Spring 2006

MICROECONOMIC THEORY COMPREHENSIVE EXAMINATION

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SECTION A (Select 8 of 9. True or false, taking the time to explain why in detail)

1. When income elasticities are all constant and equal, they must all be equal to 1.

2. A cartel serves the industry just like a social planner for an economy. Therefore, the outcome from a cartel must be efficient.

3. The six axioms about preferences over gambles imply that the indifference curves in the set of gambles are straight lines.

4. Because the profit function $\pi(p, w)$ (where $p$ is the output price and $w$ is the vector of input prices) is convex, a firm prefers uncertain prices to certain prices.

5. In a two good model if one good is an inferior good the other good must be a luxury good.

6. From general equilibrium viewpoints, providing citizenship to millions of illegal immigrants who hold low pay jobs now will further decrease wage levels.

7. Two distinct price vectors on a simplex may represent the same relative prices because of the Walras Law.

8. Validation of the Coase Theorem makes the government’s intervention in externality provision unnecessary.

9. The von Neumann-Morgenstern utility function is invariant with respect to monotone transformations.

SECTION B (Answer 3 of 4. Short answers.)

10. If a consumer has a utility function $u(x_1, x_2) = x_1 x_2^2$, what fraction of her income will she spend on good 2?


12. Prove that a monopolist charges a higher price in a less elastic market.

13. Prove: when the utility function $u(x)$ is homogeneous of degree $\alpha > 0$, the expenditure function $e(p, u) = u^{1/\alpha} e(p, 1)$. 
SECTION C (Answer 2 of 3. Longer answers.)

14. Suppose the production function \( y = f(x) \) exhibits constant returns-to-scale. Show that long-run average and long-run marginal cost are constant and equal. Show that for every level of the fixed input, short-run average cost and long-run average cost are equal at the minimum level of short-run average cost. Illustrate your results in the cost-output plane.

15. Suppose that people derive utility from two goods – housing (H) and all other consumption goods (C).

(a) Show a typical consumer’s allocation of his or her income between H and C.
(b) Suppose that the government decides that the level of housing show in part a (say, \( H^* \)) is substandard and requires that all people buy \( H^{**} > H^* \) instead. Show that this law would reduce this person’s utility.
(c) One way to return this person to the initial level of utility would be to give him or her extra income. On your graph, show how much extra income this would require.
(d) Another way to return this person to his or her initial level of utility would be to provide a housing subsidy that reduces the price of housing. On your graph, show this solution as well.

16. Consider a two-agent two-good economy with money. Each agent has a utility function

\[
U^h(x_1, x_2) = x_1^2 + x_2^2, \ h = 1, 2.
\]

The economy has endowments \( \omega = (\omega_1, \omega_2) = (4, 2) \). Each agent has monetary income (money is the numeraire for the economy) \( L^h = 3 \).

Please show that the competitive equilibrium does not exist for this economy.
Henderson/M. Jones/Yang

Write legibly in blue or black ink, not pencil. Begin each question on a new page.

SECTION A (Select 8 of 9. True or false, taking the time to explain why in detail)

True or False:

1. Equivalent variation correctly compares two mutually exclusive alternatives.

2. If preferences are locally non-satiated, then the consumer’s optimal consumption bundle must exhaust her income, i.e., \( p \cdot x^* = y \).

3. A consumer maximizes his/her utility when the marginal rate of substitution between every pair of commodities is equal to the ratio of prices of those two commodities.

4. Seeking maximum goods for the greatest number of people is a universal welfare principle.

5. If \( \lambda(w, y) \) is the lagrangian multiplier associated with the competitive firm’s cost-minimization problem, then \( mc(w, y) = \lambda(w, y) \).

6. A discriminating monopoly is never more profitable than a simple monopoly.

7. The cartel in oil industry is “environmental-friendly” because its high price setting.

8. A Pareto Optimal allocation is always Pareto Superior to any non-Pareto optimal one.

9. All agents in an economy would suffer from “pirating” copy-right materials.

SECTION B (Answer 3 of 4. Short answers.)

10. In light of the recent economic slump, suppose the U.S. federal government comes to the rescue of the airline industry by subsidizing the purchase of new capital. Thus, the price of capital paid by airline companies falls from \( r \) to \( r' \) (where \( r > r' \)). Describe and show graphically the impact of this policy on the airline industry's demand for capital and labor in the short-run and long run.
11. Suppose two consumers have the utility functions $U^1 = x_1^{1/3} y_1^{2/3}$, $U^2 = x_2^{2/3} y_2^{1/3}$. $x = x_1 + x_2$ and $y = y_1 + y_2$ represent the total amount of goods available. Please find the equation representing the contract curve for these two consumers.

12. What is the Slutsky Equation? Identify the various components and say something about their significance with respect to the possibility of Giffen goods.

13. Prove that the Walrasian equilibrium is in core.

SECTION C (Answer 2 of 3. Longer answers.)

14. A firm has the cost function
   \[ c(y) = \begin{cases} 
   y^2 + 1 & \text{if } y > 0 \\
   0 & \text{if } y = 0 
   \end{cases} \]
   Let $p$ be the price of output, and let factor prices be fixed. If $p = 2$, how much output will the firm produce? What about if $p = 1$?

15. Suppose a firm produces only one output and that the input prices are all fixed. Hence the profit function only depends on the price of output, $p$. Denote the long-run profit function by $\pi_L(p)$. Denote the short-run profit function by $\pi_S(p, z)$, where $z$ is some factor that is fixed in the short run. Let $z(p)$ be the long-run profit-maximizing demand for this input. Finally, let $p^*$ be some given output price, and let $z^* = z(p^*)$. Show that $\frac{dy_s(p^*)}{dp} \geq \frac{dy_s(p^*, z^*)}{dp}$, where $y_s(p^*)$ is the firm’s long-run optimal output given $p^*$, and $y_s(p^*, z^*)$ its short-run optimal output given $p^*$ and $z^*$. Interpret the implications of this result.

16. There are two countries, two commodities, and two households in each country. The following are the endowment situation:

<table>
<thead>
<tr>
<th></th>
<th>Country A</th>
<th>Country B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumer 1</td>
<td>(0, 12)</td>
<td>(0, 12)</td>
</tr>
<tr>
<td>Consumer 2</td>
<td>(12, 0)</td>
<td>(12, 0)</td>
</tr>
</tbody>
</table>

   Each consumer in A has the utility function $U = x_1 y_1$ and each consumer in B has $U = x_1 (x_2 + 8)$. Assume that $p = 1$ in each country. (i) Find the Walrasian equilibrium for each country, assuming they are not allowed trade across country; (ii) Find a world allocation that leaves every individual better off than under the no-trade equilibrium of (i).