Prof. Kuhn, Question 1 – Endogenous separations

A worker, who previously had been a farmer, quits his permanent position in order to move to a new city. He arrives in the new city and starts out unemployed and looking for work. During unemployment, workers receive an unemployment benefit of b. Because the job market is going well, unemployed workers find a job next period for sure. However, it is not clear if the worker is productive in a given job until he has started to work. There are two possibilities: the match between the job and the worker is unproductive in which case the worker gets paid a wage of w_L, or the match between job and worker is productive in which case the worker receives a higher wage of $w_H > w_L$. At the end of every period the worker has the opportunity to quit his current job. If he quits, he will be unemployed in the coming period.

Denote the probability that a worker who just started a job (coming from unemployment) is productive in that job with π_{uh}, and the probability that the worker is unproductive with $\pi_{ul} = 1 - \pi_{uh}$. Once a worker finds himself in a high productivity job, the chance that the job will still be high productivity next period is π_{hh}, and so there is a chance of $\pi_{hl} = 1 - \pi_{hh}$ that the match becomes low productivity. Similarly, π_{ll} is the probability that a low-productivity match stays low productivity (and $\pi_{lh} = 1 - \pi_{ll}$ the probability that it becomes a high-productivity match next period). Note that the worker does not know what next period's productivity is when making the decision whether to quit his job (just as he does not know which type the job will be when accepting the job out of unemployment).

The worker has linear utility, does not save, and discounts future periods with a factor β.

1. State the value functions for the worker, making it clear what the different states are and which choices the worker has in each state. (Hint: I suggest you use three functions and name them U, V^H, and V^L).

Now assume that the parameters of the problem are such that the worker will always want to quit a low-paying job, but will always want to stay in a high-paying job.

2. State condition(s) that need to be satisfied such that this is actually the worker's optimal strategy (the condition(s) don't have to be in terms of parameters only).

3. Use the worker's optimal strategy to find the worker's value function when employed in the high-productivity job explicitly, ie write it as a function of parameters only. Note: This involves a bit of algebra.

4. Write out the transition matrix for the stochastic transitions between uneployment, high-productivity job and low productivity job. If there are a unit mass of identical workers, how can we use the transition matrix to find the number of workers currently unemployed, in a low-type job and in a high-type job? Find this stationary distribution or explain how one can find it.

5. Use the parameter values $\pi_{hh} = 1 = \pi_{ll}$. Re-state the condition(s) from 2. which imply that the worker quits low-productivity jobs and remains in high-productivity jobs, this time as a function of parameters only.

Prof. Kuhn, Question 2 – A few basics

Two biologists, Jess and George, are far away on an expedition of infinite duration. Both biologists receive a stream of endowments $\{e_{it}\}_{t=0}^{\infty}$ every period, where $i \in \{J, G\}$. The endowment streams are such that an aggregate endowment of E is given alternatingly to one of the two; that is, one of the two biologists receives E in all even periods, and the other receives E in all odd periods. They trade goods for all periods once at the beginning of time 0 (both are price-takers and denote with p_t the price of a good in period t in units of the time-0 good). Jess and George have differentiable, strictly increasing and strictly concave period-utility functions $u^i(c_{it})$ and their respective lifetime utility is given by

$$\sum_{t=0}^{\infty} \beta^t u^i(c_{it})$$

(note that the utility functions u^J and u^G are not necessarily identical).

1. Carefully define a competitive equilibrium in this economy and characterize it.
2. Show that, even though their individual endowments fluctuate, in equilibrium, both biologists' consumption profile is constant over time. Who consumes more, and why? Solve for both biologists' consumption.

Now assume that George's utility depends on Jess' consumption level. In particular, George derives utility from his consumption relative to Jess, i.e. his utility function is now $u^G(c_{Gt}, c_{Jt})$ where $\partial u^G(c_{Gt}, c_{Jt}) / \partial c_{Jt} < 0$ and u^G is still increasing and concave in c_{Gt}.

3. Re-state any equations that have changed in the definition or in the characterization of the competitive equilibrium.
4. Whose consumption increases and whose consumption decreases compared to the previous case? Are consumption profiles still constant over time?

Now consider the problem of a Social Planner who puts a social welfare weight of λ_J on Jess' welfare and a weight of $1 - \lambda_J$ on George's welfare.

5. State the Planner's problem and derive the first-order necessary condition(s).
6. Show that under the Planner's solution the consumption profiles are constant.
7. Use the utility functions $u^J(c_{Jt}) = \log c_{Jt}$ and $u^G(c_{Gt}) = \log c_{Gt} - \phi \log c_{Jt}$. Under which utility weight λ_J does this lead to the same consumption profile as in part 2? Does it depend on ϕ?