Macroeconomics Comprehensive Examination RETAKE, Summer 2007

Part I. Answer all of the following questions concisely. Do not waste time deriving things, unless I ask you to specifically.

1) Consider an open economy with a floating exchange rate. Assume that:

- the price level is fixed; expected future inflation is always zero
- there is "static exchange-rate expectations," that is the expected future change in the exchange rate is always zero.
- the country's net exports are not affected by its own output Y: net exports is simply an increasing function of the real exchange rate $NX(\varepsilon)$.

Using graphs and words, explain what happens to *output*, the *exchange rate*, and the *domestic interest rate* if there is an increase in the foreign interest rate i^* assuming that:

a) the economy is a "large" economy with imperfect international capital mobility, and the central bank holds the money supply M at a fixed value.

b) the economy is a "small" open economy with perfect international capital mobility, and the central bank holds the money supply M at a fixed value.

c) the economy is a "large" open economy with imperfect capital mobility, and the central bank conducts monetary policy by following an interest-rate rule $r = r(Y, \pi)$ *where* $r_Y > 0$, $r_\pi > 0$.

2) Consider the Ramsey (Ramsey-Cass-Koopmans) model, assuming that:

$$U = \int_{t=0}^{\infty} e^{-\rho t}\, u(c_t)\, dt \quad \text{where} \quad u(c) = \frac{c^{1-\theta}}{1-\theta}$$

The rate of growth in TFP (total factor productivity) is g. The rate of population growth is n.

a) Recall the graph depicting the steady-state loci of the model (points where $\dot{c} = 0$ and $\dot{k} = 0$), the stable arm (saddle path), and the long-run steady state. Using such a graph, show what happens to the economy over time in response to each of the events described below. Assume that the economy is initially in a long-run steady state. Use the following symbols to label points:

(1) to label the point that is the initial LRSS before the event
(2) to label the point that is the combination of c and k immediately after the event
(3) to label a combination of c and k some time after the event, but before the new LRSS
(4) to label the point that is the new LRSS after the event.

i) An increase in ρ.
ii) An increase in θ.

b) Using graphs with time on the horizontal axis and consumption per *person* on the vertical axis, show what happens over time to *consumption per person* when there is an increase in ρ. Use t_0 to denote the point in time that the event occurs.

3) Consider an economy where:

$$\pi_t = {}_{t-1}\pi_t^e + \alpha y_t$$

$$y_t = -\beta_t(r_t - \bar{r}) + \varepsilon_{1_t} + \varepsilon_{2_t} \quad \textit{where } \beta_t = \bar{\beta} + \varepsilon_{3_t}$$

where y is the output gap and r is the real interest rate. ε_1, ε_2 *and* ε_3 are three i.i.d. random variables (mean zero, no serial correlation), uncorrelated with each other.

The central bank sets r_t to minimize a loss function:

$$L = E\left[\frac{1}{2}y_t^2 + \frac{1}{2}(\pi_t - \pi^*)^2\right]$$

The public *cannot* observe any of the shocks ε_{1_t}, ε_{2_t} or ε_{3_t} at the time that it forms its inflation expectation ${}_{t-1}\pi_t^e$. But the central bank knows all of the public's inflation expectations at the time it sets the interest rate r_t.

For each of the following cases, tell me *in words* (and in equations *too* if you want):

i) how the inflation rate π behaves over time: does it vary from period to period, or remain equal to π^*?

ii) how the real interest rate r behaves over time: does it vary from period to period, or remain equal to $\bar{r}$?

iii) whether the correlation between the inflation rate π and the real interest rate r is positive, negative, or zero.

a) The public has rational expectations. It knows the central bank's loss function and desired inflation rate π^*. The central bank can observe ε_{1_t}, ε_{2_t} *and* ε_{3_t} before it sets the period's real interest rate r_t.

b) The public has rational expectations. It knows the central bank's loss function and desired inflation rate π^*. The central bank can observe ε_{1_t} *and* ε_{3_t}, but *not* ε_{2_t}, before it sets the period's real interest rate r_t.

c) The public has rational expectations. It knows the central bank's loss function and desired inflation rate π^*. The central bank can observe ε_{1_t} *and* ε_{2_t}, but *not* ε_{3_t}, before it sets the period's real interest rate r_t.

d) The public does *not* have rational expectations. Its inflation expectation is described by:

$${}_{t-1}\pi_t^e = \pi^* + \varepsilon_{4_t}$$

where ε_4 is an i.i.d. random variable uncorrelated with the other ε's. The central bank observes ε_{1_t}, ε_{2_t}, ε_{3_t} *and* ε_{4_t} before it sets the period's real interest rate r_t.

4) Consider an economy like Romer's "baseline" real business cycle model. For simplicity, assume the population is fixed, with one person in each household, and there is just one household in the economy (N=1, H=1). The person acts to maximize:

$$E\left[\sum_{t=0}^{\infty} e^{-\rho t}\left(\ln(c_t) - \theta l_t^2\right)\right]$$

where $K_{t+1} = K_t + Y_t - C_t - \delta K_t$ $\qquad$ $Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha}$

The technology parameter A is has a long-run trend growth rate g. It may also be subject to random shocks. c is consumption per household. l is the fraction of its time that a household supplies as labor. (Since we've assumed N = H = 1, l = L, but that doesn't matter here.) Denote the real wage by $w = \partial Y/\partial L$. Denote the real interest rate, or return to capital after depreciation, as $r = \partial Y/\partial K - \delta$.

a) *Derive* the nonstochastic long-run steady-state real interest rate r^* and capital per efficiency-unit of labor k^* in terms of the relevant parameters of the model.

b) Write an expression that shows LRSS consumption per efficiency-unit of labor in terms of k^* and parameters.

5) Consider an RBC model with "indivisible labor" and "consumption insurance." A person must work a fraction l_0 of his time, or not at all. A household acts to maximize:

$$E\left[\sum_{t=0}^{\infty} e^{-\rho t}\left(\frac{\ln(c_t)}{(1-l_t)} + \ln(1-l_t)\right)\right] \qquad \textit{where } \gamma > 0$$

Let c_1 denote consumption allocated to a worker (a person assigned to work in a period), and c_2 denote consumption allocated to a nonworker (a person assigned to not work in a period). Will c_1 be greater than, less than, or equal to c_2? Explain your answer.

6) Consider two models of aggregate supply and the Phillips curve: the Lucas supply function and the Calvo model.

a) Assuming rational expectations, one of the two models implies that recessions (and booms) are never predictable. That is, the expected value for next period's output gap must always be equal to zero. Explain, using an equation or equations.

b) Assuming rational expectations, one of the two models implies that there must be a boom (positive output gap) today if the central bank announces that it will lower its target inflation rate in the future, and the announcement is believed by the public. Explain, using an equation or equations.

7) Recall that "real rigidity" means a low value of φ in the equation:

$$p_i^* = Constant + p + \varphi y$$

where y is the output gap, p is the price level (average price charged by all other firms), and p_i^* is an individual firm's profit-maximizing price *before* accounting for any costs or constraints on price adjustment. Consider each of the following possible characteristics of an economy. State whether it tends to increase real rigidity (make φ smaller), decrease real rigidity, or have no effect on the degree of real rigidity. Explain your answer. Do *not* assume that the economy is described by Romer's baseline Model of Imperfect Competition.

a) Because they sell in oligopolistic product markets, most firms do not have fixed capital stocks that minimize cost at their usual production levels. Instead, most firms have capital stocks that are *larger* than the capital stocks that would minimize production costs.

b) During recessions, consumers spend more time searching for the lowest price on whatever they intend to buy.

c) The labor market is characterized by "indivisible labor" and "consumption insurance." A person must work a fraction l_0 of his time or not at all. A lottery assigns individual workers to work or not work on a particular day.

8) In a comment on the Mankiw-Reis "sticky information" model, Robert King said that the model is inconsistent with "one of the central facts that new Keynesian macroeconomics has long stressed, which is the tendency for many nominal prices to stay constant for substantial periods of time." How is the model inconsistent with this fact?

9) In their paper "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," Christiano, Eichenbaum and Evans assume that the representative agent's intertemporal utility function displays "habit formation."

a) Explain what this assumption means in terms of the marginal utility of consumption in a particular period.

b) Explain how this assumption affects the response of consumption to a brief, temporary, unexpected increase in the real interest rate above the long-run steady-state value.

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Part II. Answer **BOTH** questions.

Required Question 1

Bennett McCallum analyzed the following model in the Federal Reserve Bank of St. Louis *Review* in July/August 2001. The model has been simplified considerably in several different ways.

In period $t = 1$, a consumer maximizes lifetime utility,

$$\sum_{t=1}^{\infty} \beta^{t-1} u(c_t)$$

subject to inter-temporal budget constraints of the following form:

$$w_t + \frac{m_{t-1}}{1+\pi_t} + b_t = c_t + m_t + \frac{b_{t+1}}{1+r_t} + \psi(c_t, m_t)$$

All variables are defined in real terms. m is money, b is bonds, and c is consumption. r is the real interest rate, π is the rate of inflation, and w is the real wage (labor supply is assumed to be fixed at one unit).

Note: McCallum's model has a tricky notation for money and bonds. In period $t = 1$, the consumer already chose m_0, b_1 in the past, but now chooses m_1, b_2, c_1 (as well as the future values of these variables).

1) State Bellman's equation. Make clear what are the states and controls.
2) Derive the Euler equation for consumption.
3) Does this model produce a money demand equation? Explain and interpret.
4) Under what conditions would this model produce an IS curve that does not contain money? Explain and interpret.
5) What does Fisher's identity look like in this model?
6) What assumptions would you make about the function ψ ? What does this function measure? i.e. how do you interpret this model in economic terms?
7) How does your finding in 4) relate to discussions from the macro theory courses?

Required Question 2

Answer the following questions:

1) Write down a standard linear quadratic dynamic programming problem. State all mathematical assumptions clearly.

2) Economic models are not typically of the linear quadratic form. How then could linear quadratic dynamic programming be useful in macroeconomics?

3) How would you derive the algebraic Riccati equation? Don't actually do it! But, it must be *crystal clear* that you know how to do it and that you could do it if you had time. Here, you just explain the basic steps and you must prove that you understand what you are doing and why you are doing it.

4) A key concept is *certainty equivalence*. In the context of linear-quadratic models, discuss certainty equivalence.