Spring 2005

MICROECONOMIC THEORY COMPREHENSIVE EXAMINATION #2

Henderson/M. Jones/Yang

Write legibly in blue or black ink, not pencil. Begin each question on a new page. Number the questions you are answering on the cover of each blue book.

SECTION A (Select 8 of 9. True or false, taking the time to explain why in detail)

1. If we observe a consumer choosing (x_1,x_2) when (y_1,y_2) is available one time, we are justified in concluding that $(x_1,x_2) > (y_1,y_2)$ (where > means strictly preferred to).

2. A college football coach says that given any two linemen A and B, he always prefers the one who is bigger and faster. This preference relation is transitive and complete.

3. An individual cannot be indifferent between a flat tax and a commodity tax.

4. Risk averse persons must prefer a safe outcome over a risky one.

5. The "steady-state" is an equivalent statement for the Walrasian equilibrium.

6. To encourage energy saving and to fight global warming, carbon tax (i.e. taxing carbon contents in fossil fuels) is better than Btu tax (i.e. taxing the heat units in commercial energy). [Btu stands for British thermal unit].

7. That utility function is homogenous of degree zero in prices is a necessary condition for the holding of the Walrasian Law.

8. Public goods can be provided efficiently by private sectors.

9. If the utility function is homogeneous, then the cross price effects for any two goods are symmetric, i.e., $\frac{\partial x_i(p,y)}{\partial p_j} = \frac{\partial x_j(p,y)}{\partial p_i}$.

SECTION B (Answer 3 of 4. Short answers.)

10. Graph the following indifference curve and determine whether it obeys the assumption of diminishing marginal rate of substitution: $u = (x^2 + y^2)^{1/2}$.

11. You are asked to analyze the impacts of SARS or avian flu on the economy. Please outline a general equilibrium framework to do the job.

12. Consider preferences defined over the nonnegative orthant by $(x_1,x_2) > (y_1,y_2)$, if and only if $x_1 + x_2 < y_1 + y_2$. Do these preferences exhibit local nonsatiation? If these are the only two consumption goods and the consumer faces positive prices, will the consumer spend all of his income? Explain.

13. State and prove the Slutsky Equation

SECTION C (Answer 2 of 3. Longer answers.)

14. A consumer consumes two goods and his utility function has the "additively separable" form $u(x_1,x_2) = f(x_1) + g(x_2)$, with $f' > 0$ and $g' > 0$. By analyzing the first- and second- order conditions of his utility maximization problem, find a *sufficient and necessary condition* under which good 1 is an inferior good. Interpret your finding.

15. What is the Negishi theorem? Please state and prove it. In addition, please indicate its implication.

16. Consider the following constant elasticity of substitution function: $a\frac{x^d}{d} + b\frac{y^d}{d}$.

(a) Show that the function is homothetic. How does the marginal rate of substitution depend on the ratio y/x?
(b) Show that the marginal rate of substitution is strictly diminishing for all values of $d < 1$.
(c) Show that if x = y, the marginal rate of substitution for this function depends only on the relative sizes of a and b.

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SECTION A (Select 8 of 9. True or false, taking the time to explain why in detail)

1. If a good has many close substitutes, then the price elasticity of demand is always elastic.

2. Price elasticity of demand is necessarily non-positive (do not consider the absolute value definition).

3. If for all prices, the quantity demanded in market A is greater than the quantity demanded in market B, then the price elasticity of demand in market A can never be the same as in market B for all prices.

4. If the price elasticity of demand is elastic, the response to an increase in price will necessarily cause expenditure to fall.

5. Efficient provision of public good does not necessarily make all agents better off than their inefficient market equilibrium positions.

6. Magnitudes of impacts are always larger in general equilibrium than in partial equilibrium framework.

7. Smuggling (legal commodities, not illegal drugs) is detrimental to domestic economy.

8. Per capita public expenditure is likely higher in "blue" states (such as New York) than in "red" states (such as Texas).

9. If preferences are strictly convex, then the consumer's optimal bundle is unique.

SECTION B (Answer 3 of 4. Short answers.)

10. If a linear demand curve shifts parallel to the right, what happens to the price elasticity of demand at a given price?

11. Please conduct a brief welfare analysis of an oligopolistic market using externality arguments.

12. Suppose a consumer buys only two goods, food and shelter, and that they buy these in fixed proportions – one unit of food for each unit of shelter. Consider the Hicksian demand curve for food: what is the price elasticity of demand along this curve? What is the income elasticity of demand for food?

13. Real-valued function is called **superadditive** if $f(z^1 + z^2) \geq f(z^1) + f(z^2)$. Show that every cost function c(w, y) is superadditive is input prices w. Use this to prove that the cost function is nondecreasing in w without requiring it to be differentiable.

SECTION C (Answer 2 of 3. Longer answers.)

14. An investor's initial wealth is $W_0 > 0$. His utility function is u(x) where x denotes his net wealth and $u' > 0$, $u'' < 0$. There is a risky asset which has n possible rates of return, $r_1, \ldots, r_n$. The probability that r_i will be the realized rate of return is given by $p_i \geq 0$, and $\sum_{i=1}^{n} p_i = 1$. Let x* be the optimal amount of money that the investor invests in the risky asset.

(i) Find a necessary and sufficient condition under which x* = 0. Interpret.

(ii) Assume $E(r_i) > 0$. Show that $\frac{\partial x^*}{\partial W_0} > 0$ if and only if the investor exhibits decreasing absolute risk aversion.

15. Consider an exchange economy consisting of two consumers with utility function $u_i(x_{i1}, x_{i2})$. Show what would be the set of Pareto optimum allocations in the following cases, assuming positive total initial endowment of both goods, i.e. $(\omega_1, \omega_2) > 0$.

(i). $u_1(x_{11}, x_{12}) = x_{11} + 0.5x_{12}$ and $u_2(x_{21}, x_{22}) = 2x_{21} + x_{22}$.
(ii) $u_1(x_{11}, x_{12}) = \text{Min}\{x_{11}, x_{12}\}$ and $u_2(x_{21}, x_{22}) = \text{Min}\{0.5x_{21}, x_{22}\}$.
(iii) $u_1(x_{11}, x_{12}) = \text{Min}\{x_{11}, x_{12}\}$ and $u_2(x_{21}, x_{22}) = x_{21} + x_{22}$.
(iv) $u_1(x_{11}, x_{12}) = x_{11} + x_{12}$, if $x_{11} < \omega_1$ and $x_{12} < \omega_2$;
$u_1(x_{11}, x_{12}) = -1$ if $x_{11} \geq \omega_1$ and $x_{12} \geq \omega_2$, and
$u_2(x_{21}, x_{22}) = 1$ for every $(x_{11}, x_{12}) \geq 0$.

16. Consider a market with n goods. If the budget share of good i is denoted by s_i, and we denote the income elasticity of demand as $e_{i,I}$ and the cross-price elasticity of demand as $e_{i,j}$, use this notation to show:

(a) Homogenity: $\sum_{j=1}^{n} e_{i,j} + e_{i,I} = 0$

(b) Engel aggregation: $\sum_{i=1}^{n} s_i \cdot e_{i,I} = 1$

(c) Cournot aggregation: $\sum_{i=1}^{n} s_i \cdot e_{i,j} = -s_j$