

Prof. Kuhn, Question 1

Consider a farmer whose cow has recently passed away. After a number of periods on the farm, the farmer decides to switch jobs. He therefore moves to the nearest city and becomes a worker. Upon arrival in the city the worker starts out unemployed. In any period in which he is unemployed, the worker receives unemployment benefits of b .

In the city there are many potential employers. As long as the worker is unemployed, he applies to a new job and receives one new job offer every period. The only way in which job offers differ from each other is in the wage w paid by the employer; the distribution of wages is given by the differentiable cdf $F(w)$ with support $w \in [w_{min}, w_{max}]$. The worker then decides whether to accept or reject the job offer.

- If the worker rejects the offer, then next period he will still be unemployed and apply to a different potential employer, getting a new wage offer from the distribution F .
- If the worker accepts the offer, then next period he will start working for the employer and from then on receive the wage w . The worker will then work for the same employer permanently (for simplicity: employment lasts an infinite number of periods).

The worker has a discount factor of $0 < \beta < 1$ and does not save or borrow. The utility function $u(c)$ indicates how much utility the household derives from a given level of consumption c . Denote the worker's value function when he is unemployed with U and the value function when he has a job with V .

1. What are the state variable(s) for each of the two value functions? What is the choice the worker makes in each case?
2. Write down the value functions when the worker is employed and unemployed, respectively.
3. Find the utility value of employment by writing the value function for employment V in terms of its state variables and the parameters of the problem. (In other words, find a non-recursive way to express it.)
4. Using the result from 3. and the unemployment value U , show that there is a cutoff value \bar{w} for the wage offer such that the worker will accept the job whenever he receives an offer greater than the reservation wage $w \geq \bar{w}$.
5. Characterize \bar{w} as much as you can. Hint: You will have to write out the expected value of unemployment.
6. What is the effect of an increase in b on the reservation wage \bar{w} ? Explain the economic intuition and use your result from 5. to show this formally.

Question 2

Consider the standard neoclassical growth model without labor. That is, there is a unit measure of identical households who derive utility from consumption according to the lifetime utility function

$$E \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right]$$

where the discount factor $\beta \in (0, 1)$. Households own capital k_t and supply 1 unit of labor inelastically. In every period t , they rent out these factors of production to firms and receive real factor payments r_t and w_t for every unit of capital and labor provided, respectively. Capital depreciates at rate $\delta \in (0, 1]$. In addition, households can trade a one-period risk-free bond b_{t+1} among each other (there are no initial bond holdings, ie $b_0 = 0$). Such a bond can be bought (or sold) at price p_t at time t , and pays off one unit of the good at time $t + 1$ for sure.

A representative, profit-maximizing firm uses the input factors capital K_t and labor L_t to produce a single output good Y_t according to $Y_t = A_t F(K_t, L_t)$ where F is a constant returns to scale production function and A_t is aggregate productivity following a stochastic process over time.

1. Define a competitive equilibrium in this economy (be sure to write down all components explicitly).
2. Characterize the equilibrium by deriving the first-order optimality conditions for firms and households.
3. Find the Euler equation for the economy's capital stock. To do this combine the relevant equilibrium conditions such that capital is the only endogenous variable in the equation. (Hint: you may find it useful to use the fact that the firm is perfectly competitive and operates with constant returns to scale).

Now use the following parameter values: Capital depreciates fully with $\delta = 1$, households have log utility $u(c) = \log c$, and the production function is $F(K, L) = K^\alpha L^{1-\alpha}$. The random variable A_t follows an i.i.d. two-point process: In every period, with probability π^h productivity is $A_t = A^h$, and with probability $1 - \pi^h$ it is $A_t = A^l$, with $A^h > A^l$.

4. Show that the policy function for capital is given as $g(k, A) = \alpha \beta A k^\alpha$.
5. What is the price of the risk-free bond? Find a closed-form expression. Use it to find out how the bond price depends on: current productivity A_t ? The discount factor β ? The current capital stock k_t ? The probability of a good shock π^h ?
6. Is the risk-premium on equity in this economy positive, negative or equal to zero? In other words, is the expected return on capital higher than, lower than, or equal to the expected return on bonds? Explain intuitively, bonus points if you can show it formally.