4) As of about 1800, India and sub-saharan Africa had similar levels of real output per person, but population density (number of people per acre) was much higher in India. Does this tell you anything about relative technological development in India *versus* sub-saharan Africa? Explain.

5 pts. This is about the Malthusian model. Consider a portion of Africa of a certain size - a certain number of square miles - and a portion of India of the same size. Higher population density in India means that the Indian land section has a bigger populaiton. If you assume India and Africa had about the same "subsistence" real wage levels, then the higher population in the Indian section indicates a higher level of technological development. In terms of the graph I used, India has an MPL curve that is farther to the right, so its equilibrium population is bigger.

5) Using equations, prove the "dynamic inconsistency of optimal monetary policy" proposition. That is, show that a central bank that aims to keep inflation at an optimal level, *and* keep output above the natural rate, will end up achieving neither. For simplicity, assume the optimal level of inflation is zero and the central bank directly sets the level of *inflation*.

6 pts. Write down an expectations-augmented Phillips curve and a loss function. Take expected inflation as given and choose optimal inflation rate. Then impose rational expectations equilibrium, that is $\pi = \pi^e$. See that resulting inflation rate is greater than zero.

$\pi = \pi^e + \beta(y - \bar{y})$ or $y = \bar{y} + \alpha(\pi - \pi^e)$

$L = \frac{1}{2}(y - y^*)^2 + \frac{1}{2}\pi^2$ $\quad y^* > \bar{y}$ ← key

$= \frac{1}{2}(\bar{y} + \alpha(\pi - \pi^e) - y^*)^2 + \frac{1}{2}\pi^2$

$\frac{\partial L}{\partial \pi} = 0 \rightarrow \pi = \frac{\alpha^2}{1+\alpha^2}\pi^e + \frac{\alpha}{1+\alpha^2}(y^* - \bar{y})$

In REE, $\pi = \pi^e$

$\rightarrow \pi = \alpha(y^* - \bar{y})$ equivalent to $\pi = \frac{1}{\beta}(y - \bar{y})$

6) Suppose China and the U.S. are the only two countries in the world. Between China and the U.S., there is trade in goods and services and financial assets. The exchange rate is floating. In each country, the level of net exports depends only on the exchange rate. Each country has a central bank that sets the country's real interest rate attempting to keep output at the natural rate. For simplicity, assume "static exchange rate expectations," and that in both countries expected inflation is always equal to zero. In your answers and graphs, make sure you make clear whether a variable is Chinese or American (e.g., whether "the interest rate" is the Chinese interest rate or the American interest rate).

This is about the old-fashioned Keynesian open-economy models. There is no LM curve in either country, because the central banks do not fix money supplies.

a) For many years, China's government authorities have been buying up foreign assets. What does this tell you about the nature of international capital mobility? That is, based on the fact that China's authorities have been buying up foreign assets, would you guess that international capital mobility is *perfect* or *imperfect*? Explain. *2 pts. This is about "reserve gain" RG. Recall that if intl. capital mobility is perfect, RG has no effect on the exchange rate - it just substitutes official holdings of foreign assets for private holdings of foreign assets. So if the authorities have been engaging in RG, intl. capital mobility must be imperfect. If intl. capital mobility is imperfect, RG tends to raise (depreciate) the exchange rate.*

b) Suppose the United States is in a "liquidity trap." That is, the central bank has cut the interest rate as low as it possibly can, but output is still less than the natural rate of output. Suddenly, China's authorities stop buying foreign assets. Does this have any effect on output in the U.S., assuming your answer to a) is correct? Explain. *4 pts. The liquidity trap in the US means that the US central bank cannot lower the US interest rate to boost output up to the natural rate. The US central bank would not want to raise the US interest rate. Positive RG for China has tended to depreciate China's currency, hence appreciate the US dollar, reduce US net exports and reduce US output. If China stops buying foreign assets, this effect ceases: the dollar depreciates, US net exports increase, US output increases. (A potential complication: if China lowers its interest rate at the same time it ceases to buy foreign assets, the resulting increase in CF for the US may prevent dollar depreciation.)*

7) Unless the price level can "jump," output cannot always remain at the natural rate. Demonstrate with an example. *6 pts. See notes.You cannot demonstrate this point with an IS/LM graph. IS/LM can show what the price level must do to maintain natural-rate output in response to a jump in the money supply M. That is not what I am asking about.*

8) Suppose an economy can be described by three equations, where *y* is the "output gap":

- A Friedman-Phelps Phillips curve: $\pi_t = {}_{t-1}\pi_t^e + y_t$ *note the coefficient on y here is 1*

- An AD curve: $y_t = a(m_t - p_t)$

- Money supply: $m_t = \bar{m} + \epsilon_t$ where ϵ is a mean-zero i.i.d. random variable ${}_{t-1}\epsilon_t^e = 0$

Assume the economy is in "rational expectations equilibrium."

a) Derive expressions for y_t and π_t.
4 pts. To do this, recall that $\pi_t = p_t - p_{t-1}$ *and* ${}_{t-1}\pi_t^e = {}_{t-1}p_t^e - p_{t-1}$ *so* $p_t = {}_{t-1}p_t^e + y_t$.
Write an expression for p_t *taking as given* ${}_{t-1}p_t^e$ and ϵ_t.
Then apply rational expectations equilibrium: solve for p^e *assuming* $p^e = E[p]$ *using the fact that* $E_{t-1}[\epsilon_t] = 0$.

Then take that value for p^e, *put it in with realized* ϵ *and get realized* p_t.

To get π_t, *use* $\pi_t = p_t - p_{t-1}$.

To get y, use $y_t = p_t - {}_{t-1}p_t^e$.

$p_t - p_{t-1} = p_t^e - p_{t-1}^e + y \rightarrow p_t = p_t^e + y_t$

$p_t = p_t^e + a(\bar{m} + \epsilon_t - p_t) \rightarrow p_t = \frac{1}{1+a} p_t^e + \frac{a}{1+a}(\bar{m} + \epsilon_t)$

$p_t^e = E[p_t] = \frac{1}{1+a} p_t^e + \frac{a}{1+a} \bar{m} \rightarrow p_t^e = \bar{m}$

$p_t = \frac{1}{1+a} \bar{m} + \frac{a}{1+a}(\bar{m} + \epsilon) = \bar{m} + \frac{a}{1+a} \epsilon_t$

$y_t = p_t - p_t^e = \frac{a}{1+a} \epsilon_t$

$\pi = p_t - p_{t-1} = \frac{a}{1+a}(\epsilon_t - \epsilon_{t-1})$

b) Consider the correlation between output and inflation in this economy. Is the correlation positive, negative or zero? *2 pts. Positive.*

9) One of the models we learned explains both of the following facts.
- According to many studies, the marginal product of capital in small business is always higher than the real interest rate on government bonds.
- People who take out loans to start small businesses are, on average, wealthier than otherwise-similar people who do not start small businesses.

Explain.

6 pts. The model of financial market imperfection due to asymmetric information and costly state verification in Romer can account for these facts if one assumes that model describes most "small business." The model implies that the required expected return to a business project is higher than the safe interest rate, because the expected return must be enough to cover the expected verification cost . It also implies that whether or not a loan is made depends on a potential entrepreneuer's wealth, as well as the expected return to the project and the government-bond interest rate. A wealthier person might be able to get a loan and start a business, while a less-wealthy person with a project of the same quality (same expected return) can't.

Note this is not due to the uncertainty in the rate of return on a potential project. It's due to asymmetric information and costly state verification.