPE.*

The assumptions listed in the question make aggregate LS super-elastic. So...

7) Consider a simple "New Keynesian" IS-LM model of the type described by King in "The New IS-LM Model: Language, Logic and Limits." Recall that such a model implies there is no lag in the effect of the real interest rate on output, and the real interest rate r that enters the IS curve is $r_t = i_t - {}_t\pi^e_{t+1}$.

Assume that the "Solow residual" in the aggregate production function always grows at a constant rate g. There are no shocks to households' preferences. Government expenditure is always a constant fraction of output. You have time-series data on real GDP, the nominal interest rate i and expected inflation ${}_t\pi^e_{t+1}$ (from a survey, say).

Note that these assumptions mean:

$$y_t = {}_t y^e_{t+1} - \beta(r_t - \bar{r})$$
$$= {}_t y^e_{t+1} - \beta(i_t - {}_t\pi^e_{t+1} - \bar{r})$$

a) **3 pts.** Suppose monetary policy is run by chimps. Each period t, the chimps set the nominal interest rate i_t so that the real interest rate r_t is equal to the natural rate of interest plus a totally random, mean-zero disturbance.

You run an OLS regression. The dependent (left-hand-side) variable is the rate of growth in real GDP from period t to period $t+1$. The independent (right-hand-side) variable is the real interest rate in period t, that is $(i_t - {}_t\pi^e_{t+1})$. (There is also a constant in the regression.) Would you expect to find a positive, negative, or zero value for the coefficient on the real interest rate? Positive.

With $r_t = \bar{r} + \nu_t$, ← monkey disturbance

then $y_t = {}_t y^e_{t+1} - \beta(\bar{r} + \nu_t - \bar{r})$

$y_{t+1} = {}_t y^e_{t+1} + \varepsilon_{t+1}$ ← error in expectation. No reason to think it's correlated with ν

$$y_{t+1} - y_t = \beta \nu_t - \varepsilon_{t+1}$$

Hence both r_t and $y_{t+1} - y_t$ are + correlated with ν_t.

So in regression...

b) Now suppose monetary policy is run by a committee of humans who act to minimize an "as if" loss function (that is, the central bank acts as if the desired output level were the natural rate of output):

$$L = \frac{1}{2}E[y_t^2] + \frac{1}{2}E[(\pi_t - \pi^*)^2] \text{ where } y \text{ is the output gap.}$$

where y denotes the output gap. When the central bank sets i_t, it knows the public's expectations for future inflation π^e_{t+1}. Finally, suppose that the public is subject to irrational "inflation scares." Each period, the public's expected future inflation π^e_{t+1} is equal to the central bank's target inflation rate π^* *plus* a totally random, mean-zero disturbance. You run the same regression described in a). Would you expect to find a positive, negative, or zero value for the coefficient on the real interest rate? *Positive again.*

Assuming central bank knows y^e_{t+1} (public's expectation of y_{t+1}) then

$$L = \frac{1}{2}\left(y^e_{t+1} - \beta(r_t - \bar{r})\right)^2 + \frac{1}{2}\Big(\underbrace{\pi^* + \nu_t}_{\pi^e_{t+1}} + \alpha\left(y^e_{t+1} - \beta(r_t - \bar{r})\right) - \pi^*\Big)$$

To simplify notation, say $\bar{r} = 0$

$$\frac{\partial L}{\partial r} = 0 = (y^e - \beta r_t)(-\beta) + (\nu_t + \alpha y^e - \alpha\beta r_t)(-\alpha\beta)$$

Solve for r_t

$$r_t = \frac{1}{\beta} y^e_{t+1} + \frac{\alpha}{1+\alpha^2}\nu_t$$ hence r_t (+) corr'ted with y^e, ν_t

Meanwhile, from $y_t = y^e_{t+1} - \beta r_t$ and r_t from above

$$\Rightarrow y^e_{t+1} - y_t = y^e_{t+1} + \frac{\alpha\beta}{1+\alpha^2}\nu_t$$

Assuming $y_{t+1} = y^e_{t+1} + \varepsilon_t$,

error in expectation of y_t. No reason to think it's correlated with ν_t

$$y_{t+1} - y_t = y^e_{t+1} + \frac{\alpha\beta}{1+\alpha^2}\nu_t + \varepsilon_t$$

hence $(y_{t+1} - y_t)$ is (+) corr'ted with y^e, ν_t

So in regression...

c) **3 pts.** Continuing to consider case b) (monetary policy run by humans), what would happen if you run an OLS regression with the *nominal* interest rate, rather than the real interest rate, as the independent variable? *Would the value of this coefficient be the same as the one you find in b)? Movements in i will be bigger than movements in r. Hence regression coefficient will be same sign, but smaller magnitude.*

$$i_t = \pi^e_{t+1} + r_t = \pi^* + v_t + r_t$$

$$= \pi^* + v_t + \frac{1}{\beta} y^e + \frac{\alpha}{1+\alpha^2} v_t$$

Note v_t has bigger effect on i,

So movements in i_t are bigger relative to movements in $(y_{t+1} - y_t)$.

So in regression...

8) In "Resuscitating Real Business Cycles," King and Rebelo (K & R) make assumptions that allow for variable capital utilization. In "Nominal Rigidities" (2005), Christiano, Eichenbaum and Evans (CEE) also make assumptions that allow for variable capital utilization. K&R's assumptions imply that capital utilization is especially low when the real interest rate is especially low (below the LRSS value). CEE's assumptions imply that capital utilization is especially HIGH when the real interest rate is especially low.

a) **3 pts.** What is the key difference in assumptions that creates this difference in implications?

K & R assume that when you run your capital harder - higher capacity utilization - it depreciates faster. CEE assume that there is a "maintenance" cost, in current output, incurred when you run your capital, and this maintenance cost is higher when you run the capital harder..

b) **3 pts.** Why do K & R want capital utilization to be lower when the real interest rate is low? *Within an RBC model, K & R are trying to get output/employment to fall without technological regress. In an RBC model, a low realization of productivity growth - slower growth, not necessarily an absolute drop in productivity - results in an extra-low real interest rate. Low interest rate reduces capital utilization, which tends to reduce output and employment. See notes.*

c) **3 pts.** Why do CEE want capital utilization to be higher when the real interest rate is low? *CEE are in a new-Keynesian model, where business cycles are caused by shocks to monetary policy. Low interest rate causes high output. But CEE need to add a sort of real rigidity: they need to keep real MC from rising much when output is high. By making cap. util. rise when the interest rate is low, they make cap. util. rise when output is high, which mutes the increase in* *real MC associated with the increase in output.*

9) It has been observed that if one runs an OLS regression of the future change in inflation on the current output gap, like

$$\pi_{t+1} - \pi_t = \textit{Constant} + \beta\, y_t \quad \text{(where } y \text{ is the output gap)}$$

the coefficient β appears to be insignificantly different from zero, or positive.

a) **4 pts.** This fact is often described as being inconsistent with "New Keynesian" models of pricesetting and inflation. Explain, using an equation or equations.

Simplest to use Rotemberg/Calvo $\pi_t = {}_t\pi^e_{t+1} + \alpha\, y_t$
which means ${}_t\pi^e_{t+1} = \pi_t - \alpha\, y_t$
which means $\pi_{t+1} = \pi_t - \alpha\, y_t + \varepsilon_{t+1}$ *where* ε *is the error in the expectation*

which means $\pi_{t+1} - \pi_t = -\alpha\, y_t + \varepsilon_{t+1}$

If expectations are rational, ε_{t+1} *must be uncorrelated with anything people can see at time t, such as output. So in an OLS regression, we should see the true value of the coefficient on output, which is negative.*

b) **3 pts.** In "New Keynesian Economics and the Phillips Curve" (1995), John Roberts concluded that the time-series behavior of inflation *is* consistent with New Keynesian models of inflation. How? Explain. *Roberts uses survey results for* ${}_t\pi^e_{t+1}$. *Survey expectations do not seem to be perfectly rational.*

c) **3 pts.** Some people have argued that the apparent relation between inflation and labor's share of national income shows that New Keynesian models of inflation are correct. How? Explain.

NK models imply that current inflation depends on expected future inflation and "real marginal cost," that is MC/P. You get the standard NKPC by assuming that real MC is positively correlated with the output gap. Under some assumptions, labor's share of income is positively correlated with real MC (see the notes). If you put labor's share in your regression instead of the output gap, you get a negative coefficient in the type of regression described in a).

e) **3 pts.** What assumption is made by Christiano, Eichenbaum and Evans (2005) that allows their model to reproduce the time-series behavior of inflation? *A seller who is not adjusting his price (wage) this period does not hold his price fixed: instead he "indexes" it, that is sets it equal to the previous period's price level scaled up by the previous period's average inflation rate.*