

# MICROECONOMIC THEORY COMPREHENSIVE EXAMINATION

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Write legibly in blue or black ink, not pencil. Begin each question on a fresh page. Number the questions you are answering on the cover of each blue book.

**SECTION A** (Select 8 of 10. True or false, taking the time to explain why in detail.)

1. "If a prisoners' dilemma is repeated 100 times, the players are sure to achieve their cooperative outcome."
2. "The elementary utility function  $u(y) = 1 - e^{-\lambda y}$  over income,  $y$ , implies a constant measure of *relative* risk aversion:  $R_r = \lambda$ , for all  $\lambda > 0$ ."
3. "The fact that all factors are paid their value of marginal productivity can not guarantee product exhaustion in a competitive industry (or economy), unless all production functions are homogenous of the first degree."
4. "If a consumer receives income in goods, then the demand function is homogenous of degree zero in prices."
5. "If we impose a constant per unit tax on the output of a monopolist then price will rise by exactly half the amount of the tax."
6. "Assuming a profit maximizing firm then the firm's demand curve for an input will always show that more is bought at a lower price."
7. "If we imagine Bertrand duopolists but assume that the quantity one can sell will be positively related to the price of its rival then in equilibrium price will be set equal to marginal cost."
8. "If a person can borrow only at interest rates that exceed what he can lend at then she will make any investment that given the borrowing rate will increase the present value of her wealth."
9. "If the government gives some of a good free to each consumer then obviously total consumption of the good must increase even if anyone who takes the government gift can not supplement expenditures on the good out of their own pocket."
10. "If a firm has monopsony power and can discriminate among two equally productive factors, then it will pay the one in more elastic supply a lower wage."

**SECTION B** (Answer 3 of 5. Short answers.)

11. Two boy scouts are in the vicinity of an old lady that is looking for help crossing the street. Only one person is needed to help her (more are okay but no better than one). Each boy has to choose simultaneously whether to do so, and each will get pleasure worth a 4 from her success in crossing the street, no matter who actually helps her. However, each helper will bear a cost of 1 (the value of time consumed in helping).
- (a) Set the payoff bimatrix of this game and find all pure- and mixed-strategy Nash equilibria.
- (b) Under the mixed-strategy, what is the probability that the old lady will have to cross the street by herself?

12. A consumer with infinitely long life horizon is endowed with 1 unit of a commodity that she consumes over her lifetime. The commodity is perfectly storable and she will receive no more than she has now. Consumption of the commodity in time  $t$  is denoted  $c_t$ , and her lifetime utility is given by

$$u(c_0, c_1, c_2, \dots) = \sum_{(t=0)}^{\infty} \beta^t \ln(c_t), \quad \text{where } 0 < \beta < 1.$$

Find the optimal level of consumption in each period.

13. Explain why, generally, if a monopolist faces two different types of consumers that he can not charge different prices that his profit maximizing two part tariff involves charging an entrance fee and a usage fee, but the latter exceeds the marginal cost of production.
14. Suppose there are three identical firms each with the same constant marginal cost of production which is \$5. Suppose that the market demand curve is  $Q = 125 - 5P$  and that each is a Cournot quantity setter. What is the equilibrium output of each firm and its profit? How does this contrast with the output that would occur in a competitive industry under the same conditions?
15. A consumer's utility function is given by  $u(x_1, x_2) = \min \{ax_1, bx_2\}$ , with  $a > 0$  and  $b > 0$ .

Prices are  $p_1$  and  $p_2$ , and income is  $m$ .

- (a) What are the Marshallian and Hicksian demand functions?
- (b) What is the expenditure function?
- (c) What is the indirect utility function?

**SECTION C (Answer 2 of 4)**

16. A principal hires an agent to work for him. The agent's utility function is

$u(w, a) = \sqrt{w} - a$ , where  $w$  stands for wage and  $a$  for effort. The agent can choose from two possible levels of effort: "high effort", which means  $a=5$ , or "low effort," which means  $a=0$ . The agent's reservation utility is 5. The probability distribution of the principal's gross profits ( $S_i$ ) conditional on the agent's effort level is given by:

	$S_1 = \$0$	$S_2 = \$500$
$a = 0$	4/5	1/5
$a = 5$	1/2	1/2

Effort levels are NOT observable. So the agent's wage can only depend on gross profit, which is observable.

- (a) What is the optimal way to induce the agent to choose  $a = 0$ ?
  - (b) What is the optimal way to induce the agent to choose  $a = 5$ ?
  - (c) What is the optimal contract to offer this agent?
  - (d) Is it true to say that the principal would have been better off if effort levels were observable? Why?
17. Coke and Pepsi are the two dominant firms in the cola industry. The market size is \$10 billion annually. Each firm can choose whether to advertise. Advertising costs \$1 billion per year for each firm that chooses to do so. If only one firm advertises, it captures the whole market. If both firms advertise, they split the market evenly. If neither advertises, they again split the market evenly, but without the expense of advertising.
- (a) Write down the payoff bimatrix and determine what kind of game is being played here between Coke and Pepsi. (*Battle of the Sexes? Prisoners' Dilemma? Matching Pennies? Chicken? Etc.*)
  - (b) What is the Nash equilibrium solution if the game is played only once?
  - (c) Alternatively, suppose the game is played year after year with no end in sight. Under what conditions can one anticipate no advertising in the cola industry?

18. It costs \$1 to make a pound of cheese and \$2 to make a bottle of wine, and there is only one producer of each on an isolated island. If the cheese-maker charges  $p_c$  per pound and the wine-maker charges  $p_w$  per bottle, their respective weekly sales ( $q_c$  thousand pounds of cheese and  $q_w$  thousand bottles of wine) are given by the following demand functions:

$$Q_c = 10 - p_c - 0.5p_w \quad \text{and} \quad q_w = 12 - 0.5p_c - p_w$$

- (a) If the two producers independently set prices, find the two (Bertrand) reaction curves and the corresponding Nash equilibrium prices.
- (b) Alternatively, if the two producers collude and set prices jointly to maximize their combined profit, show that the prices will be lower and consumers will be better off (hence, the deadweight loss under monopoly is smaller than under the kind of oligopolistic competition in part (a) above).
- (c) What is the intuitive reason for this seemingly strange result? (Explain.)
19. Two editions (hardcover and softcover) of a new book are to be published  $t$  years apart. Copyright arrangements stipulate that publication dates and prices must be announced (by the publisher) in advance. There are two types of readers. Type I have a reservation price of \$24 and type II have a reservation price of \$16. At any given time, however, all readers are indifferent between hardcover and softcover copies of the same book. (To keep the analysis simple, assume negligible production cost and equal number of both types of readers.)

Determine the profit maximizing prices of the two editions ( $p_H$  and  $p_S$ ), and the optimal waiting period,  $t$ , between their publication dates assuming the annual discount rate for the publisher is 10% ( $r = .1$ ), whereas the annual discount rate for readers (who are less patient) is 30% ( $R = .3$ ). [Suggestion: use continuous compounding.]