Spring 2006
MACROECONOMIC THEORY COMPREHENSIVE EXAM

Hanes/B. Jones

SECTION A – ANSWER ALL QUESTIONS

1) Consider the Ramsey (Ramsey-Cass-Koopmans) model, assuming that:

$$U = \int_{t=0}^{\infty} e^{-\rho t}\, u(c_t)\, dt \quad \text{where} \quad u(c) = \frac{c^{1-\theta}}{1-\theta}$$

The rate of growth in TFP (total factor productivity) is g.

Recall the graph depicting the steady-state loci of the model (points where $\dot{c} = 0$ and $\dot{k} = 0$), the stable arm (saddle path), and the long-run steady state. Using such a graph, show what happens to the economy over time in response to each of the events described below. Assume that the economy is initially in a long-run steady state. Use the following symbols to label points:

(1) to label the point that is the initial LRSS before the event
(2) to label the point that is the combination of c and k immediately after the event
(3) to label a combination of c and k some time after the event, but before the new LRSS
(4) to label the point that is the new LRSS after the event.

a) An increase in g.

b) A sudden one-time in-migration of workers who arrive with no capital, but have exactly the same preferences and behavior as the natives.

c) Consider how the event described in b) would affect an economy that can be described by the Solow model with a fixed savings rate. Would the response to the event of this economy over time be exactly the same as the response you described in b)? Carefully describe any differences.

2) Consider a closed-economy IS-LM model with a fixed price level and a fixed money supply M, described by the following expressions:

$$\frac{M}{P} = L(i,Y) \quad where \quad L_i<0,\ L_Y>0$$

$$Y = E(Y,r,G,T) \quad where\ 0 < E_Y < 1\ ,\ E_r < 0\ ,\ and\ .\ r = i - \pi^e$$

Derive expressions showing the effect of a change in expected inflation on output Y and the nominal interest rate i, assuming the money supply M remains fixed.

3) Consider two economies that can be described by the baseline RBC model, where the deviation from trend in the productivity parameter A is:

$$A_t = \rho_A\, A_{t-1} + \varepsilon_{A,t}$$

In economy I, $\rho_A = 0.1$.
In economy II, $\rho_A = 0.5$.

In which economy, I or II, would you expect to observe bigger variations in employment L, as measured by the magnitude of the initial response to productivity shocks? Explain.

4) Consider an open economy with a fixed money supply M and a floating exchange rate. Let ε denote the nominal exchange rate (as in class, ε is dollars per euro, so ε increases if the dollar depreciates). Assume static exchange-rate expectations ($\dot{\varepsilon}^e/\varepsilon = 0$). In the "short run," the price level is fixed. Suppose there is a sudden, unexpected increase in the foreign interest rate i^*.

a) Assuming imperfect capital mobility, what happens to output Y, the exchange rate ε, and the domestic interest rate i in the short run? Illustrate with a graph.

b) Assuming perfect capital mobility, what happens to output Y, the exchange rate ε, and the domestic interest rate i in the short run? Illustrate with a graph.

c) Continuing to consider case b) (perfect capital mobility), suppose that the original level of output in the economy (before the increase in i^*) was the natural rate of output, and the natural rate of output does not change when i^* rises. In the "long run," the economy must return to the natural rate of output. Explain how this can happen. In the long run, is the (nominal) exchange rate ε the same as, greater than, or smaller than its original value (before the increase in i^*)?

5) Consider an economy where

y is the output gap and $r_t = i_t - {}_t\pi^e_{t+1}$ is the real interest rate

$y_t = -\beta_t(r_t - \bar{r})$ where $\bar{r}$ is the natural rate of interest

Each period, the central bank sets the interest rate to minimize an "as if" loss function (that is, the central bank acts as if the desired output level were the natural rate of output):

$L = \frac{1}{2} y_t^2 + \frac{1}{2} (\pi_t - \pi^*)^2$ where y is the output gap.

When the central bank sets i_t, it knows the public's expectations for future inflation.

a) Assume that "aggregate supply" follows the "Lucas supply function." Can $\pi_t > \pi^*$ be a "rational expectations equilibrium"? Explain.

b) Assume that "aggregate supply" is derived from the Calvo or Rotemberg model of pricesetting. Can $\pi_t > \pi^*$ be a "rational expectations equilibrium"? Explain.

6) Consider the basic model Romer uses to discuss "menu costs," with imperfectly competitive product markets and a perfectly competitive labor market. Suppose one changed the assumptions of this model to introduce "indivisible labor" plus "employment lotteries" plus "consumption insurance" to the model. For any given magnitude of menu costs, would this change make it harder or easier to maintain a "fixed-price equilibrium"? Explain.

7) Consider a simple "New Keynesian" IS-LM model of the type described by King in "The New IS-LM Model: Language, Logic and Limits." Recall that such a model implies there is no lag in the effect of the real interest rate on output, and the real interest rate r that enters the IS curve is $r_t = i_t - {}_t\pi^e_{t+1}$.

Assume that the "Solow residual" in the aggregate production function always grows at a constant rate g. There are no shocks to households' preferences. Government expenditure is always a constant fraction of output. You have time-series data on real GDP, the nominal interest rate i and expected inflation ${}_t\pi^e_{t+1}$ (from a survey, say).

a) Suppose monetary policy is run by chimps. Each period t, the chimps set the nominal interest rate i_t so that the real interest rate r_t is equal to the natural rate of interest plus a totally random, mean-zero disturbance.

You run an OLS regression. The dependent (left-hand-side) variable is the rate of growth in real GDP from period t to period $t+1$. The independent (right-hand-side) variable is the real interest rate in period t, that is $(i_t - {}_t\pi^e_{t+1})$. (There is also a constant in the regression.) Would you expect to find a positive, negative, or zero value for the coefficient on the real interest rate?

b) Now suppose monetary policy is run by a committee of humans who act to minimize an "as if" loss function (that is, the central bank acts as if the desired output level were the natural rate of output):

$L = \frac{1}{2} E[y_t^2] + \frac{1}{2} E[(\pi_t - \pi^*)^2]$ where y is the output gap.

where y denotes the output gap. When the central bank sets i_t, it knows the public's expectations for future inflation ${}_t\pi^e_{t+1}$. Finally, suppose that the public is subject to irrational "inflation scares." Each period, the public's expected future inflation ${}_t\pi^e_{t+1}$ is equal to the central bank's target inflation rate π^* *plus* a totally random, mean-zero disturbance.

You run the same regression described in a). Would you expect to find a positive, negative, or zero value for the coefficient on the real interest rate?

c) Continuing to consider case b) (monetary policy run by humans), what would happen if you run an OLS regression with the *nominal* interest rate, rather than the real interest rate, as the independent variable? Would the value of this coefficient be the same as the one you find in b)?

8) In "Resuscitating Real Business Cycles," King and Rebelo (K & R) make assumptions that allow for variable capital utilization. In "Nominal Rigidities" (2005), Christiano, Eichenbaum and Evans (CEE) also make assumptions that allow for variable capital utilization. K&R's assumptions imply that capital utilization is especially low when the real interest rate is especially low (below the LRSS value). CEE's assumptions imply that capital utilization is especially HIGH when the real interest rate is especially low.

a) What is the key difference in assumptions that creates this difference in implications?

b) Why do K & R want capital utilization to be lower when the real interest rate is low?

c) Why do CEE want capital utilization to be higher when the real interest rate is low?

9) It has been observed that if one runs an OLS regression of the future change in inflation on the current output gap, like

$$\pi_{t+1} - \pi_t = Constant + \beta\, y_t \quad \text{(where } y \text{ is the output gap)}$$

the coefficient β appears to be insignificantly different from zero, or positive.

a) This fact is often described as being inconsistent with "New Keynesian" models of pricesetting and inflation. Explain, using an equation or equations.

b) In "New Keynesian Economics and the Phillips Curve" (1995), John Roberts concluded that the time-series behavior of inflation *is* consistent with New Keynesian models of inflation. How? Explain.

c) Some people have argued that the apparent relation between inflation and labor's share of national income shows that New Keynesian models of inflation are correct. How? Explain.

e) What assumption is made by Christiano, Eichenbaum and Evans (2005) that allows their model to reproduce the time-series behavior of inflation?

SECTION B – ANSWER ALL QUESTIONS

Problem 1: Optimal Control

An agent solves the following problem:

$$\text{Max} \int_0^T e^{-\rho t}\left(ac(t)-\frac{b}{2}c^2(t)\right)dt$$

subject to:

$$\dot{a} = ra - c$$

where $r(t)$ is the real interest rate and a and b are both positive constants.

A) Write out the Hamiltonian.

B) State all necessary conditions for optimality. You may assume interior solutions and non-satiation in your answer.

C) Suppose $r(t) = \rho, \forall t \in [0,T]$. What are the optimal solutions for all variables (controls, states, co-states)?

D) If the consumer is satiated (at some point), then what does this imply about the co-state variable? Interpret in economic terms.

E) What model, models, or well-known theories of consumption can you use to explain/interpret you results in C? Do so.

Problem 2: Dynamic Programming

A firm maximizes:

$$\sum_{t=0}^{\infty} \frac{\Pi_t k_t - I_t (1+I_t)}{(1+r)^t}$$

subject to the following constraint:

$k_t = k_{t-1} + I_t$.

The firm chooses investment, I, where r is a fixed rate, and Π_t is known for all t. Starting capital, k_{-1}, is also known.

A) Write down Bellman's equation, clearly identifying your states and controls.

B) Derive the Euler equation for the firm.

C) Interpret the Euler equation in economic terms. Hint: you may wish to define $Q_t = 1 + I_t$ in your answer.

Short Answer

1. Explain briefly why Sidrauski's model is not characterized by economic efficiency.

2. Explain briefly why a model with external economies of scale (e.g. "learning by doing") is not characterized by economic efficiency.

3. Explain in economic terms what the "Keynes-Ramsey" rule means.

4. Show how different rates of time discount (with the same initial level of wealth or capital per person) lead to different consumption trajectories. In your answer include a graph of two consumption trajectories illustrating the point.

5. What is the main idea of the "certainty equivalence" principle?

Spring 2006
MACROECONOMICS COMPREHENSIVE EXAM RE-TAKE

Hanes/B. Jones
SECTION A

ANSWER ALL QUESTIONS

1) Consider the Malthusian model of economic growth, with a constant subsistence wage σ. One implication of this model is that, over the long run, the degree of improvement in technology can be inferred from population growth. Explain.

2) Consider the Ramsey (Ramsey-Cass-Koopmans) model, assuming that:

$$U = \int_{t=0}^{\infty} e^{-\rho t}\, u(c_t)\, dt \quad \text{where} \quad u(c) = \frac{c^{1-\theta}}{1-\theta}$$

The rate of growth in TFP (total factor productivity) is g.

Recall the graph depicting the steady-state loci of the model (points where $\dot{c} = 0$ and $\dot{k} = 0$), the stable arm (saddle path), and the long-run steady state. Using such a graph, show what happens to the economy over time in response to each of the events described below. Assume that the economy is initially in a long-run steady state. Use the following symbols to label points:

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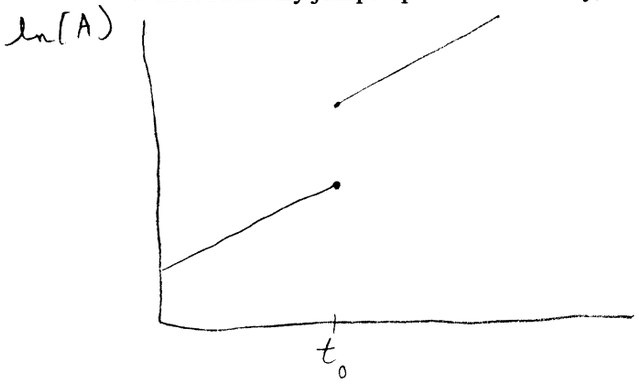
(3) to label a combination of c and k some time after the event, but before the new LRSS
(4) to label the point that is the new LRSS after the event.

a) An increase in θ.

b) A sudden one-time increase in the productivity parameter A. That is, A is growing at rate g, then it suddenly jumps up discontinuously, then it continues to grow at rate g, like this:



c) For part b), draw a graph that shows the path over time of consumption per person. The vertical axis of your graph should be the log of consumption per person. The horizontal axis should be time. The sudden increase in A takes place at time t_0.

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3) Consider a closed-economy IS-LM model with a fixed price level and a fixed money supply M, described by the following expressions:

$$\frac{M}{P} = L(i,Y) \quad where \quad L_i<0,\ L_Y>0$$

$$Y = E(Y,r,G,T) \quad where\ 0 < E_Y < 1\ ,\ E_r < 0\ ,\ E_G > 0 \quad and \quad r=i-\pi^e$$

a) Derive expressions showing the effect of a change in government expenditure G on output Y and the nominal interest rate i, assuming the money supply M and expected inflation π^e and the price level P remain fixed. If there is an increase in G, does output Y increase, decrease, or remain the same?

b) Suppose this economy did not have a fixed price level, but was instead characterized by an expectations-augmented Friedman-Phelps Phillips curve:

$$\pi_t = {}_{t-1}\pi_t^e + \alpha\, y_t$$

Assuming the money supply M and expected inflation π^e are fixed, what would you expect to happen to the price level and output as a result of an increase in G? Illustrate your answer with a graph or graphs.

4) Consider an open economy with perfect capital mobility, a fixed money supply M, and a fixed price level P, described by the Mundell-Fleming model. Let ε denote the nominal exchange rate (as in class, ε is units of domestic currency per foreign currency, so ε increases if the home currency depreciates). The expected future rate of depreciation in the exchange rate is $\dot{\varepsilon}^e/\varepsilon$. The foreign interest rate is i^*.

Suddenly, there is an increase in $\dot{\varepsilon}^e/\varepsilon$ from zero to some positive number. That is, people had been expecting no future change in the exchange rate. Now they suddenly expect the exchange rate to depreciate in the future.

a) Assuming the exchange rate is floating, use a graph to show what happens to output and the current exchange rate in response to the sudden expectation of *future* depreciation.

b) Now assume the central bank is trying to maintain a fixed exchange rate $\bar{\varepsilon}$. In order to maintain this fixed exchange rate, what must the central bank do in response to the sudden expectation of *future* depreciation? What happens to output? Use a graph to illustrate your answer.

5) Consider an open economy with imperfect capital mobility, a fixed price level P, and static exchange-rate expectations, that is $\dot{\varepsilon}^e/\varepsilon = 0$. (As before, ε is dollars per euro, so ε increases if the dollar depreciates). There is not a fixed money supply: instead the central bank maintains a fixed nominal interest rate $\bar{i}$. Suddenly, the government of this country begins to purchase foreign bonds (bonds issued by the foreign country's government or businesses).

Explain what happens to the country's exchange rate ε and output Y. Use a graph to illustrate your answer.

6) Consider an economy where

y is the output gap and $r_t = i_t - {}_t\pi^e_{t+1}$ is the real interest rate

$y_t = -\beta_t(r_t - \bar{r})$ where $\bar{r}$ is the natural rate of interest

$\pi_t = {}_{t-1}\pi^e_t + \alpha\, y_t + \varepsilon_t$ where ε_t is an i.i.d. random variable

The central bank sets the interest rate to minimize a loss function:

$$L = \tfrac{1}{2} E[(y_t - a)^2] + \tfrac{1}{2} E[(\pi_t - \pi^*)^2]$$

When the central bank sets i_t, it knows the public's expectations for future inflation, but it does not know the value of ε_t. Neither does the public.

Find the "rational expectations equilibrium" of this economy. Write down expressions for:

- the output gap y

- inflation π

- the real interest rate r

that will prevail in the rational expectations equilibrium.

7) Consider a simple "New Keynesian" IS-LM model of the type described by King in "The New IS-LM Model: Language, Logic and Limits." Assume that the central bank sets the real interest rate to minimize an "as if" loss function (that is, the central bank acts as if the desired output level were the natural rate of output):

$$L = \tfrac{1}{2} E[y_t^2] + \tfrac{1}{2} E[(\pi_t - \pi^*)^2]$$ where y is the output gap.

where y denotes the output gap. When the central bank sets i_t, it knows the public's expectations for future inflation ${}_t\pi^e_{t+1}$ and output ${}_ty^e_{t+1}$.

a) How would the central bank respond to a temporary increase in the share of output consumed by the government? Explain.

b) How would the central bank respond to a perceived increase in the rate of technological progress? Explain.

8) What is the surprising result of the Caplin-Spulber model?

9) How does the Phillips curve implied by the Taylor model of "overlapping contracts" with fixed durations *differ* from the Phillips curve implied by the Calvo model or the Rotemberg model?

10) In their paper "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," why do Christiano, Eichenbaum and Evans assume the household's utility function is characterized by "habit formation"?

11) Consider a "new Keynesian" model where each firm has a monopoly in the production of a good g and the labor market is perfectly competitive with a market-clearing wage W per unit of labor L. Each firm's production function is:

$Y_g = \frac{1}{2} L_g$ where L_g is the number of labor units used by the firm and Y_g is the number of units of output produced.

Each household i has a utility function:

$$U_i = C_i^{\alpha}(M/P)^{1-\alpha} - \frac{1}{\lambda} L_i^{\lambda} \quad \textit{where} \quad \alpha < 1, \quad \lambda > 1$$

and:

$$C_i = \left(\sum_{g=1}^{T} C_{gi}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \quad \textit{where} \quad \eta > 1 \text{ and } T \text{ is a very large number}$$

C_{gi} is the number of units of good g consumed by household i. The number of labor units supplied by the household is L_i

a) What does T stand for?

b) What is the elasticity of labor supply?

c) What is the elasticity of demand for any particular good?

d) What is the profit-maximizing price of any good g, expressed as a function of the wage W?

e) What is the number of labor units that a household will supply?

f) Suppose that the value of the parameter η tends to increase when aggregate output is higher. Will this phenomenon tend to make it harder or easier to maintain a "fixed price equilibrium" with a "menu cost" of given size? Explain.

SECTION B

Essay Questions:

Answer all parts of the following questions:

Question 1

One of the major developments in macroeconomics in the 1990's was a model that treated technical progress (productivity growth) as the expansion of the number of intermediate inputs to the production function.

Write an essay addressing the following issues:

i) Why is the expansion of the number of intermediate inputs equivalent to an improvement in the productivity of final goods producing firms?

ii) Why and how does this model of technical progress relate to the types of market structures that macroeconomists are interested in?

iii) What are the efficiency implications of this type of model?

iv) Why should we consider the development of this model to be significant to the profession?

Question 2

In an essay, list several major concepts of convergence, provide a definition of each one, provide an overview of the empirical evidence on these concepts, and compare and contrast them.

Question 3

Provide several different examples of inefficiency in macroeconomic models (i.e. one example of several different types of inefficiency, not many examples of the same basic idea) and discuss each one in detail. Focus on economic intuition and policy relevance.

Math Problem:

Answer all parts of the following problem:

A) A firm maximizes the following objective function

$$\int_0^{\infty} e^{-rt}\left\{F(K,L) - wL - I\left(1+\phi(I/K)\right)\right\}dt$$

subject to the following constraints:

$$\dot{K} = I - \delta K$$

given initial capital. The discount factor r is constant. The function, ϕ, is assumed to have the following properties: $\phi(0) = 0$, $\phi' > 0$, and $\phi'' \geq 0$. F is neoclassical.

Write down the Hamiltonian.

B) Find all necessary conditions for optimality.

C) Convert these conditions to intensive form in terms of $k = K/L, i = I/L, q = \lambda e^{rt}$, where λ is the usual costate variable.

D) Characterize the steady state for k and q.

E) Draw the phase plane for the model assuming that ϕ is an affine function: i.e. $\phi(x) = bx$, for some positive b.