

Spring 2002-2

**MICROECONOMIC THEORY COMPREHENSIVE EXAMINATION**Greene/M. Jones/Ofek

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Write legibly in blue or black ink, not pencil. Begin each question on a fresh page. Number the questions you are answering on the cover of each blue book.

**PART 1** (Select 8 of 10. True or false, taking the time to explain why in detail.)

1. A risk averter will invest more in a riskless asset than in a risky asset that offers the same expected return.
2. If the price of an input rises, even if it has a negative output elasticity, a competitive firm must use less of it.
3. If a per unit excise tax of  $T$  per unit is imposed on a monopolist, price will rise by exactly  $1/2 T$ .
4. If duopolists act according to Bertrand's assumptions price must fall to marginal cost in equilibrium.
5. Assume an industry with a large number of firms that act as competitors and one dominant firm that acts as a von Stackelberg leader. A subsidy to the competitive producers will lower the price and the leader's profits.
6. The use of second degree price discrimination by a monopolist lowers consumer welfare.
7. If a certain person was originally consuming no  $X$ , a subsidy would be more effective than a voucher in getting him to consume some  $X$ .
8. In a two-good ( $X$  and  $Y$ ) economy, if at a certain price ratio demand equals supply in the market for  $X$  it follows that equilibrium is also achieved in the market for  $Y$ .
9. Suppose that the Lespeyre quantity index exceeds one, then the consumer is worse off.
10. The reason that for a two good world we assume convex indifference curves is that there is a declining marginal utility of each good.

**PART 2 (Answer 3 of 5. Short answers.)**

1. A risk-neutral homeowner has the choice between:

- (i) ordering supplies of heating oil for delivery next winter at a price to be paid then of \$1 per gallon.
- (ii) buying oil next period at the then prevailing price which, with equal probability, may be either \$0.90 or \$1.10 per gallon.

Bearing in mind that quantity demanded depends on price, which of the two options will he choose?

2. Derive the profit function for a firm with the Cobb-Douglas technology,  $y = x_1^a x_2^b$ . What restrictions on  $a$  and  $b$  are required to ensure that the profit function is well-defined? Explain.

3. Prove: If good  $j$  is inferior but not Giffen, then we must have  $\frac{\partial x_j(p, y)}{\partial p_k} > 0$  for at least some  $k \neq j$ .

4. A consumer consumes two goods, food ( $x_1$ ) and water ( $x_2$ ). His utility function is given by

$u(x_1, x_2) = (x_1 - \bar{x}_1)^a (x_2 - \bar{x}_2)^{1-a}$ , where  $0 < a < 1$ ,  $\bar{x}_1 > 0$ , and  $\bar{x}_2 > 0$ . His income is  $y$ , the prices of food are water are  $p_1$  and  $p_2$ , respectively.

a. Find this consumer's Marshallian demand functions for the two goods.

b. Give an economic interpretation of the parameters  $\bar{x}_1$  and  $\bar{x}_2$ . Then comment on the salient feature of this consumer's consumption pattern based on your interpretation. (Hint: we are NOT interested in the sign any partial derivative.)

5. Suppose that the market demand curve is of the form that  $Q = 1000 - 5P$ . Assuming there are five firms all of whom have a total cost  $= 2q$ , and that they all behave as Cournot players find equilibrium price output and profit.

**PART 3 (Answer 2 of 4. Longer answers.)**

1. Marshall's Law of Demand states that if the conditions of competition are fulfilled, then more will be bought and sold in long run competitive equilibrium only at a lower price. This holds even if there are upward sloping portions of a demand curve. Explain why this is so.
2. A local government has a traffic problem and is determined to resolve it by reducing the number of people who park illegally. It has to make a policy choice between two options:
  - (i) increasing by 10% the probability that illegal parking will result in a conviction;
  - (ii) raising by 10% the fine which is imposed once convicted.
  - (a) If law breakers are risk averse, which policy should be enacted?
  - (b) Alternatively, suppose that the real problem is not the traffic, but the budget, and the government is attempting to resolve it by using the proceeds from the fine to help cover its deficit. What change, if any, would you make to your answer in part (a) above?
3. Air Shangri-la is the only airline allowed to fly between the islands of Shangri-la and Nirvana. There are two types of passengers, tourist and business. Business travelers are willing to pay more than tourists. The airline, however, cannot tell directly whether a ticket purchaser is a tourist or a business traveler. The two types do differ, though, in how much they are willing to pay to avoid having to purchase their tickets in advance. (Passengers do not like to commit themselves in advance to traveling at a particular time.)

More specifically, the utility levels of each of the two types net of the price of the ticket,  $P$ , for any given amount of time  $W$  prior to the flight that the ticket is purchased are given by

$$\text{Business: } v - \theta_B P - W,$$

$$\text{Tourist: } v - \theta_T P - W$$

where  $0 < \theta_B < \theta_T$ . (Note that for any given level of  $W$ , the business traveler is willing to pay more for his ticket. Also, the business traveler is willing to pay more for any given reduction in  $W$ .) The proportion of travelers who are tourists is  $\lambda$ . Assume that the cost of transporting a passenger is  $c$ .

Assume in (a) to (d) that Air Shangri-la wants to carry both types of passengers.

- a. Draw the indifference curves of the two types in  $(P, W)$ -space. Draw the airline's isoprofit curves. Now formulate the optimal (profit-maximizing) price discrimination problem mathematically that Air Shangri-la would want to solve. (Hint: impose nonnegativity of prices as a constraint, since, if it charged a negative price, it would sell an infinite number of tickets at this price.)
- b. Show that in the optimal solution, tourists are indifferent between buying a ticket and not going at all.

- c. Show that in the optimal solution, business travelers never buy their ticket prior to the flight and are just indifferent between doing this and buying when tourists buy.
- d. Describe fully the optimal price discrimination scheme under the assumption that they sell to both types. How does it depend on the underlying parameters  $\lambda$ ,  $\theta_B$ ,  $\theta_T$ , and  $c$ ?
- e. Under what circumstances will Air Shangri-la choose to serve only business travelers?

4. In the following game,

		Player II			
		$t_1$	$t_2$	$t_3$	$t_4$
Player I	$s_1$	0, 0	2, 3	1, 1	4, 2
	$s_2$	1, 4	0, 1	2, 2	3, 0

- a. What strategies survive elimination of dominated strategies?
- b. Find the pure-strategy Nash equilibria (if any).
- c. Solve for the mixed-strategy Nash equilibria (if any) and find the expected payoff to each player.
- d. Obtain the maximum (security) strategy of player I. What is the expected payoff to each player if I is determined to play his maximum strategy and II knows it?

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## SECTION A (Select 8 of 10. True or false, taking the time to explain why in detail.)

1. In a perfect capital market, a change in an individual's intertemporal preferences will not influence his decision whether or not to undertake a given investment project.
2. An inferior equilibrium point like *Bottom-Right* in the following game is perfectly consistent with the notion of a pure-strategy Nash equilibrium but it rules out the possibility of having a mixed-strategy Nash equilibrium in this kind of games.

	Left	Right
Top	2,2	0,0
Bottom	0,0	1,1

3. If a majority of people are risk averse then risky occupations will command higher wages than safe jobs.
4. Osso buco is a very tasty dish made with veal shanks. Veal Milanese is also a tasty dish made of veal cutlets. Suppose only if the demand for Veal Milanese rises, the price of veal shanks rises also.
5. In the case of differentiated products sold by two duopolists, it still is the case that price will tend to fall to marginal cost if they behave according to the Bertrand assumption.
6. Even if the quantity demand of a good is positively related to its price, it cannot always be so.
7. Suppose two factors of production are equally productive for some monopsonist who uses both and pays them different prices. A law, or some other device that prevents such factor price discrimination, will lead to a contraction in the usage of the factor in more elastic supply.
8. Suppose that people consume positive amounts of some private good at market prices. Suppose the government begins to provide some free of charge but prohibits those who accept it any supplementation of the government provisions. All consume more of the good and the total consumption rises.
9. If we imagine the isoquants for a Cobb Douglas production function, none of them could ever conceivably have positively sloped sections.
10. If a person's monetary income rises more quickly than a Paasche price index based on his own patterns of consumption, he is better off.

**SECTION B (Answer 3 of 5. Short answers.)**

1. A ship is overdue in port and a shortage of water develops. The limited supplies available are divided amongst all those on board. One of the crew receives 225 pints which is his supply of water from today, day 1, until the ship docks. His utility function is  $U(q) = 600q - 2.5q^2$  each day, where  $q$  is daily consumption of water (in pints). The probability of making landfall at the end of day 1 is 0.6, at the end of day 2 is 0.3 and at the end of day 3 is 0.1. How many pints of water does the sailor allocate to consumption on each of the three days?
2. Two identical firms operate in the same market. The payoff to both firms is  $\pi_j$  if they produce the level of output that maximizes their joint profits and  $\pi_c$  if they produce the Cournot level of output. The maximum payoff that one player can get if the other chooses the joint profit maximizing output is  $\pi_d$  reducing, in consequence, the payoff to its (losing) counterpart down to  $\pi_l$ .
  - (a) Describe the situation in a 2x2 bimatrix game. Show that this game is equivalent to the Prisoners' Dilemma.
  - (b) Suppose the game in part (a) above is to be repeated indefinitely with each player adopting the punishment (grim) strategy of reverting to the Cournot game if the other player defects from the joint profit-maximizing strategy. How large can the discount rate,  $r$ , that assures cooperation, be?
  - (c) Alternatively, suppose that the same game is to be repeated only 10 times. Is a punishment strategy optimal in this case? Explain.
3. Explain why, generally, if a monopolist faces two different types of consumers that he cannot charge different prices, that his profit maximizing two part tariff involves charging an entrance fee and a usage fee, but the latter exceeds the marginal cost of production.
4. Explain whether you can be certain that in a monopolistically competitive market there is too much variety or too many different brands from an efficiency perspective.
5. Suppose there are two types of used cars: peaches and lemons. A peach is worth \$3,000 to a buyer and \$2,500 to a seller. A lemon is worth \$2,000 to a buyer and \$1,000 to a seller. There are twice as many lemons as peaches. Buyers can't tell the quality of a given car, but sellers can.
  - (a) Draw demand and supply curves for used cars. In equilibrium, what will be the price of a used car? What types of cars will be traded? Is this an efficient outcome? Why?
  - (b) Suppose, instead, that there are twice as many peaches as lemons. Then how would your answer to (a) change?

**SECTION C (Answer 2 of 4. Longer answers.)**

1. An investor with income of  $w = \$10,000$ , from which he derives utility according to the function  $u(w) = \sqrt{w}$ , is offered a project with equal probability chance of making \$12,000 or losing \$8,000.
  - (a) Will the investor choose to undertake the project?
  - (b) Alternatively, suppose the project in question is to be equally shared between two investors both with the income and preferences above. Will they choose to invest on this basis?
  - (c) What is the cost of the risk associated with this project in (a) above? What is the total cost (i.e. summed over the two individuals) of risk in (b)?
  
2. Given the payoffs bimatrix for the game

		Player II		
		$t_1$	$t_2$	$t_3$
Player I	$s_1$	6, 0	-1, - 100	0, 1
	$s_2$	2, 2	0, 3	1, 1

- (a) Find the pure strategy point of Nash equilibrium.
- (b) Show that each of the following strategy pairs is a mixed strategy point of Nash equilibrium. What are the players' expected payoffs in each?

$$P = \left( \frac{1}{2}, \frac{1}{2} \right), \quad Q = \left( \frac{1}{5}, 0, \frac{4}{5} \right) \quad \text{and} \quad P' = \left( \frac{1}{101}, \frac{100}{101} \right), \quad Q' = \left( \frac{1}{5}, \frac{4}{5}, 0 \right)$$

- (c) If player I is a long term player who could publicly make a credible commitment of always playing the same mixed strategy that assigns to  $s_1$  probability  $p = \alpha$ , what  $\alpha$  would he choose? What would be the players' expected payoffs under this strategy?

3. Consider the following moral hazard insurance model. Let a consumer's utility be  $u(w, e) = \sqrt{w} - e$ , where  $w$  denotes net wealth, and  $e$  denotes effort. Let her initial wealth be  $w_0 = \$100$ , and suppose that there are but two loss levels,  $L = 0$  and  $L = \$51$ . The consumer can exert two effort levels,  $e = 0$  and  $e = 1/3$ . The loss probabilities are given by the following entries.

	$L = 0$	$L = 51$
$e = 0$	$1/3$	$2/3$
$e = 1/3$	$2/3$	$1/3$

- Find the consumer's reservation utility assuming that there is only one insurance company and that the consumer's only other option is to self-insure.
  - What effort level will the consumer exert if no insurance is available?
  - Show that if effort is observable, then it is optimal for the insurance company to offer a policy that induces high effort.
  - Show that the policy in (c) will not induce high effort if effort is unobservable.
  - Find the optimal policy when effort is unobservable.
  - Argue that the symmetric information solution Pareto dominates that with asymmetric information.
4. Suppose a consumer has no money income but has an initial *endowment* of goods  $w = (w_1, \dots, w_n)$ . Goods can be bought and sold at market prices  $p = (p_1, \dots, p_n)$ . So essentially the consumer's income is the market value of his endowment, or  $y = pw$ . Suppose he chooses his consumption bundle  $x$  to maximize his utility in the usual way.
- Derive the Slutsky equation in this setting, i.e., a mathematical expression for  $\frac{\partial x_i}{\partial p_j}$  which shows explicitly the substitution and income effects.
  - Use a graph to illustrate the income and substitution effects in this setting.
  - True or false:* In this setting, if good  $i$  is normal, then  $\frac{\partial x_i}{\partial p_i} < 0$ . Explain