January 2007
Macro comprehensive exam Hanes' questions ANSWERS

1) Consider the Ramsey (Ramsey-Cass-Koopmans) model, with a rate of growth g in the labor-efficiency parameter A, and a rate of population growth n.

Assume that the economy is initially in a long-run steady state. Define c and k in the usual way.

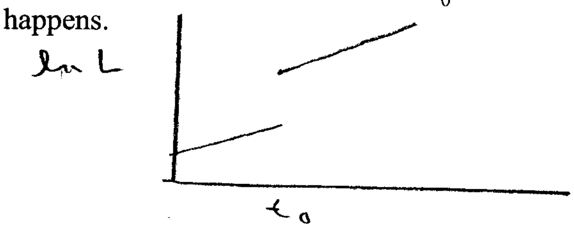
a) Suppose there is an unexpected, one-time upward jump (increase) in the population, with no change in the steady growth rate n.

i) 3 pts Recall the phase diagram with k on the horizontal axis and c on the vertical axis. Using such a graph, show what happens to the economy in response to the event. Use the following symbols to label points:

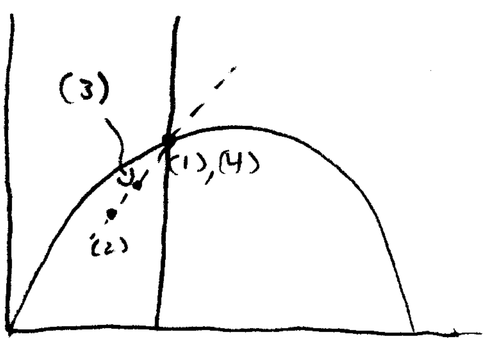
(1) to label the point that is the initial LRSS before the event
(2) to label the point that is the combination of c and k immediately after the event
(3) to label a combination of c and k some time after the event, but before the new LRSS
(4) to label the point that is the new LRSS after the event.

ii) 3 pts Consider what happens over time to consumption per person C/L. Draw a graph with the log of (C/L) on the vertical axis and time on the horizontal axis. Use t_0 to denote the point in time that the event occurs. Show what happens.

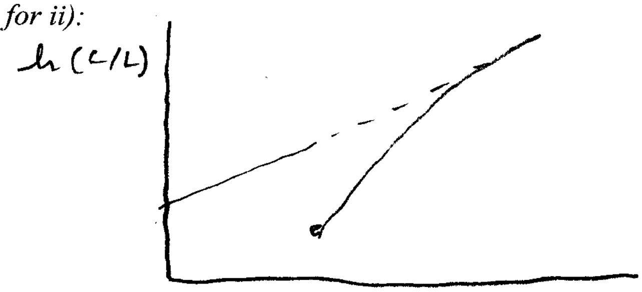
This question means that what happens to L is:



so the answer to i) is:



and for ii):



Note that C/L eventually returns to the same level it would have been on without the shock. This is because the path for A was not affected by the shock, so the same value for c in LRSS means the same value for Ac = C/L.

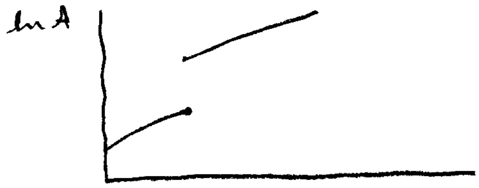
b) Suppose there is an unexpected, one-time upward jump (increase) in A, with no change in its steady growth rate g. Answer as for part a). That is:

i) Using a phase diagram, show what happens to the economy in response to the event, with:

(1) to label the point that is the initial LRSS before the event
(2) to label the point that is the combination of c and k immediately after the event
(3) to label a combination of c and k some time after the event, but before the new LRSS
(4) to label the point that is the new LRSS after the event.

ii) Draw a graph with the log of (C/L) on the vertical axis and time on the horizontal axis. Use t_0 to denote the point in time that the event occurs. Show what happens.

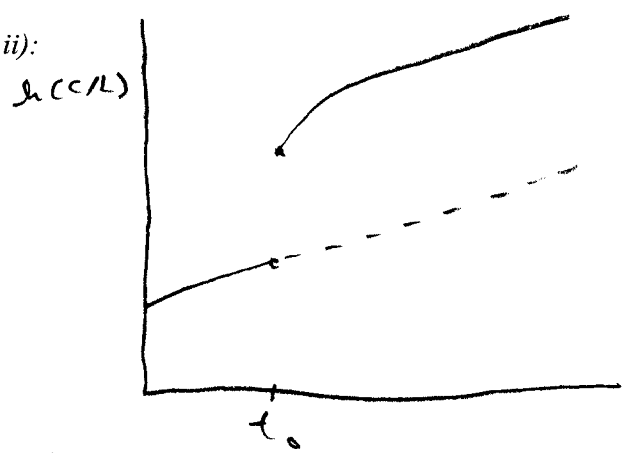
What happens to A is:



so i):

same as a), i)

and ii):



LRSS C/L is at a higher level than it would have been without the shock. This is because A is at a higher value than it would have been before the shock, so the same value of c means higher Ac = C/L.

2) Consider a closed-economy IS-LM model with a fixed price level and a fixed money supply *M*, described by the following expressions:

$$\frac{M}{P} = L(i,Y) \quad where \quad L_i<0,\ L_Y>0$$

$$Y = E(Y,r,G,T) \quad where \quad 0 < E_Y < 1\ ,\ E_r < 0\ ,\ E_G>0,\ E_T<0$$

As usual, *r* denotes the real interest rate. Consider the effect of an exogenous change in expected inflation π^e. Derive an expression showing the effect of a change in π^e on output *Y*, and an expression showing the effect of a change in π^e on the nominal interest rate *i*.

6 pts.

$$i = \pi^e + r \qquad \frac{\partial i}{\partial \pi^e} = 1 + \frac{\partial r}{\partial \pi^e} \qquad \frac{\partial r}{\partial \pi^e} = \frac{\partial i}{\partial \pi^e} - 1$$

$$Y = E(Y, r, G, T) \qquad \frac{M}{P} = L(i, Y)$$

$$\frac{\partial Y}{\partial \pi^e} = E_Y \frac{\partial Y}{\partial \pi^e} + E_r \frac{\partial r}{\partial \pi^e} \qquad 0 = L_i \frac{\partial i}{\partial \pi^e} + L_Y \frac{\partial Y}{\partial \pi^e}$$

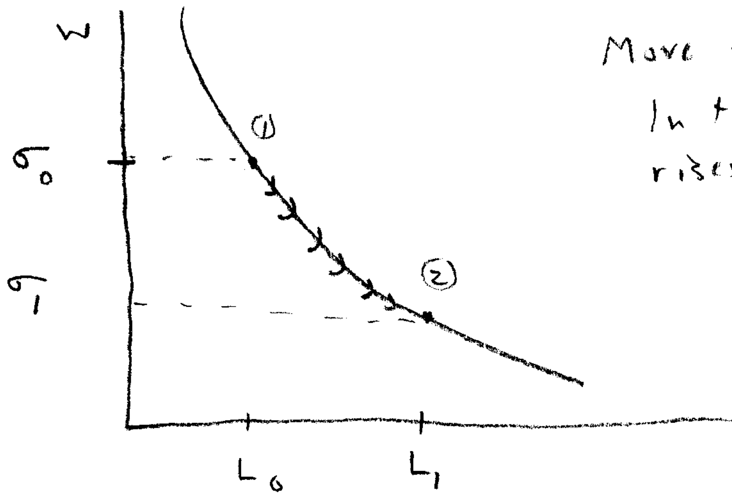
$$\frac{\partial Y}{\partial \pi^e} = E_Y \frac{\partial Y}{\partial \pi^e} + E_r \left(\frac{\partial i}{\partial \pi^e} - 1\right) \qquad \Rightarrow \frac{\partial i}{\partial \pi^e} = -\frac{L_Y}{L_i}\frac{\partial Y}{\partial \pi^e}$$

end up with

$$\frac{\partial Y}{\partial \pi^e} = \frac{-E_r}{(1-E_Y) + E_r \frac{L_Y}{L_i}} = \frac{-1}{\frac{1-E_Y}{E_r} + \frac{L_Y}{L_i}} > 0$$

to get $\frac{\partial i}{\partial \pi^e}$, stick in here

3) Consider the Malthusian model of economic growth, with the rate of population growth described by: $n = G(w - \sigma)$ $\qquad G'(\,) > 0$
where w is the real wage received by peasants.
Assume the economy is initially in its long-run steady state.
a) Describe what happens to the population and the real wage if there is a permanent decrease in the parameter σ. Use *words in full sentences* and a graph with the real wage on the vertical axis, population on the horizontal axis.



Move gradually from 1 to 2
In the long run, the population rises and the real wage falls.

b) Consider an economy where the population's birth rate b (births per capita, per unit time) and mortality rate d (deaths per capita, per unit time) are given by:

$$b = c + fw \qquad\qquad d = h - jw$$

where c, f, h and j are all positive parameters.

i) Is this economy a special case of the Malthusian model, or is it fundamentally different from the Malthusian model? Explain, using *words in full sentences* and an equation or equations.
ii) Suppose that improvements in public order *reduce* the death rate that would occur at any given real wage. Explain how this would affect the peasants' standard of living in the long run, using *words in full sentences* and an equation or equations. *6 pts total. This is a special case of the Malthusian model. The rate of population growth n is equal to the birth rate minus the death rate. The subsistence wage σ is the value of w that makes n = 0, which is to say b=d. So:*

$$n = c + fw - (h - jw)$$

$$n = c - h + (f + j)w$$

Setting $n = 0$ gives σ:

$$0 = c - h + (f + j)\sigma$$

$$\Rightarrow \sigma = \frac{h - c}{f + j}$$

$$G(w - \sigma) = c - h + (f + j)\Big(w - \underbrace{\frac{h-c}{f+j}}_{\sigma}\Big)$$

$$G'(\,) = f + j > 0$$

As you can see from the above expressions, a decrease in h causes a decrease in σ, with results as in a).

4) Consider an economy where

$$y_t = -\beta r_t$$

$$\pi_t = {}_{t-1}\pi_t^e + \alpha y_t + \varepsilon_t$$

In these expressions, y is the output gap and r_t is the *difference* between the real interest rate and the natural rate of interest. The real interest rate is $i_t - {}_t\pi_{t+1}^e$.

ε_t is an i.i.d. random variable. As of period (t-1), the expected value of ε_t is zero.

The central bank sets the nominal interest rate i_t. When it sets i_t, it knows the value of ε_t and the public's inflation expectations. The central bank acts to minimize a loss function:

$L = \frac{1}{2}E[y_t^2] + \frac{1}{2}E[(\pi_t - \pi^*)^2]$ where y is the output gap.

a) Derive the value of r_t that the central bank sets taking ${}_{t-1}\pi_t^e$ as given. *3 pts. It was important to notice that the central bank observes ε_t before it sets i_t, so there's no uncertainty from the central bank's point of view; the central bank's expected value for ε_t is ε_t. Hence:*

$$L = \tfrac{1}{2}(-\beta r)^2 + \tfrac{1}{2}(\pi^e - \alpha\beta r + \varepsilon - \pi^*)$$

$$\frac{\partial L}{\partial r} = 0 = (-\beta r)(-\beta) + (\pi^e - \alpha\beta r + \varepsilon - \pi^*)(-\alpha\beta)$$

$$\Rightarrow r = \frac{\alpha}{(1+\alpha^2)\beta}(\pi^e - \pi^* + \varepsilon)$$

b) Now suppose the economy is in rational expectations equilibrium. Derive the value of r_t that the central bank sets in rational expectations equilibrium, and the values of y_t and π_t that will result. *3 pts.*

$$\pi = \pi^e - \alpha\beta r + \varepsilon = \pi^e - \frac{\alpha^2}{(1+\alpha^2)}(\pi^e - \pi^* + \varepsilon) + \varepsilon$$

In REE,

$$\pi^e = E[\pi] = \pi^e - \frac{\alpha^2}{(1+\alpha^2)}(\pi^e - \pi^*)$$

(from public's point of view, $E_{t-1}[\varepsilon_t] = 0$

$$\Rightarrow \pi^e = \pi^*$$

hence

$$r = \frac{\alpha}{(1+\alpha^2)\beta}\varepsilon$$

$$y = -\beta r = -\frac{\alpha}{(1+\alpha^2)}\varepsilon$$

$$\pi = \pi^* + \alpha y + \varepsilon = \pi^* - \frac{\alpha^2}{1+\alpha^2}\varepsilon + \varepsilon$$

π^e in REE

$$= \pi^* + \left(1 - \frac{\alpha^2}{1+\alpha^2}\right)\varepsilon$$

$$= \pi^* + \frac{1}{1+\alpha^2}\varepsilon$$

c) Write down an equation that gives the nominal interest rate i_t as a function of the same period's inflation rate π_t, denoting the natural rate of interest by $\bar{r}$. *4 pts. I should have said, "in rational expectations equilibrium." I gave full credit for answers that were correct outside REE. But to give the asnwer for REE:*

$$i_t = \bar{r} + r_t + {}_t\pi^e_{t+1}$$ In REE, ${}_t\pi^e_{t+1} = \pi^*$, $r_t = \frac{\alpha}{(1+\alpha^2)\beta}\varepsilon$

so

$$i_t = \bar{r} + \frac{\alpha}{(1+\alpha^2)\beta}\varepsilon + \pi^*$$

How can it be put in terms of π?

In REE, $\pi = \pi^* + \frac{1}{1+\alpha^2}\varepsilon$

so $\varepsilon = (1+\alpha^2)(\pi_t - \pi^*)$

hence

$$i_t = \bar{r} + \frac{\alpha}{\beta}(\pi_t - \pi^*) + \pi^*$$

c) Now consider a model with "habit formation" in the utility function, so that:

$$E\left[\sum_{t=0}^{\infty} e^{-\rho t}\left(\frac{(c_t - bc_{t-1})^{1-\theta}}{1-\theta} - zl_t^2\right)\right]$$

i) Using appropriate mathematical expressions and words in full sentences, explain why this change to the felicity function tends to create "persistence" in consumption. *4 pts. In this utility function, last period's consumption is positively related to the marginal utility of consumption this period:*

$$\frac{\partial \text{felicity}}{\partial c_t} = (c_t - bc_{t-1})^{-\theta}$$

$$\frac{\partial^2}{\partial c_t \partial c_{t-1}} = -\theta(c_t - bc_{t-1})^{-\theta-1}(-b) > 0$$

note this is not $\frac{\partial^2}{\partial c_t^2}$

Thus, at any given values for lifetime income and the current rela interest rate, this period's consumption will be higher (lower) if last period's consumption was higher (lower).

ii) In order for this habit-formation model to have a long-run steady state, the value of the felicity-function parameter θ must be one, so that the utility function is equivalent to:

$$E\left[\sum_{t=0}^{\infty} e^{-\rho t}\left(\ln(c_t - bc_{t-1}) - zl_t^2\right)\right]$$

Using appropriate mathematical expressions and words in full sentences, explain why this is true. *4 pts. Using the fact that $c_{t-1} = e^{-g}c_t$, you can derive an expression that gives the value of θ in terms of parameters alone, which implies that θ equals one. An expression which has values of consumption in it - c_t, c_{t-1}, c_{t-2} - won't do the trick.*

Intratemporal f.o.c. is:

$$(c_t - bc_{t-1})^{-\theta} w_t = 2zl_t$$

From $c_{t-1} = e^{-g} c_t$,

$$(c_t - be^{-g}c_t)^{-\theta} w_t = \ldots$$

$$(1 - be^{-g})^{-\theta} c_t^{-\theta} w_t = \ldots$$

From $z_{t+1} = z_t$ and $\frac{c_{t+1}}{c_t} = \frac{w_{t+1}}{w_t} = e^{g}$

$$(e^{g})^{-\theta} e^{g} = 1$$

$$e^{g} = e^{g\theta}$$

Take logs

$$g = g\theta$$

$$1 = \theta$$

6) Consider an economy like Romer's baseline real business cycle model. For simplicity, assume that the population is fixed and there is one person per household. The immortal representative-agent person-household acts to maximize:

$$E\left[\sum_{t=0}^{\infty} e^{-\rho t}\left(\frac{c_t^{1-\theta}}{1-\theta} - z l_t^2\right)\right]$$

where z is a parameter and notation is as usual: l is the fraction of his time that a household-person supplies as labor and c is his consumption.The technology parameter A is has a long-run trend growth rate g. Let r denote the real interest rate, equal to the return to capital after depreciation, and w denote the real wage per unit of labor (not per efficiency-unit of labor).

a) Write down the "intratemporal first-order condition" that relates a person's consumption c in a period to the same period's real wage w and labor-supply fraction l. *3 pts.*

$$\frac{\partial \text{Felicity}}{\partial c_t} w_t = -\frac{\partial \text{felicity}}{\partial l_t}$$

$$c_t^{-\theta} w_t = 2 z l_t$$

b) In a nonstochastic long-run steady state, both the real wage and consumption must be growing at rate g. Using this fact and your answer to a), demonstrate that the value of the felicity-function parameter θ must be one, so that the utility function is equivalent to:

$$E\left[\sum_{t=0}^{\infty} e^{-\rho t}\left(\ln(c_t) - z l_t^2\right)\right]$$

4 pts.

From a),

$$\frac{c_{t+1}^{-\theta} w_{t+1}}{c_t^{-\theta} w_t} = \frac{2 z l_{t+1}}{2 z l_t}$$

$$\left(\frac{c_{t+1}}{c_t}\right)^{-\theta} \frac{w_{t+1}}{w_t} = \frac{2 z l_{t+1}}{2 z l_t}$$

Across two periods,

$$\frac{(1-be^{-g})^{-\theta} c_{t+1}^{-\theta} w_{t+1}}{(1-be^{-g})^{-\theta} c_t^{-\theta} w_t} = \frac{2 z Z_{t+1}}{2 z Z_t}$$

$$\frac{c_{t+1}^{-\theta} w_{t+1}}{c_t^{-\theta} w_t} = \text{-----}$$

Proceed as for b),

7) Consider the old-Keynesian Friedman-Phelps Phillips curve $\pi_t = {}_{t-1}\pi_t^e + \beta y_t$

versus the new-Keynesian Phillips curve $\pi_t = {}_t\pi_{t+1}^e + \beta y_t$

and their implications for the time-series behavior of the output gap y and inflation π under rational expectations.

a) In what way is the new-Keynesian Phillips curve (plus rational expectations) *more* consistent with the actual time-series behavior of the output gap y in reality? Use words in full sentences *and* an equation or equations in your answer. *4 pts. The old-K P C plus RE implies that the expected value of the output gap is always zero:*

$$\pi_t = {}_{t-1}\pi_t^e + \beta y_t$$

$$E_{t-1}[\pi_t] = {}_{t-1}\pi_t^e = {}_{t-1}\pi_t^e + \beta\, {}_{t-1}y_t^e$$

rational expectations

hence ${}_{t-1}y_t^e = 0$

The future output gap must be uncorrelated with any information available today, including today's estimate of today's output gap. So there can be no serial correlation in the output gap (and one never forecasts that a current recession(boom) will continue into the future). In reality, there appears to be lots of serial correlation in the output gap (sometimes people expect a current recession(boom) to continue into the future). In new-K P C, it is possible for the expected value of the output gap to be positive or negative:

$$\pi_t = {}_t\pi_{t+1}^e + \beta y_t$$

$$E_{t-1}[\pi_t] = {}_{t-1}\pi_t^e = {}_{t-1}\pi_{t+1}^e + \beta\, {}_{t-1}y_t^e$$

$${}_{t-1}y_t^e = -\frac{1}{\beta}\left({}_{t-1}\pi_{t+1}^e - {}_{t-1}\pi_t^e\right)$$

Note: to give you full credit, I had to see some equations.

b) In what way is the new-Keynesian Phillips curve (plus rational expectations) *less* consistent with the actual time-series behavior of inflation π in reality? Use words in full sentences *and* an equation or equations in your answer. *4 pts. The NK PC implies a negative relation between the output gap and the expected change in inflation from the current period to the upcoming period:*

$$\pi_t = {}_t\pi^e_{t+1} + \beta Y_t$$

$${}_t\pi^e_{t+1} - \pi_t = -\beta Y_t$$

and rational expectations implies that the error in that expectation is uncorrelated with the current output gap:

$$\pi_{t+1} - {}_t\pi^e_{t+1} = \varepsilon_{t+1}$$

$$\pi_{t+1} - \pi_t = -\beta Y_t + \varepsilon_{t+1}$$

uncorrelated with Y_t

so an OLS regression of the change in inflation on the previous period's output gap should give a negative coefficient on the output gap. That doesn't match the data. The OK PC doesn't have this implication.

Another good answer: assuming there is a LRSS inflation rate, NK PC implies that the beginning of a recession (boom) will be associated with a downward (upward) jump in inflation. That doesn't happen.

(Another implication of NK PC is that, holding fixed current money supply, an expected future incerase in the money supply must be associated with lower output today. But this is not an answer to the question unless you relate it to the time-series behavior of inflation.)

8) In the "new Keynesian" model that Clarida, Gali and Gertler use to analyse monetary policy,

$$x_t = -\varphi r_t + {}_t\pi^e_{t+1} + g_t$$
$$\pi_t = {}_t\pi^e_{t+1} + \lambda x_t + u_t$$

where x is the output gap, g is the "spending shock," u is the "cost-push" shock, and the central bank can observe the values of both g and u when setting the real interest rate.
Consider the case where the central bank has "discretion" (it cannot "pre-commit" in its policy actions).
a) Describe what the central bank will do, and the results for the economy, if the economy is subject *only* to spending shocks - that is, if u is always zero. *4 pts. The central bank will adjust the real interest rate to completely counteract the effect of a spending shock, so spending shocks will not affect output or inflation. For example, if g is positive, the central bank will cut the real interest rate.*

b) Describe what the central bank will do, and the results for the economy, if the economy is subject *only* to cost-push shocks - that is, if g is always zero. *4 pts. The central bank will adjust the real interest rate to partly counteract the effect of a cost-push shock on inflation. For example, if u is positive, the central bank will raise the real interest rate to depress output, but not enough to completely counteract the effect of the cost-push shock on inflation. Result: a positive cost push shock is associated with a negative output gap and high inflation.*

9) In some models, there is a welfare cost - a loss of profit or utility - associated with an increase in the long-run, predictable, trend inflation rate. In other models, there is no such welfare cost. For each of the models listed below, state whether there is such a welfare cost of higher trend inflation, and *explain* why or why not.
a) Lucas supply function model. *5 pts. No welfare cost. In the LSF model, labor supply, output and consumption are affected only by unexpected changes in inflation, so a change in predictable component of inflation has no effect on these variables. There is no cost of adjusting prices, so there are no costs created by the larger price adjustments associated with a higher trend inflation rate.*
b) Rotemberg's model of a price adjustment cost that increases quadratically with the size of the adjustment. *5 pts. Yes, welfare cost. In this model, there is a cost of letting your price deviation from p*, and a price-adjustment cost that increases with the size of the adjustment. When trend inflation is higher, firms will make larger price adjustments on average every year, and thus incur larger price-adjustment costs on average.*

c) Mankiw-Reis sticky-information model. 5 pts. Two correct answers.
No welfare cost. The predictable, trend inflation rate is part of every firm's information set, so it is built into every firm's price plan: it doesn't affect any firm's relative price or the price level relative to aggregate demand; hence no effect on aggregate output or an individual firm's output.

Maybe welfare cost, maybe not. If the trend inflation rate increases, it will take time for that news to get into every firm's information set; in the meantime, the price level will lag behind aggregate demand and output will be higher. But, because the economy is full of monopolies, output must have been below the Pareto-optimal level before the increase in trend inflation, so....