Micro Comp First Round May 31, 2016

Part A: Short Questions

1. (Jones) In the utility function $u(x_1, x_2) = x_1 + \ln x_2$, are both goods normal?

2. (Jones) The elasticity of scale at point x is given by $\mu(x) = \frac{\frac{df(tx)}{f(tx)}}{\frac{dt}{t}}$ evaluated at $t = 1$. What is the elasticity of scale of the CES production function, $f(x_1, x_2) = \left(x_1^\rho + x_2^\rho\right)^{1/\rho}$?

3. (Pape) Consider this characterization how Nash proved the existence of Nash equilibria: *Nash proved the existence of Nash equilibria by describing a Nash equilibrium as fixed point in the sense of a fixed-point theorem, and applying that theorem.*
   Is this a correct characterization? If you think so, explain why; i.e. how are NE a kind of fixed point? If not, explain why not.

4. (Pape) Xavier and Yvette participate in the market for apples, bananas, and carrots. The market is in Walrasian equilibrium. You want a banana, and you offer Xavier two apples in exchange for a banana. He declines. You turn to Yvette, and you're about to offer *her* two apples for a banana. Your friend says, "Don't bother, you're wasting your time." Is your friend right? Explain.

Part B: Medium-Length Questions

5. (Jones) Consider a profit-maximizing firm that produces a good which is sold in a competitive market. It is observed that when the price of the output good rises, the firm hires more skilled workers but fewer unskilled workers. Now the unskilled workers unionize and succeed in getting their wage increased. Assume that all other prices remain constant.
   a. What will happen to the firm's demand for unskilled workers?
   b. What will happen to the firm's supply of output?

6. (Jones) There are two risky assets with independently and identically distributed returns. For each asset, there is probability p_i that the rate of return is r_i, $i = 1, 2, \dots, n$ ($\sum_{i=1}^{n} p_i = 1$). Show that an expected utility maximizer will diversify her investment (i.e., invest a positive amount of wealth in each asset) provided she is risk averse; and invest all her wealth in one of the assets if she's risk loving.

7. (Pape) Consider a standard 2x2 production model describing a small open economy: There are two CRS firms, called X and Y, which produce goods x and y at world prices, and there are a fixed total amounts of labor and capital in the economy that are allocated between the two firms in Walrasian equilibrium. Assume X is more labor-intensive than Y. Claim: because X is more labor-intensive than Y, these two events would cause the same direction of change for all endogenous variables: (a) an increase in the price of x and (b) an increase in the labor-intensity of X. Is this claim true or false? Explain.

8. (Pape) Consider this modified version of the Rubenstein bargaining game: There are two players, A and B, who are bargaining over a cake that begins at size 1 at time zero. The cake shrinks over time; after one period, only a fraction δ, $0 < \delta < 1$, remains (i.e., the usual meaning).

   At time t, Player *i* makes an offer of how to split the cake. Player *j* then either (A)ccepts the offer, and they divide the cake according to *i*'s offer and the game ends, (R)ejects the offer, in which case we go to time t+1 and Player *i* and *j* switch roles (as usual), or (E)nds the bargaining process. If Player *j* chooses to End the process, then immediately one third of the remaining cake is given to player A, one third is given to player B, one third is destroyed, and the game ends.

   Find all SPNE of this game.

Part C: Long Questions

9. (Jones) In a first-price, all-pay auction, the bidders simultaneously submit sealed bids. The highest bid wins the object and every bidder pays the seller the amount of his bid. Consider the independent private values model with symmetric bidders whose values are each distributed on [0, 1] according to the distribution function F, with density f.
   a. Find the unique symmetric equilibrium bidding function. Interpret.
   b. Do bidders bid higher or lower than in a first-price auction?
   c. Find an expression for the seller's expected revenue.
   d. For the case of uniformly distributed values and $n = 2$, show that the seller's expected revenue is the same as in the first-price auction.

10. (Jones) The owner of a firm hires an agent to manage the firm. The agent's utility function is given by $u(w, e) = \sqrt{w} - e$, where w denotes his wage and e his effort. He can choose one of 2 levels of effort: $e = 0$ or $e = 1$. The firm's profit, π, is either 20 or 70, and the probability of each profit being realized depends on the agent's effort, as follows:

	$\pi = 20$	$\pi = 70$
$e = 0$	0.6	0.4
$e = 1$	0.4	0.6

Let a wage contract be represented by (w_1, w_2), where w_1 denotes wage paid to the agent if profit is 20, and w_2 wage paid if profit is 70. Note that the agent will only accept the contract if he can at least reach his reservation utility, which equals 4.

Find the wage contract that maximizes the owner's expected net profit in each of the following situations:

   a. The owner can observe the agent's effort and so the contract can be conditioned on the effort level of the agent. How much effort does the agent exert under the optimal contract?
   b. The owner cannot observe the agent's effort and so the contract cannot be conditioned on effort. How much effort does the agent exert under the optimal contract?
   c. What interesting insight can we get from this example?

11. (Pape) Consider a game with two players, a firm **F**, which is deciding whether or not to develop a new technology; and the government **G**, which is deciding whether to provide patent protection for the technology. The firm's actions are either to *I*nvest or *N*ot invest, and the government's action is to provide a level of patent protection *p* between zero and one (inclusive), where 0 means no protection and 1 means full protection.

    For the firm, the cost of investment is *c*, which is strictly between zero and one. The social value of a **developed** technology is *1*, and this value is divided between the firm and the government according to *p*; the firm gets profits of *p* and consumers get the value *(1-p).* (The social value of undeveloped technology is 0.) The firm, of course, is seeking to maximize its own profit and assume the government is seeking to maximize consumer welfare.

    The legislature is considering three proposals: (a) that the government chooses *p* for a particular firm before the development decision by the firm; (b) that the government chooses *p* for a particular firm after the development decision; or (c) that the government chooses *p* at the same time that the development decision is made. Which policy, if any, maximizes consumer welfare? Explain!

12. (Pape) Consider the following economy: There are three consumption goods, *x*, *y*, and *z*. *x* is already in existence, and can simply be traded; *y* must be produced from the two factors of production in the economy, labor *l* and capital *k;* and *z* is already in existence but cannot be traded (so it has no price and does not "participate" in the equilibrium). There are two kinds of consumers: there are 50 capitalists, indexed *i=1,…,* 50, who are each endowed with one unit of capital, one unit of *x*, one unit of *z*, and ownership over a single firm (see below); and 50 workers, indexed *i = 51, …, 100,* who are each endowed with one unit of labor, **two** units of *x*, zero units of *z,* and do not own a firm.

    y is produced by 50 firms, each one owned by exactly one capitalist and indexed *j=1,…,50*. Production is Leontief:

    $$y_j = f(l_j, k_j) = Min[\, l_j, k_j \,]$$

    All consumers have the same utility function: $u(x,y,z) = xy + z$.
    In equilibrium, do workers or capitalists have higher utility, or does it depend? Answer carefully and explain.